Copula Models of DRAM Test and Use

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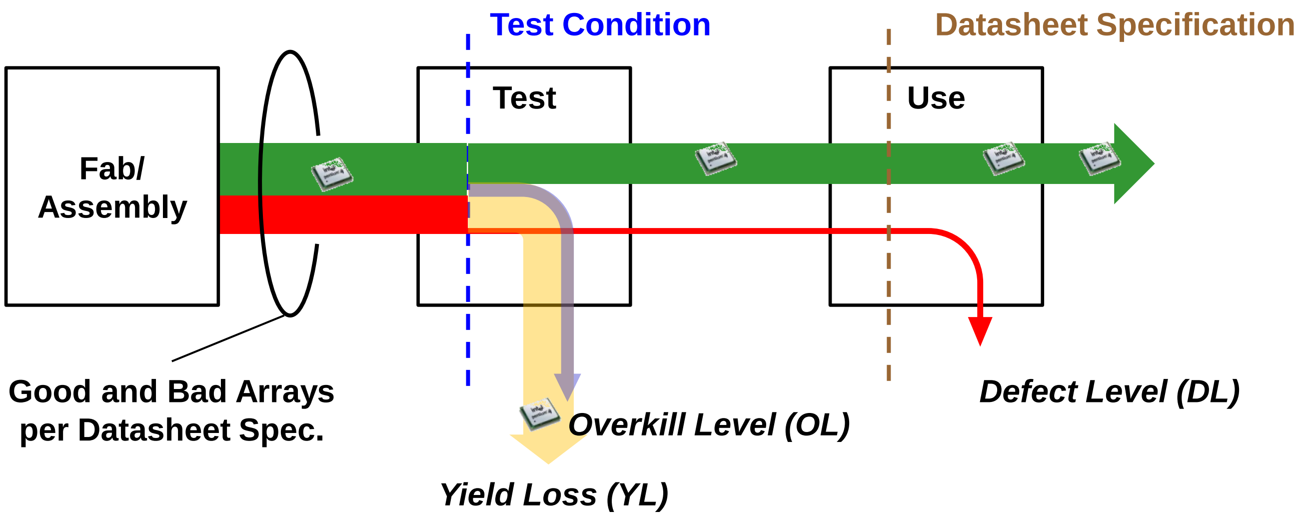
Outline

- Introduction
- DRAM Experiment in ICDT
- Fitting a Model
- Using the Model
- Final Thoughts

Motivation

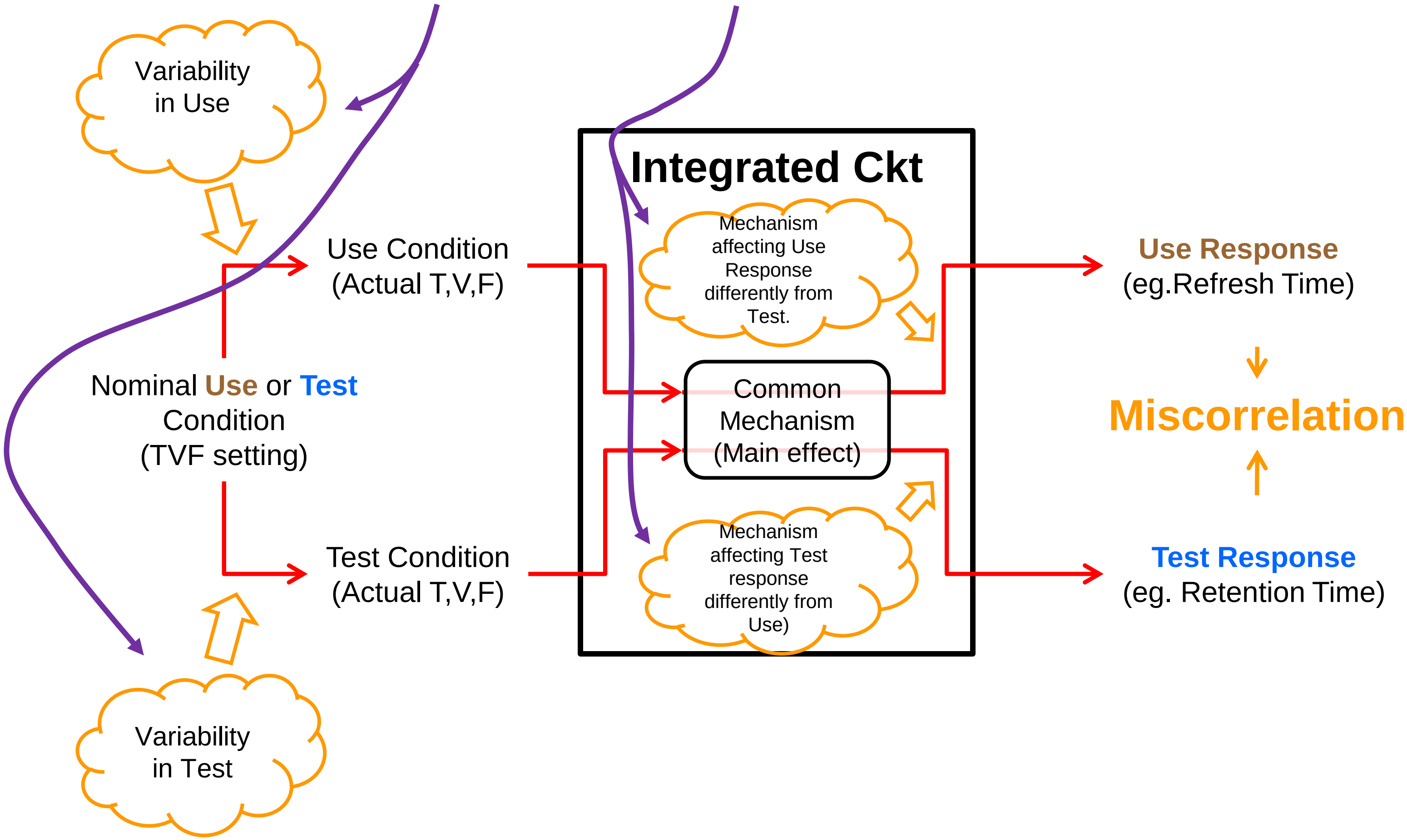
- An IC product is designed, manufactured and used.



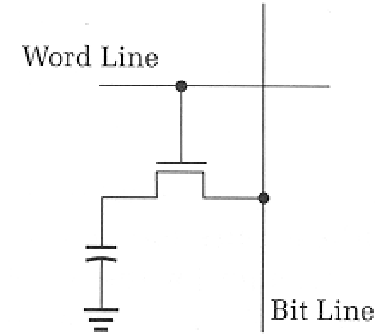
- Test is used to screen defective ICs.
- Miscorrelation between Test and Use allows bad units to reach the customer, and makes Test reject good units.
- Key measures are Yield Loss (*YL*), Overkill Level (*OL*) and Defect Level (*DL*) in end use.

Causes of Test/Use Miscorrelation

- Causes are external and internal to the IC.



Causes of Miscorrelation for a DRAM



- Internal
  - Bit data is charge stored on a capacitor, held by an “off” transistor.
  - Charge leaks away with a temperature/voltage dependent retention time, and must be periodically refreshed with a specified refresh time.
  - The transistor is vulnerable to a defect that intermittently makes it “leaky” in the “off” state, making a “VRT” bit.

“**V**ariable **R**etention **T**ime”

- External
  - Some VRT bits may pass retention Test while “normal” but fail in Use when the “leaky” state occurs.
  - A bit is Tested once, but Used many times so the probability of detecting a VRT bit in Test and Use is different.

Variable Retention Time

- A bistable atomic defect occurs everywhere in Si.

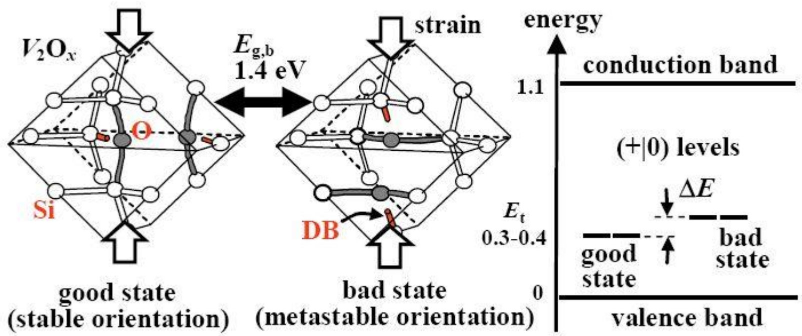
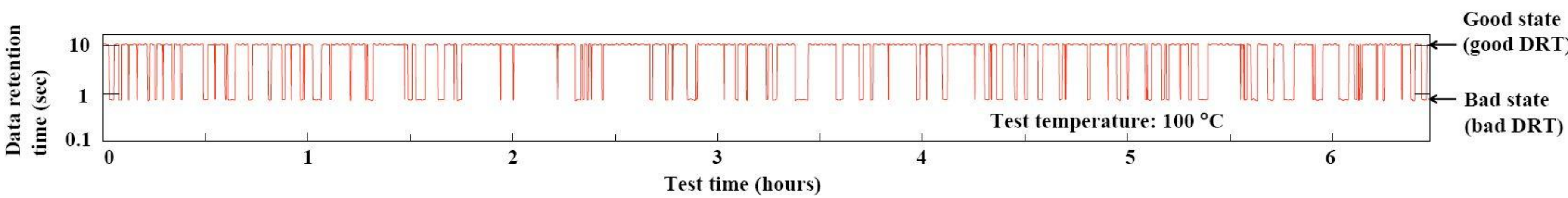


Fig. 5 V_2O_x defect model

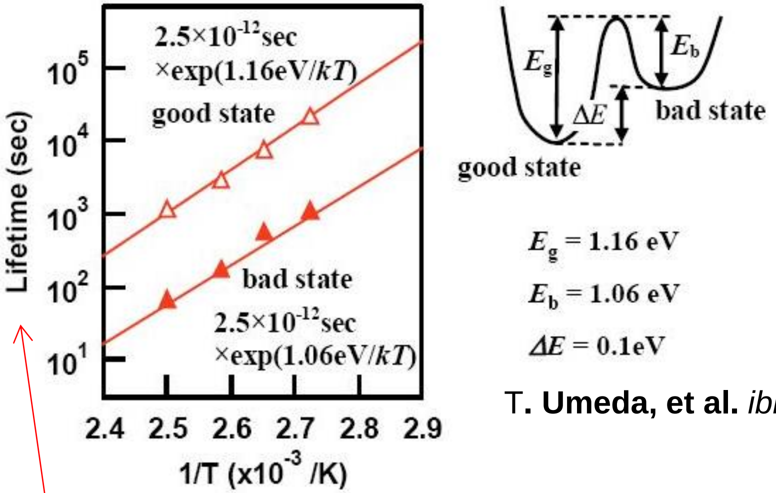
T. Umeda, et al. *"Single silicon vacancy-oxygen complex defect and variable retention time phenomenon in dynamic random access memories."* Applied Physics Letters Vol. 88 253504 (2006)

- When it falls at the near-surface gate/drain boundary of the DRAM capacitor pass xistor a bistable leakage current, and so bistable retention time, occurs.



Variable Retention Time, ct'd

- Retention times range from 100's of microseconds to seconds.
- Difference between min and max retention times varies from bit to bit.
  - In PSU experiment, 82% of measured bits were "stable" (min = max retention time).
- Bits are "stuck" in high or low retention time states for many minutes, or even hours!
- Time-in-state is thermally activated.



T. **Umeda, et al.** *ibid.*

Fig. 2 Thermal activation of the VRT.

(Time constants for exponential time-in-state distributions.)

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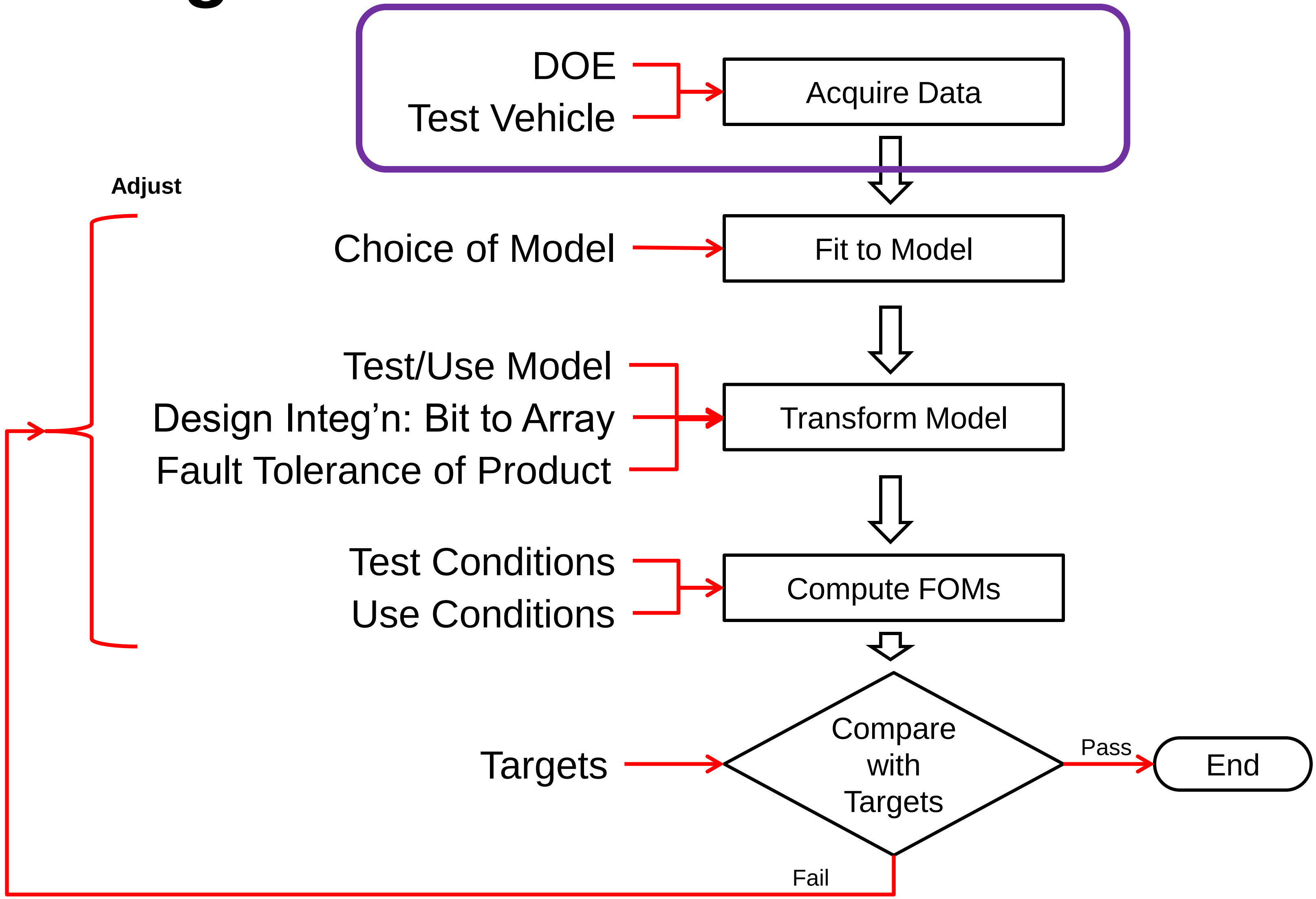
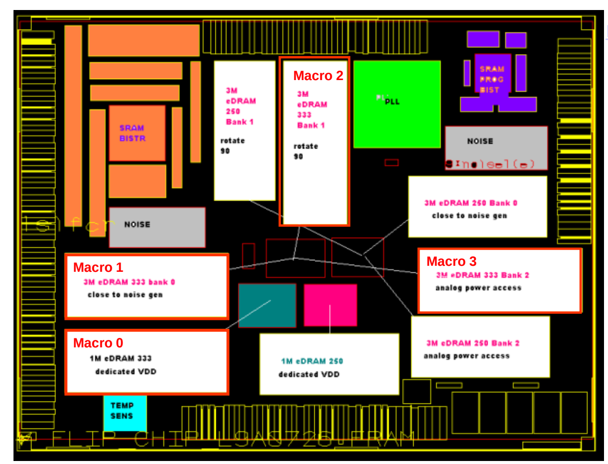
Modeling Miscorrelation



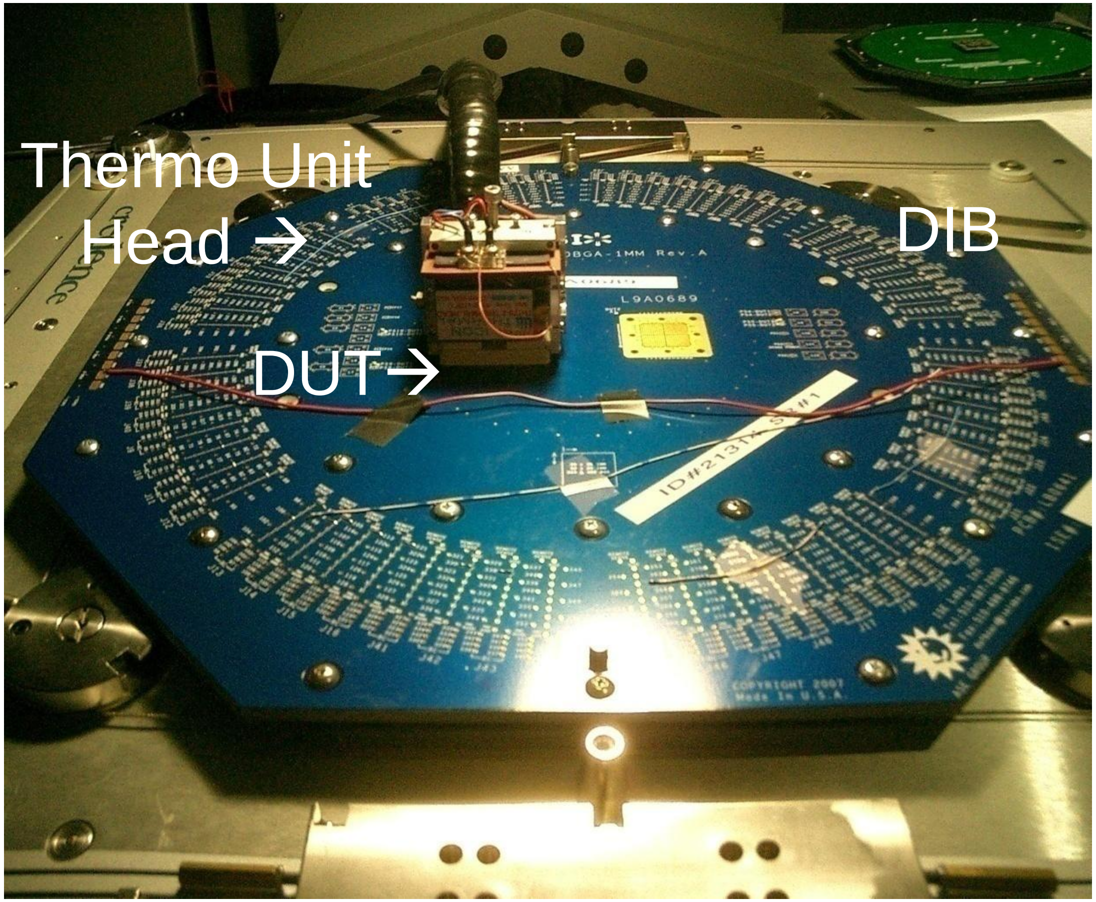
DRAM Experiment

- A 65 nm process DRAM test chip, packaged in BGA, was used.
- 10 random test chips, prescreened for gross faults, were selected.
- Each test chip unit has 4 identical arrays of 1,218,750 bits.
  - Number of bits tested = $10 \times 4 \times 1{,}218{,}750$ = 48.75 Mb
- Tested on Credence Quartet with 145 I/Os, and 7 power supplies using Silicon Thermal Powercool LB300-i for temperature control.
- Retention time for each bit was measured at..
  - 3 temperatures: 105ºC, 115ºC, 125ºC
  - 3 Vdd's: 0.8, 1.0, 1.2 volts
  - 2 Vp's: 0.4, 0.45 volts
- Repeated retention time measurements were made on each bit to characterize retention time variability.



Thanks to Satoshi Suzuki for acquiring the data!

Test Environment at PSU



- ICDT Lab at PSU.
- Credence Quartet IC tester.
- One chip per test.
- Temperature controller & sensor (thermocouple).

Temperature Controller

Repeated Bit Retention Measurement

- 12 retention times were measured 5 times for each bit.
- Retention times ranged from 60 au* to 604 au.
  (t_{ret} = 10 + i x 49.5, i =1 to 12)

** Retention times are given in arbitrary units*

X = 0 (pass), or 1 (fail). X = **X (60 au);** X = **X (604 au)**

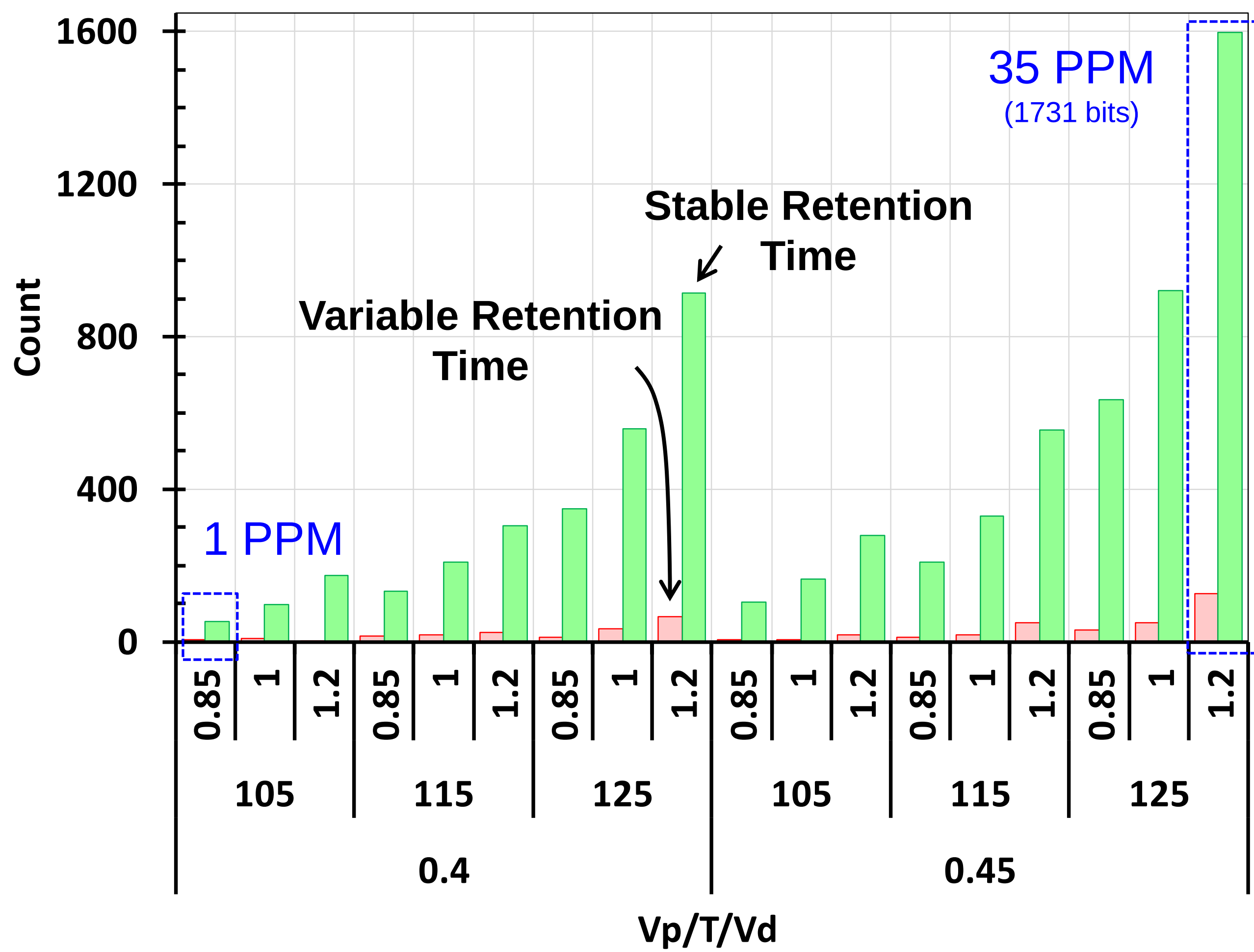
XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX
Repetition 1	Repetition 2	Repetition 3	Repetition 4	Repetition 5

- Repetitions separated by variable durations, many hours.
- Repetition ensures that any variability will be captured.
- Examples
  - 000000000111 000011111111 000000001111 000000001111 000000000111 } Variable
  - 000110111111 000001111111 000111101111 000001111111 000111111111 } Variable
  - 000000011111 000000011111 000000011111 000000011111 000000011111 } Stable
  - 000000001111 000000001111 000000011111 000000001111 000000011111 } Stable

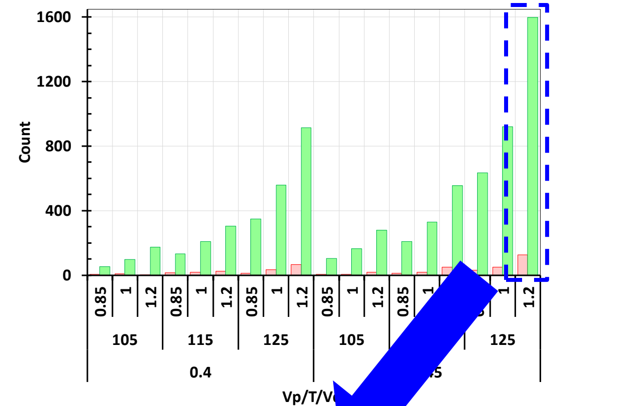
Variation ≤ 1 is regarded as “stable”.

Environmental Dependence

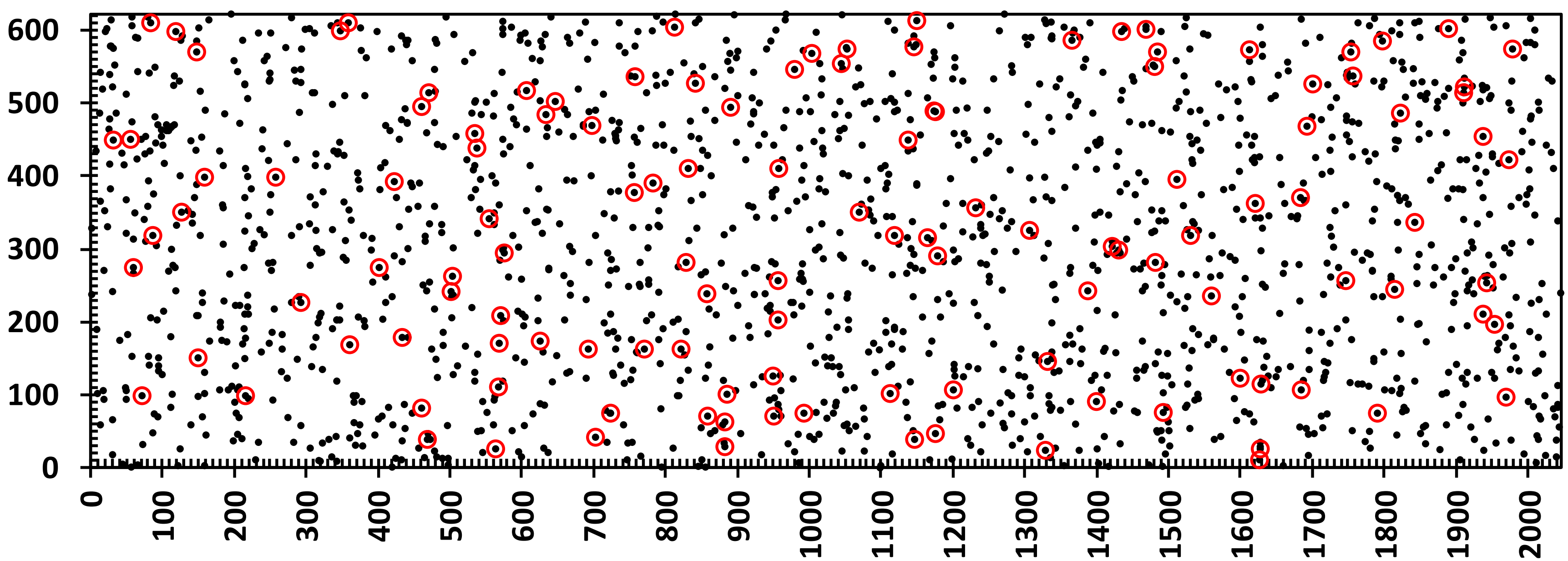
- The sample was from 48,750,000 bits (49Mb).
- The sample was bits with retention times ≤ 604 au.
- Failing bit count was 1 PPM to 35 PPM depending on environmental condition.
- 18% of bits with retention time ≤ 604 au were VRT (pink).



Random Spatial Distribution Seen

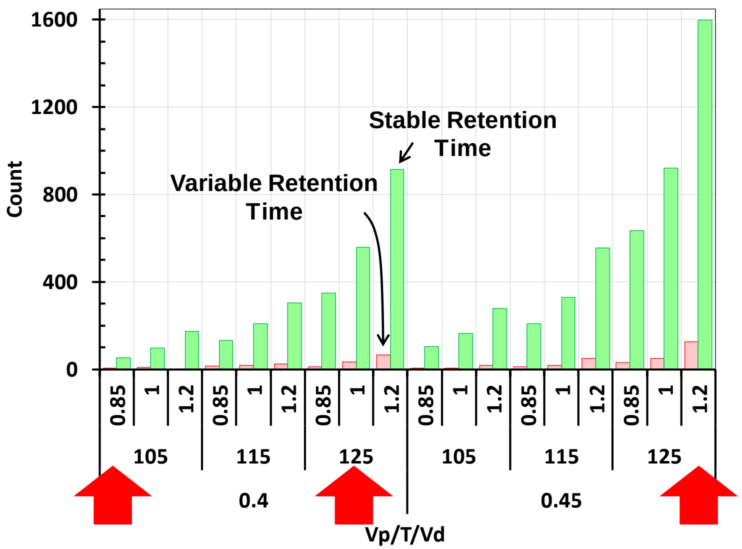


- Bits with retention times ≤ 604 au, sampled from 48,750,000 bits.
- At maximum environmental condition V_{dd} = 1.2, V_p = 0.45, T = 125 °C
- 1731 bit failures, 126 of these (red circles) had variable retention times.



Retention Time Distributions

- Tabulate data assuming $r_{Test}/r_{Use} = t_{min}/t_{max}$ or t_{max}/t_{min} with equal probability.
- Gives symmetrical model, equal margins, easy to fit.
- Will be transformed to realistic Test/Use model later.



Nominal Skew T/Vp/Vd: 105/.4/.85

r_Test (au) \ r_Use (au)	0	60	109	159	208	258	307	357	406	456	505	555	604
604	0	0	0	0	0	0	0	0	0	0	0	3	N/A
555	0	0	0	0	0	0	0	0	0	1	3	1	1
505	0	0	0	0	0	0	0	0	1	1	3	5	1
456	0	0	0	0	0	0	0	0	3	4	7	0	0
406	0	0	0	0	0	0	0	3	2	2	0	0	0
357	0	0	0	0	0	0	4	3	1	1	0	0	0
307	0	0	0	0	0	0	2	1	0	0	0	0	0
258	0	0	0	0	0	2	0	0	0	0	0	0	0
208	0	0	0	0	2	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Nominal Skew T/Vp/Vd: 125/.4/1.

r_Test (au) \ r_Use (au)	0	59.5	109	159	208	258	307	357	406	456	505	555	604
604	0	0	0	0	0	0	1	0	1	0	7	23	N/A
555	0	0	0	0	1	0	0	0	0	2	31	38	17
505	0	0	0	0	0	0	0	1	3	38	37	29	7
456	0	0	0	0	0	0	0	3	22	42	17	0	0
406	0	0	0	0	0	0	0	19	24	19	2	0	0
357	0	0	0	0	0	2	12	25	14	2	0	0	0
307	0	0	0	0	0	8	26	18	0	0	0	0	0
258	0	0	0	0	6	24	12	0	0	0	1	0	0
208	0	0	0	3	11	4	0	0	0	0	0	0	0
159	0	0	0	23	7	0	0	0	0	0	0	0	0
109	0	0	2	2	0	0	0	0	0	0	0	0	0
60	0	5	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0

Nominal Skew T/Vp/Vd: 125/.45/1.2

r_Test (au) \ r_Use (au)	0	59.5	109	159	208	258	307	357	406	456	505	555	604
604	0	0	0	0	1	0	1	1	1	1	18	69	N/A
555	0	0	0	0	0	0	0	0	1	14	104	95	59
505	0	0	1	0	0	0	1	2	10	97	92	120	20
456	0	0	0	0	0	0	1	5	46	83	86	14	2
406	0	0	0	0	0	1	4	59	80	56	6	2	3
357	0	0	0	0	0	1	29	68	53	2	1	2	0
307	0	0	0	0	0	36	56	38	2	1	1	0	0
258	0	0	0	0	15	71	20	2	1	0	0	0	0
208	0	0	0	12	38	18	0	0	0	1	0	0	0
159	0	0	8	43	9	0	0	0	0	0	0	0	1
109	0	1	19	6	0	0	0	0	1	0	0	0	0
60	0	11	2	0	0	0	0	0	0	0	0	0	0
0	5	1	0	0	0	0	0	0	0	0	0	0	0

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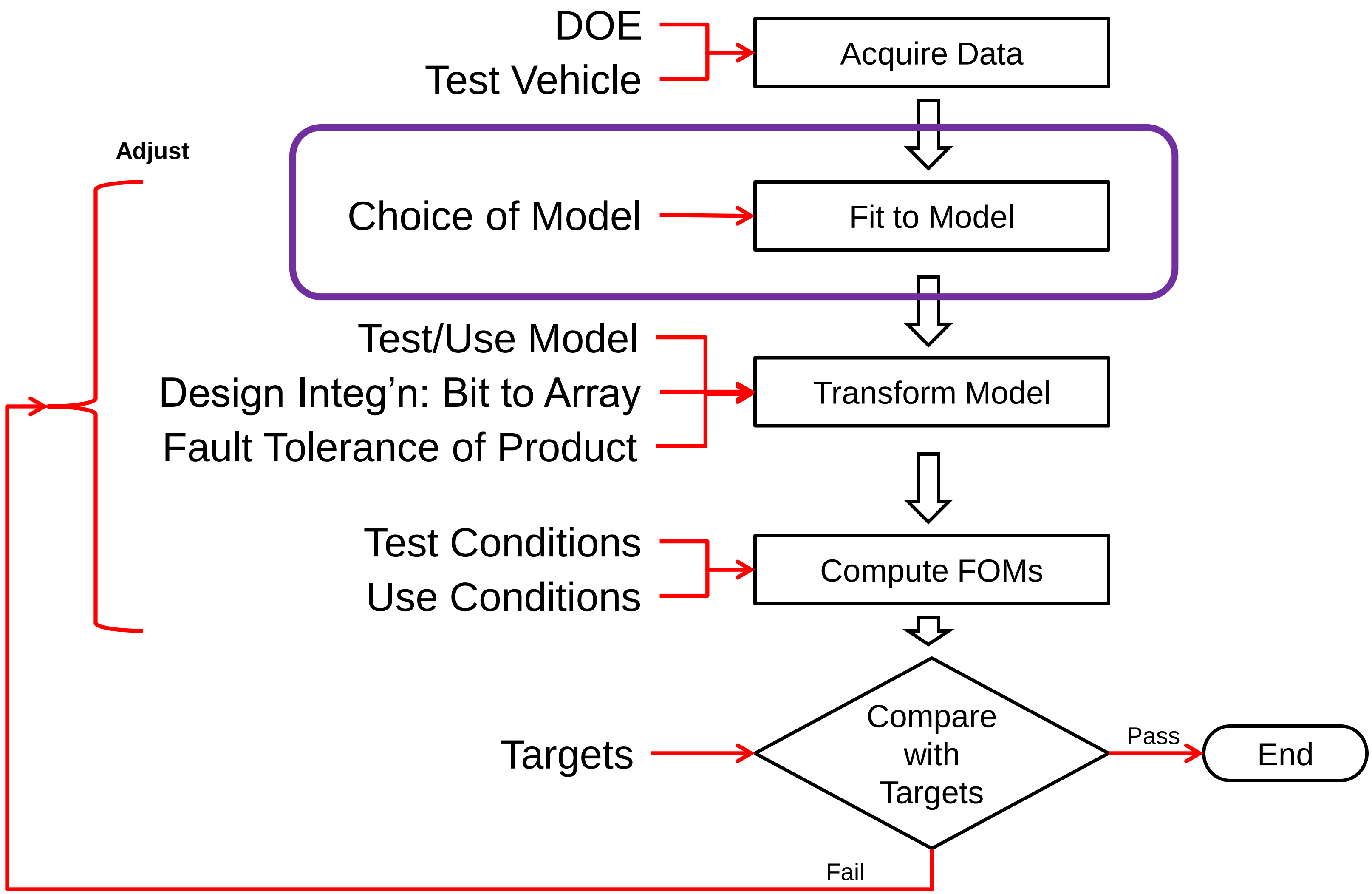
Modeling Miscorrelation



Fit of Marginal Distributions

T = 125 C
Vp = 0.45 V
Vd = 1.2 V
SS = 48750000

$$\text{Probability mass in cell} = \frac{92}{48750000} = 1.89\times10^{-6} = 1.89\ \text{DPPM}$$

Test																		
F = Cum N/SS (PPM)	Cum N	N	r (au)															
			604	0	0	0	0	1	0	1	1	1	1	18	69	N/A		
33.6	1639	273	555	0	0	0	0	0	0	0	0	1	14	104	95	59		
28.0	1366	343	505	0	0	1	0	0	0	1	2	10	97	92	120	20		
21.0	1023	237	456	0	0	0	0	0	0	1	5	46	83	86	14	2		
16.1	786	211	406	0	0	0	0	0	1	4	59	80	56	6	2	3		
11.8	575	156	357	0	0	0	0	0	1	29	68	53	2	1	2	0		
8.6	419	134	307	0	0	0	0	0	36	56	38	2	1	1	0	0		
5.8	285	109	258	0	0	0	0	15	71	20	2	1	0	0	0	0		
3.6	176	69	208	0	0	0	12	38	18	0	0	0	1	0	0	0		
2.2	107	61	159	0	0	8	43	9	0	0	0	0	0	0	0	1		
0.9	46	27	109	0	1	19	6	0	0	0	0	1	0	0	0	0		
0.4	19	13	60	0	11	2	0	0	0	0	0	0	0	0	0	0		
0.1	6	6	0	5	1	0	0	0	0	0	0	0	0	0	0	0		
				0	60	109	159	208	258	307	357	406	456	505	555	604	r (au)	Use
				5	13	30	61	63	127	112	175	195	255	308	302		N	
				5	18	48	109	172	299	411	586	781	1036	1344	1646		Cum N	
				0.1	0.4	1.0	2.2	3.5	6.1	8.4	12.0	16.0	21.3	27.6	33.8		F=Cum N/SS (PPM)	

$$\text{Marginal cumulative probability} = \frac{299}{48750000} = 6.13\times10^{-6} = 6.13\ \text{DPPM}$$

Weibull Fit of Marginal Dist'ns

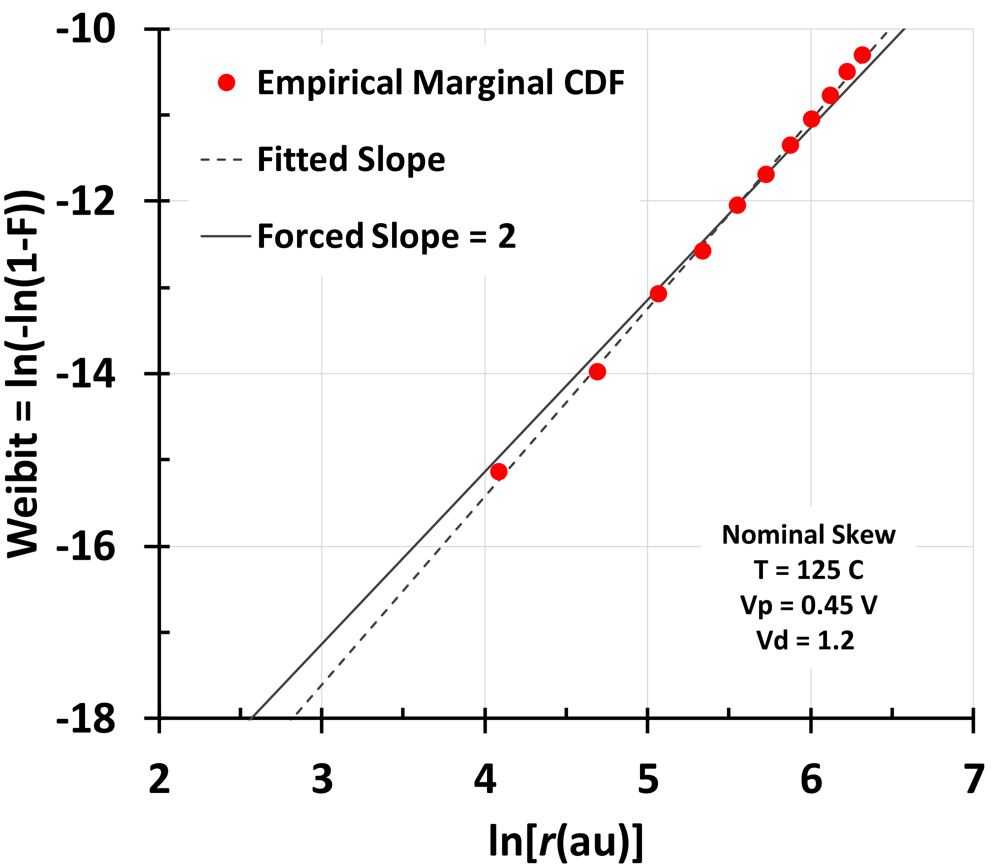
$$W = \ln\left[-\ln\left(1-F\right)\right]$$

$$F = 1-\exp\left[-\left(\frac{r}{\alpha}\right)^{\beta}\right]$$

$$W = \beta \ln r - \beta \ln \alpha$$

Slope (Forced) — Intercept (Determines α)

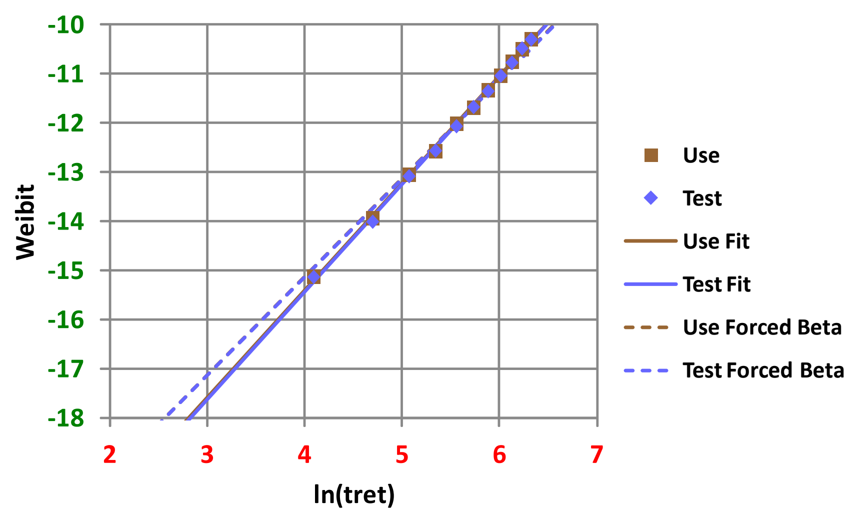
- Slope of Weibull plots is close to 2 for all environmental conditions and skews.
- Determine lnα for each of 18 environmental conditions.



T/Vp/Vd = 125/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	0	1	1	1	1	18	69	0	
6.32	555	-10.30	33.50	1633	273	11	0	0	0	0	0	0	0	0	1	14	104	**95**	59	
6.23	505	-10.49	27.90	1360	343	10	0	0	1	0	0	0	1	2	10	97	**92**	120	20	
6.12	456	-10.78	20.86	1017	237	9	0	0	0	0	0	0	1	5	46	**83**	86	14	2	
6.01	406	-11.04	16.00	780	211	8	0	0	0	0	0	1	4	59	**80**	56	6	2	3	
5.88	357	-11.36	11.67	569	156	7	0	0	0	0	0	1	29	**68**	53	2	1	2	0	
5.73	307	-11.68	8.47	413	134	6	0	0	0	0	0	36	**56**	38	2	1	1	0	0	
5.55	258	-12.07	5.72	279	109	5	0	0	0	0	15	**71**	20	2	1	0	0	0	0	
5.34	208	-12.57	3.49	170	69	4	0	0	0	12	**38**	18	0	0	0	1	0	0	0	
5.07	159	-13.09	2.07	101	61	3	0	0	8	**43**	9	0	0	0	0	0	0	0	1	
4.69	109	-14.01	0.82	40	27	2	0	1	**19**	6	0	0	0	0	1	0	0	0	0	
4.09	60	-15.14	0.27	13	13	1	0	**11**	2	0	0	0	0	0	0	0	0	0	0	
						0	**5**	1	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								13	30	61	63	127	112	175	195	255	308	302		N
								13	43	104	167	294	406	581	776	1031	1339	1641		CumN
								0.27	0.88	2.13	3.43	6.03	8.33	11.92	15.92	21.15	27.47	33.66		F
								-15.14	-13.94	-13.06	-12.58	-12.02	-11.70	-11.34	-11.05	-10.76	-10.50	-10.30		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)

Data was acquired at 18 environmental conditions. See backup slides.



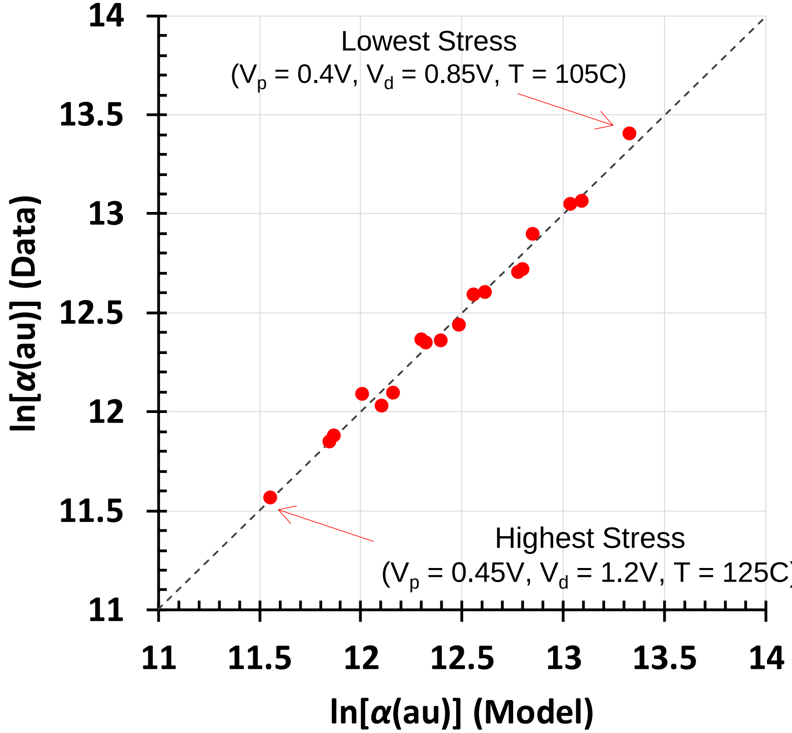
Fit lnα to an Environmental Model

- Determine $\ln\alpha_0$, a, b, Q by least-squares regression of...

... Weibull α at each environmental condition using a reference condition

$$\ln\alpha = \ln\alpha_0 + a(V_p - V_{p0}) + b(V_d - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

- Very good fit.
- lnα is a convenient measure of environmental condition.
- A given α defines a locus of statistically equivalent set points.
  - Gives useful flexibility in test programs.



Measure Scatter vs Truncation

- Kendall's Tau is a measure of the scatter.
- Environmental condition controls the truncation of data.

Nominal Skew T/Vp/Vd: 125/.45/1.2

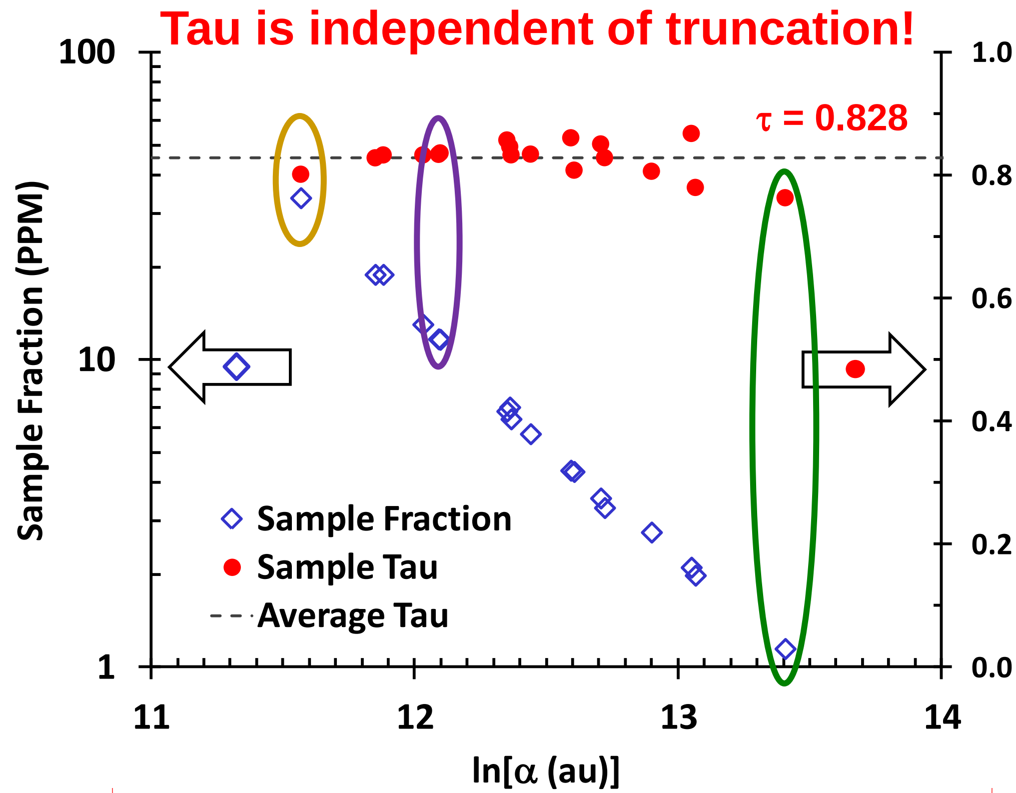
r_Test (au)													
604	0	0	0	0	1	0	1	1	1	1	18	69	N/A
555	0	0	0	0	0	0	0	0	1	14	104	95	59
505	0	0	1	0	0	0	1	2	10	97	92	120	20
456	0	0	0	0	0	0	1	5	46	83	86	14	2
406	0	0	0	0	0	1	4	59	80	56	6	2	3
357	0	0	0	0	0	1	29	68	53	2	1	2	0
307	0	0	0	0	0	36	56	38	2	1	1	0	0
258	0	0	0	0	15	71	20	2	1	0	0	0	0
208	0	0	0	12	38	18	0	0	0	1	0	0	0
159	0	0	8	43	9	0	0	0	0	0	0	0	1
109	0	1	19	6	0	0	0	0	1	0	0	0	0
60	0	11	2	0	0	0	0	0	0	0	0	0	0
0	5	1	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)

Nominal Skew T/Vp/Vd: 125/.4/1.

r_Test (au)													
604	0	0	0	0	0	0	1	0	1	0	7	23	N/A
555	0	0	0	0	1	0	0	0	0	2	31	38	17
505	0	0	0	0	0	0	0	1	3	38	37	29	7
456	0	0	0	0	0	0	0	3	22	42	17	0	0
406	0	0	0	0	0	0	0	19	24	19	2	0	0
357	0	0	0	0	0	2	12	25	14	2	0	0	0
307	0	0	0	0	0	8	26	18	0	0	0	0	0
258	0	0	0	0	6	24	12	0	0	0	1	0	0
208	0	0	0	3	11	4	0	0	0	0	0	0	0
159	0	0	0	23	7	0	0	0	0	0	0	0	0
109	0	0	2	2	0	0	0	0	0	0	0	0	0
60	0	5	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)

Nominal Skew T/Vp/Vd: 105/.4/.85

r_Test (au)													
604	0	0	0	0	0	0	0	0	0	0	0	3	N/A
555	0	0	0	0	0	0	0	0	0	1	3	1	1
505	0	0	0	0	0	0	0	0	1	1	3	5	1
456	0	0	0	0	0	0	0	0	3	4	7	0	0
406	0	0	0	0	0	0	0	3	2	2	0	0	0
357	0	0	0	0	0	0	4	3	1	1	0	0	0
307	0	0	0	0	0	0	2	1	0	0	0	0	0
258	0	0	0	0	0	2	0	0	0	0	0	0	0
208	0	0	0	0	2	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	60	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)



Environmental Condition:

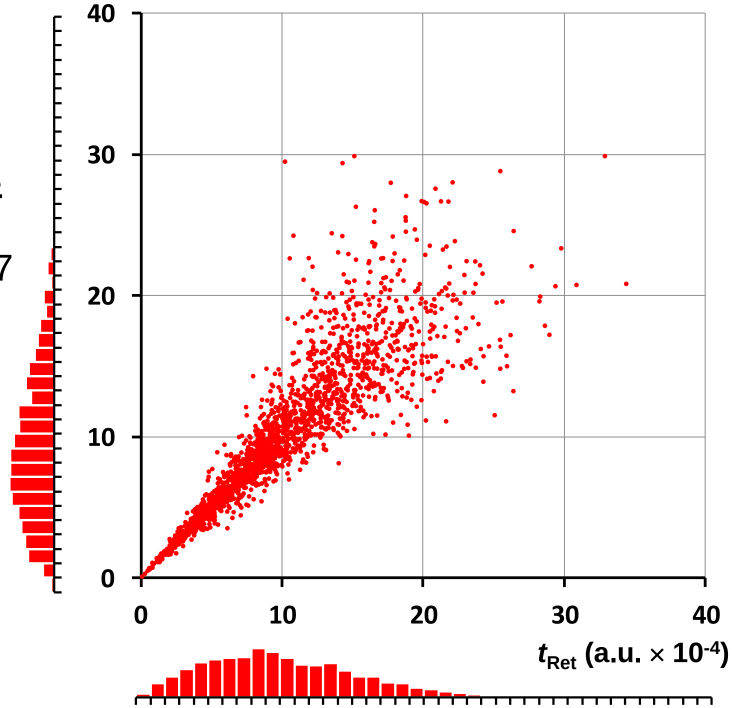
$$\ln\alpha = \ln\alpha_0 + a(V_p - V_{p0}) + b(V_d - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

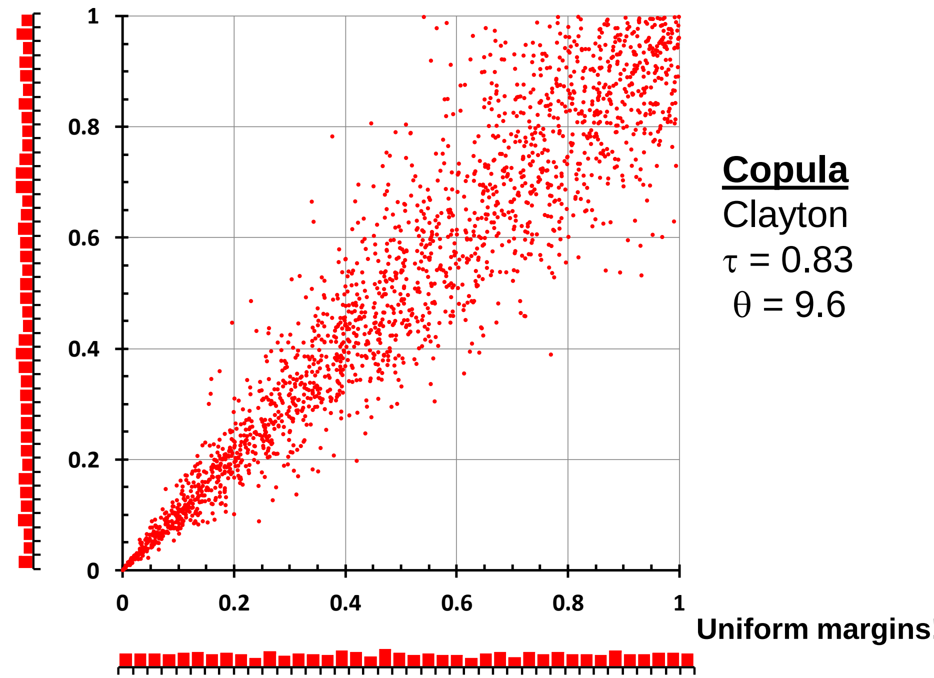
Rank Correlation from Correlation

- Correlation plot of ranks is the empirical copula pdf.
- Kendall's Tau is a measure of miscorrelation (scatter).
- Tau depends only on ranks of data.

Margins
Weibull
lnα= 11.7
β = 2

$f(t_{Ret})$
(pdf of t_{Ret})





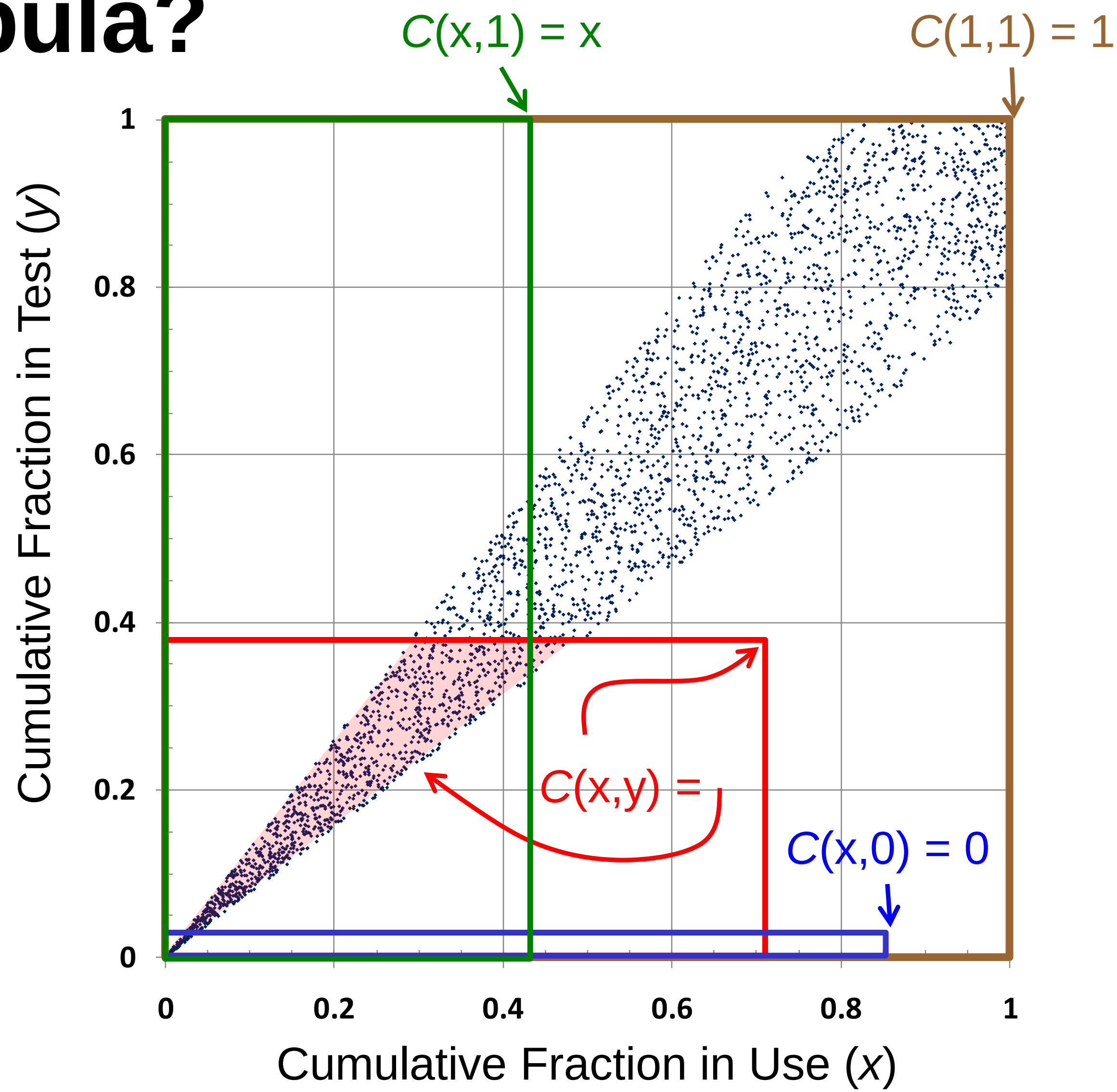
Copula
Clayton
τ = 0.83
θ = 9.6

Synthesized data with same parameters as DRAM model at maximum environmental condition.

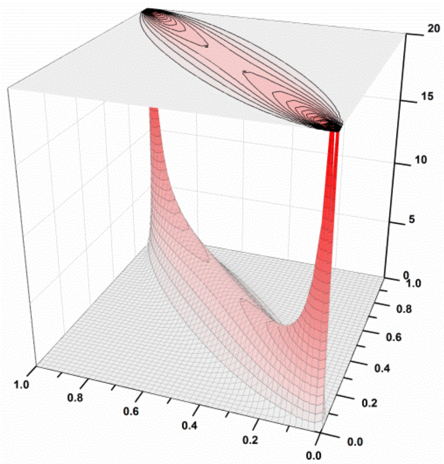
What is a Copula?

- To get the copula from the copula PDF:
  - The copula is the probability mass in rectangles [0,*x*]×[0,y] as a function of (x,y).
- Definition of a copula
  - $C(1,1) = 1$
  - $C(x,0)=C(0,y)=0$
  - $C(x,1)=x,\ C(1,y)=y$
  - 2-increasing
- Copula pdf (analytical)

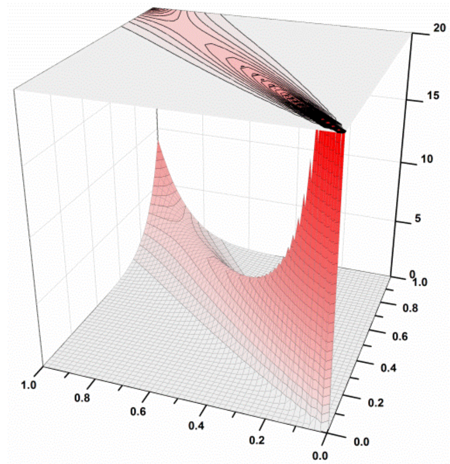
$$c(x,y) = \frac{\partial^2 C(x,y)}{\partial x \partial y}$$



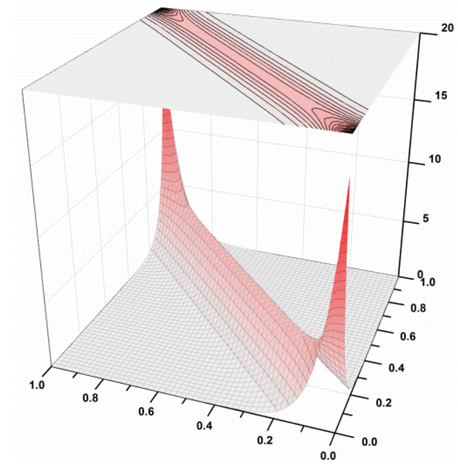
Examples of Copula PDFs



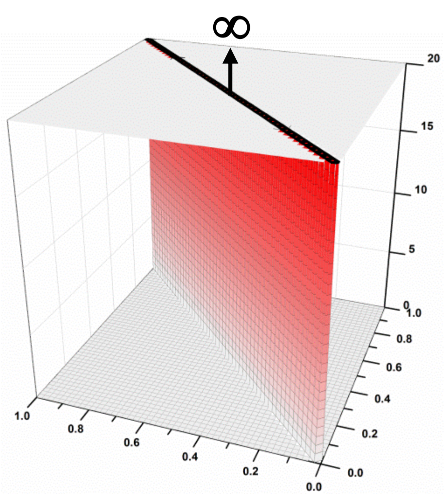
Gaussian, τ = 0.8, ρ = 0.951



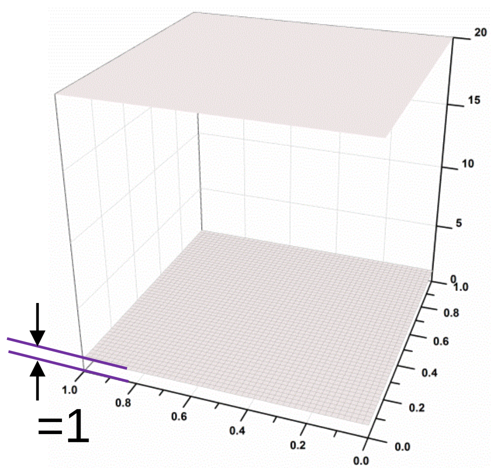
Clayton, τ = 0.8, θ = 8



Frank, τ = 0.8, θ = 17.5



Perfect Correlation, τ = 1

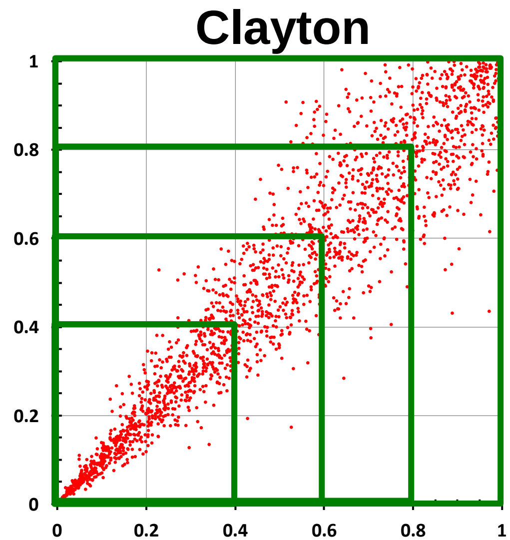


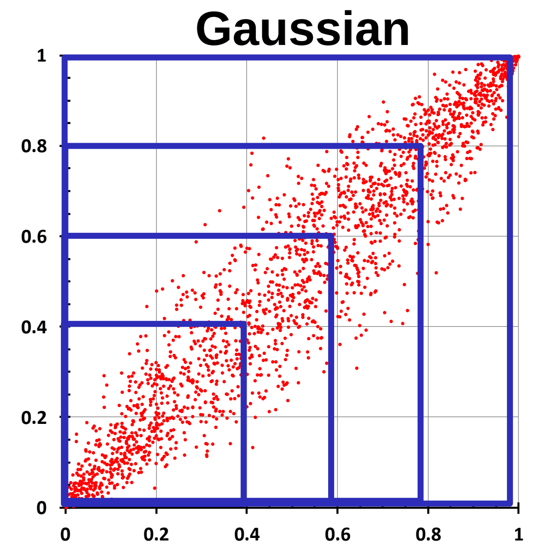
Independence, τ = 0

A copula pdf is a probability density function which integrates to unity on the unit square.

Tau vs Copula Truncation

- Dependence fades away in the tail of the Gaussian copula.
- Dependence is invariant as the Clayton copula is truncated.





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On the preservation of copula structure under truncation

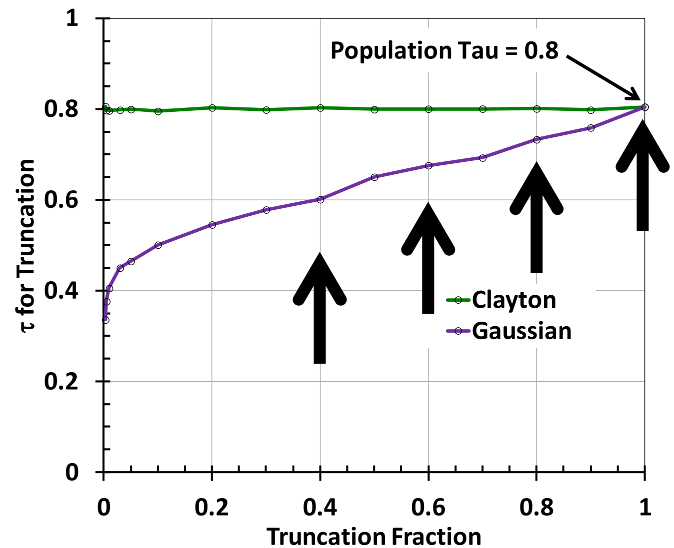
David OAKES

Key words and phrases: Archimedean copula; bivariate distribution; Clayton model; Cook–Johnson model; gamma frailty model; Kendall's tau.

MSC 2000: Primary 62H20; secondary 62P10.

Abstract: The author characterizes the copula associated with the bivariate survival model of Clayton (1978) as the only absolutely continuous copula that is preserved under bivariate truncation.

Thanks to Roger Nelsen for pointing this out!

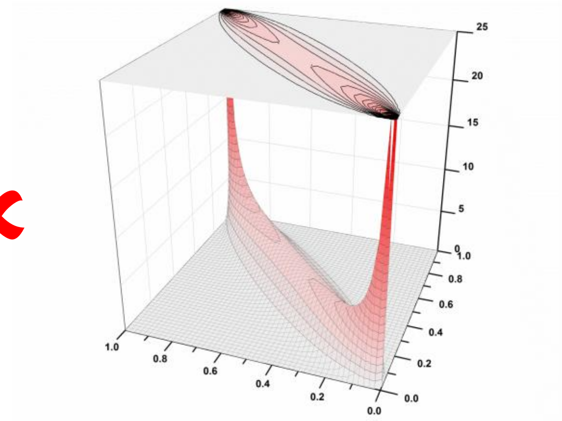


Model, Fitted and Chosen

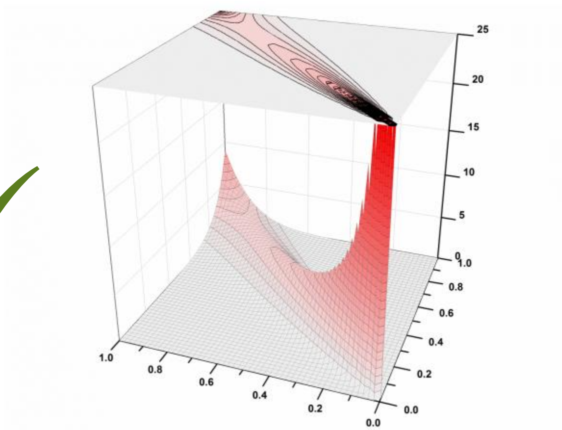
- Gaussian model rejected because it does not have the correct tail dependence.

Margin	β	2.0
	ln[a_0 (au)]	11.57
	a (V^{-1})	-5.79
	b (V^{-1})	-1.55
	Q (eV)	0.605
	V_{p0} (V)	0.45
	V_{d0} (V)	1.2
	T_0 (°C)	125.0
Dependence (ρ = 0.999305)	Sample Tau, τ'	0.828
	Clayton Copula, θ	9.74
	Gaussian Copula $(1-\rho)\times10^3$	0.695

✗



✓



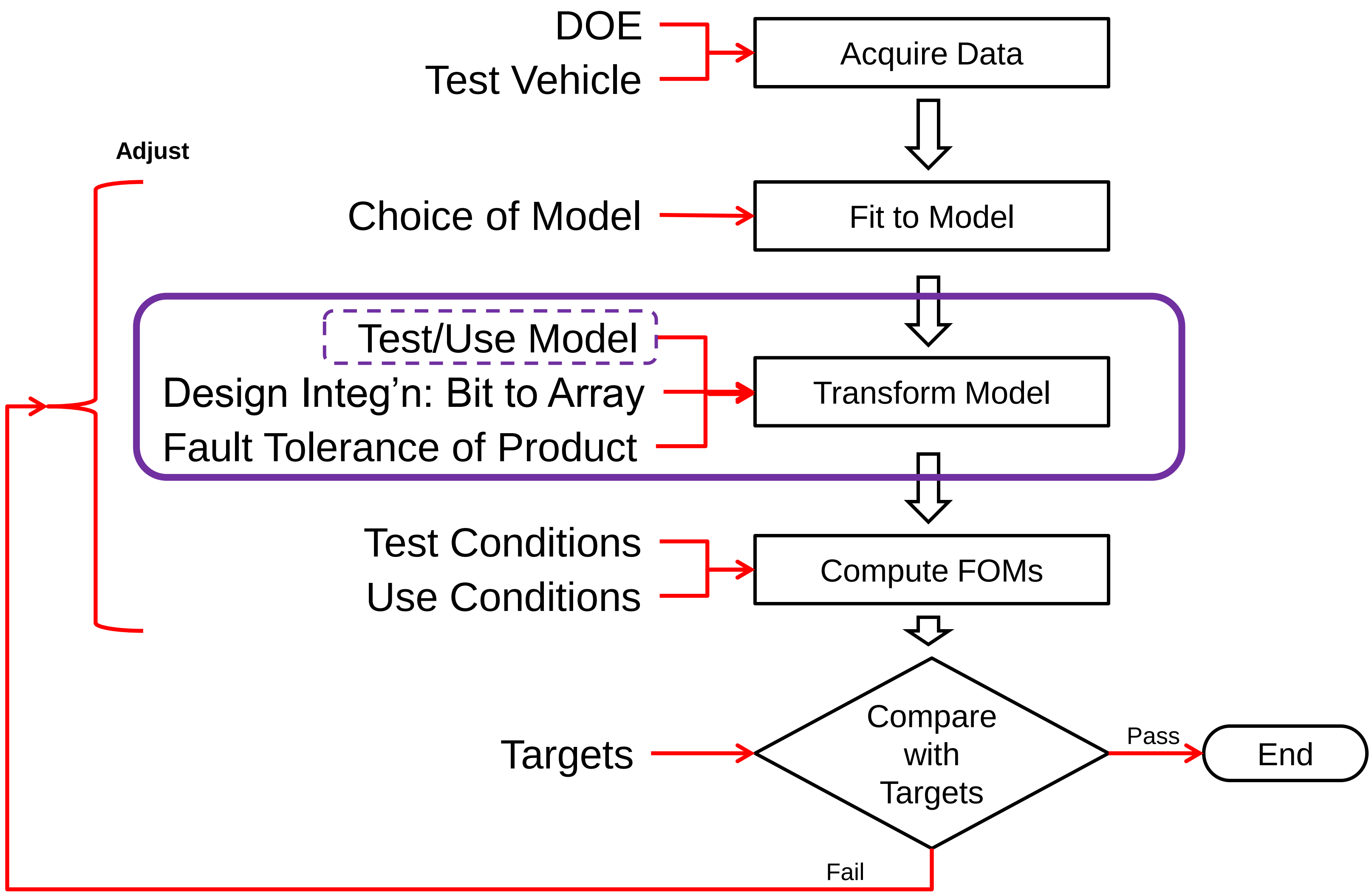
A simple single-parameter copula describes the dependency structure of the DRAM VRT phenomenon across *all* environmental conditions.

$$C(u,v)=\left(u^{-\theta}+v^{-\theta}-1\right)^{-1/\theta} \quad \theta=9.74$$

Outline

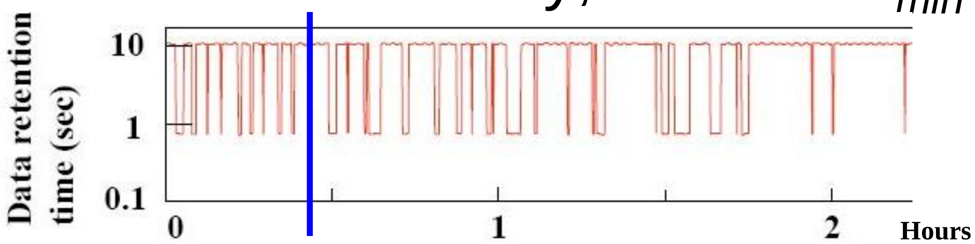
- Introduction
- DRAM Experiment in ICDT
- Fitting a Model
- Using the Model
- Final Thoughts

Modeling Miscorrelation

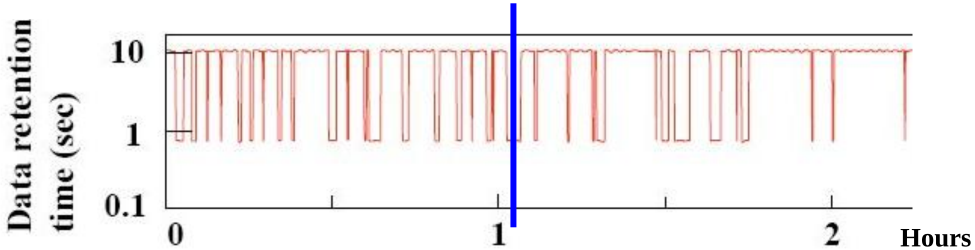


Test/Use Model for a Bit

- Test is momentary, so either r_{min} or r_{max} may be observed.



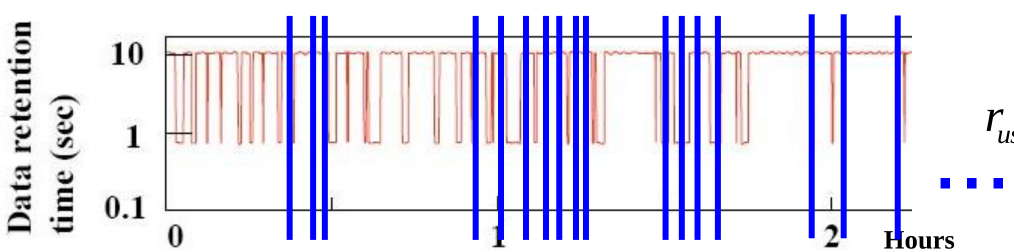
$r_{test} = r_{\max}$ Probability of occurrence: s



$r_{test} = r_{\min}$ Probability of occurrence: $(1-s)$

s = "duty cycle" of bit variability

- Use is repeated, r_{min} (worst case) will certainly occur.



$r_{use} = r_{\min}$ All the time.

Map Fitted Copula to Test/Use Model

- Each bit has r_{min} and r_{max}.
- r_{min} and r_{max} were mapped to r_1 and r_2 and fitted to an exchangeable copula.

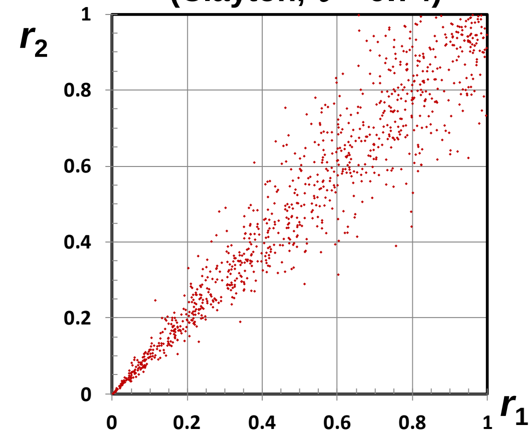
C(*x*,*y*) = *C*(*y*,*x*)

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} r_{\max} \\ r_{\min} \end{bmatrix} \quad 50\% \text{ of the time}$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} r_{\min} \\ r_{\max} \end{bmatrix} \quad 50\% \text{ of the time}$$

$$C(x,y) = \left[x^{-\theta} + y^{-\theta} - 1\right]^{-1/\theta}$$

pdf of *C*
(Clayton, θ = 9.74)

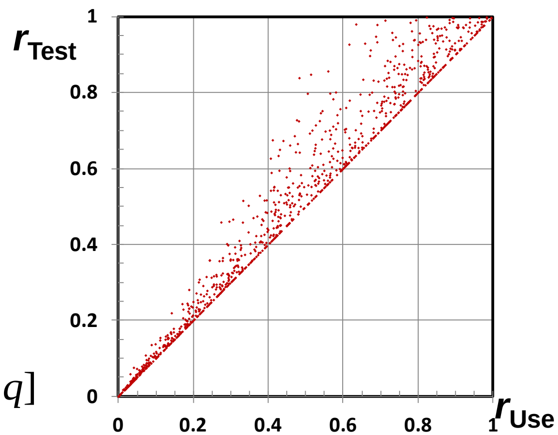


- Fitted copula maps to Test/Use model:

$$r_{test} = \begin{cases} r_{\min} = \min[r_1, r_2] & (1-s) \text{ of the time} \\ r_{\max} = \max[r_1, r_2] & s \text{ of the time} \end{cases}$$

$$r_{use} = r_{\min} = \min[r_1, r_2] \quad \text{All the time.}$$

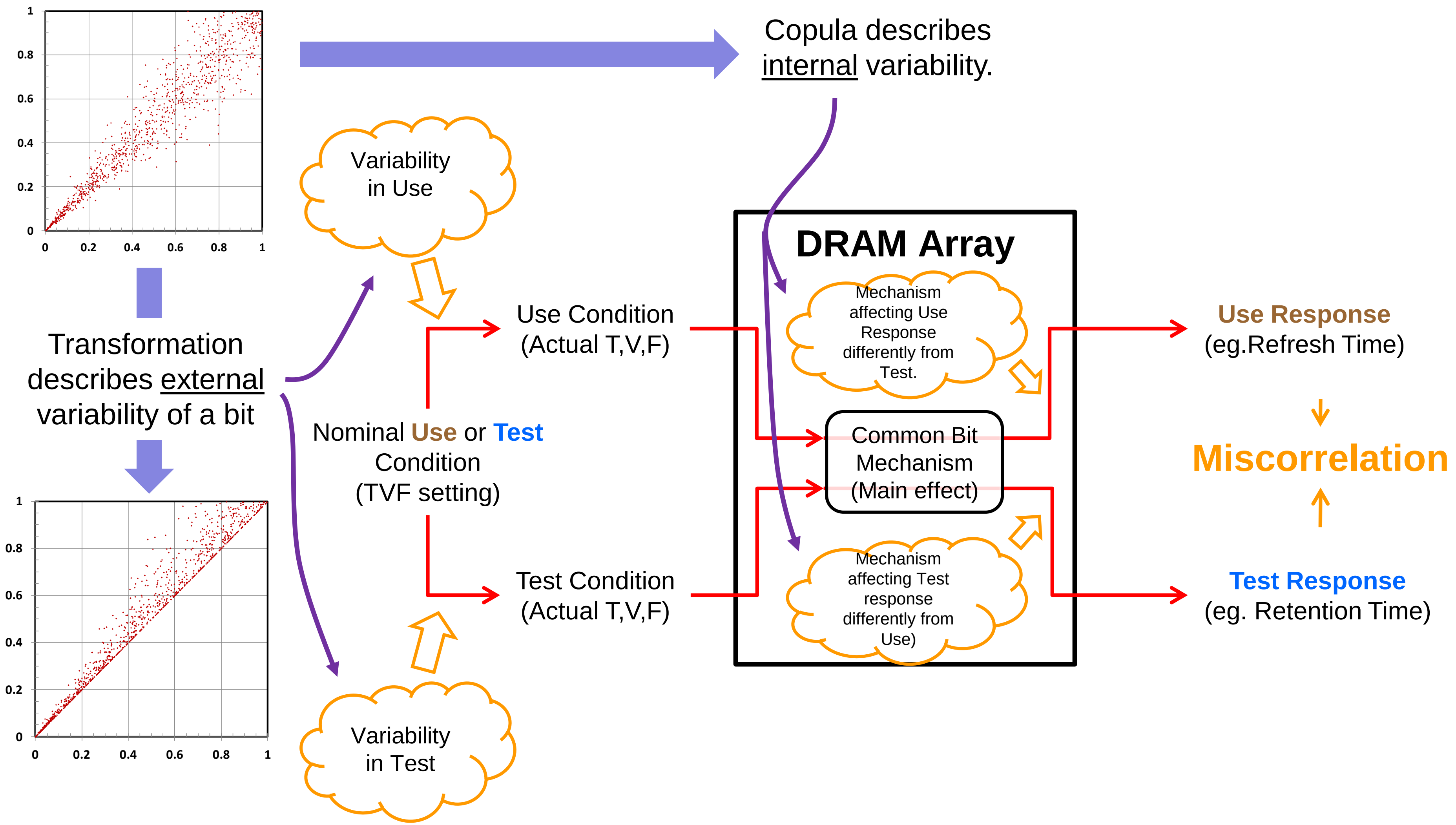
pdf of *D* (*s* = 0.7)



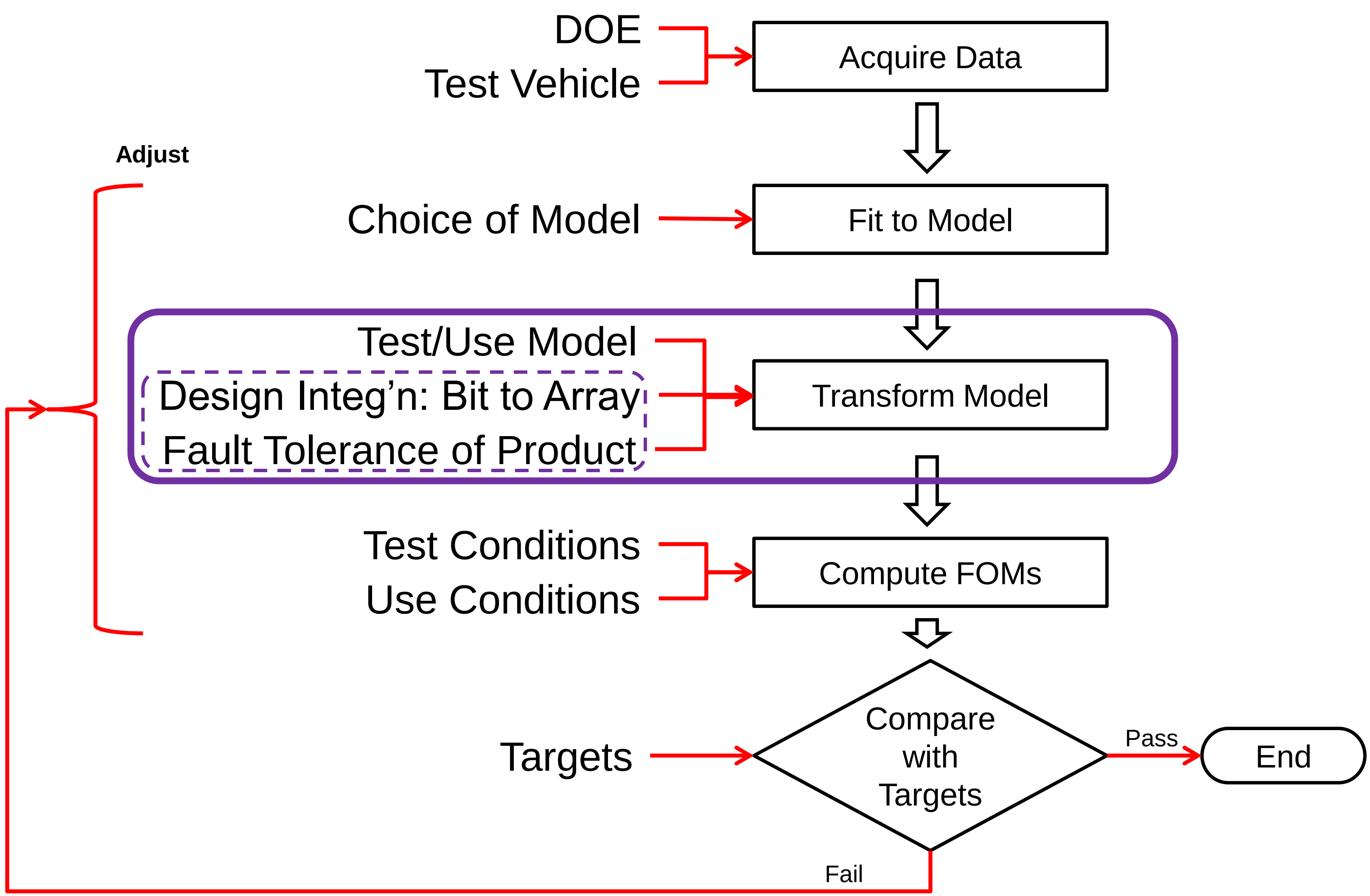
- Mathematically.. (*D* is a pseudo-copula.)

$$D(p,q) = s\left[C(p,q) + C(q,z) - C(p,z)\right] + (1-s)\left[2z - C(z,z)\right], \quad z = \min[p,q]$$

Causes of Miscorrelation



Modeling Miscorrelation



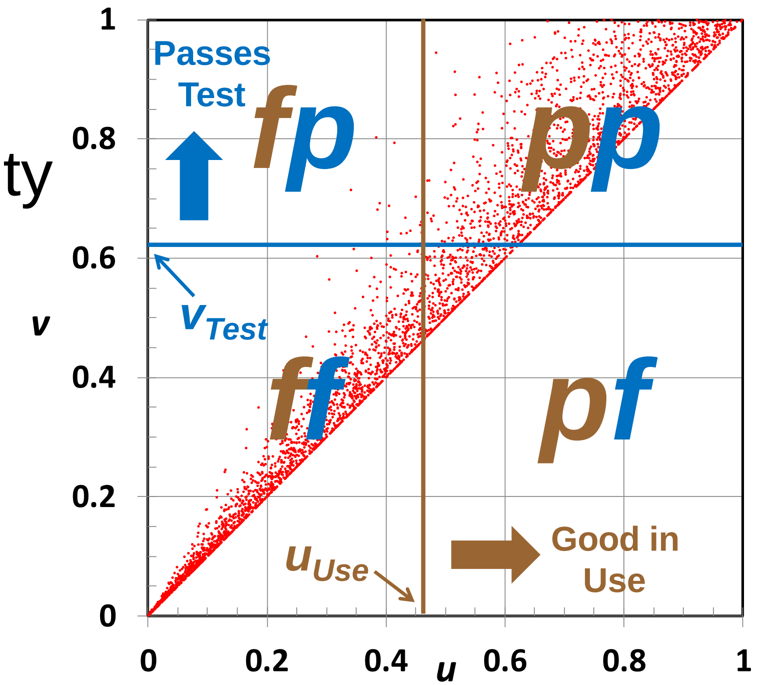
Design Integration: Bits to Arrays

- Probabilities that a bit falls into each of the 4 categories are, in terms of *D*,

(**First subscript letter** is for Use, **second subscript letter** is for Test.)

$$p_{fp} = D(u,1) - D(u,v) \qquad p_{pf} = D(1,v) - D(u,v)$$

$$p_{ff} = D(u,v) \qquad p_{pp} = 1 - p_{fp} - p_{pf} - p_{ff}$$



- If an array has *N* bits, the probability that it has *exactly* n_{fp}, n_{pf}, and n_{ff} bits in *fp*, *pf*, *ff* categories is

$$\frac{\lambda_{fp}^{n_{fp}} \exp(-\lambda_{fp})}{n_{fp}!} \frac{\lambda_{pf}^{n_{pf}} \exp(-\lambda_{pf})}{n_{pf}!} \frac{\lambda_{ff}^{n_{ff}} \exp(-\lambda_{ff})}{n_{ff}!}$$

where

$$\lambda_{fp} = Np_{fp},\ \lambda_{pf} = Np_{pf},\ \lambda_{ff} = Np_{ff}.$$

- Use of Poisson statistics is well justified.

Design Integration: Example

- Fraction of arrays perfect in Use, irrespective of Test.

$n_{fp} = n_{ff} = 0$ $\quad 0 \le n_{pf} \le \infty$

$$\text{"True Yield"} = \sum_{\substack{n_{fp}=n_{ff}=0 \\ 0\le n_{pf}\le\infty}} \frac{\lambda_{fp}^{n_{fp}} \exp(-\lambda_{fp})}{n_{fp}!} \frac{\lambda_{pf}^{n_{pf}} \exp(-\lambda_{pf})}{n_{pf}!} \frac{\lambda_{ff}^{n_{ff}} \exp(-\lambda_{ff})}{n_{ff}!}$$

$$= \exp\left[-(\lambda_{fp} + \lambda_{ff})\right] = \exp\left[-ND(u,1)\right] = \exp\left[-N\left(2u - C(u,u)\right)\right]$$

where

$$C(u,v) = \left[u^{-\theta} + v^{-\theta} - 1\right]^{-1/\theta}$$

$$u = 1 - \exp\left[-\left(\frac{r_{use}}{\alpha_{use}}\right)^{\beta}\right]$$

$$\ln \alpha_{use} = \ln \alpha_0 + a(V_{p(use)} - V_{p0}) + b(V_{d(use)} - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T_{use}} - \frac{1}{T_0}\right)$$

Fault Tolerance of Arrays

- If an array tolerates up to n_u bad bits in Use and up to n_t bad bits in Test...
- Passes Test, irrespective of Use requires..

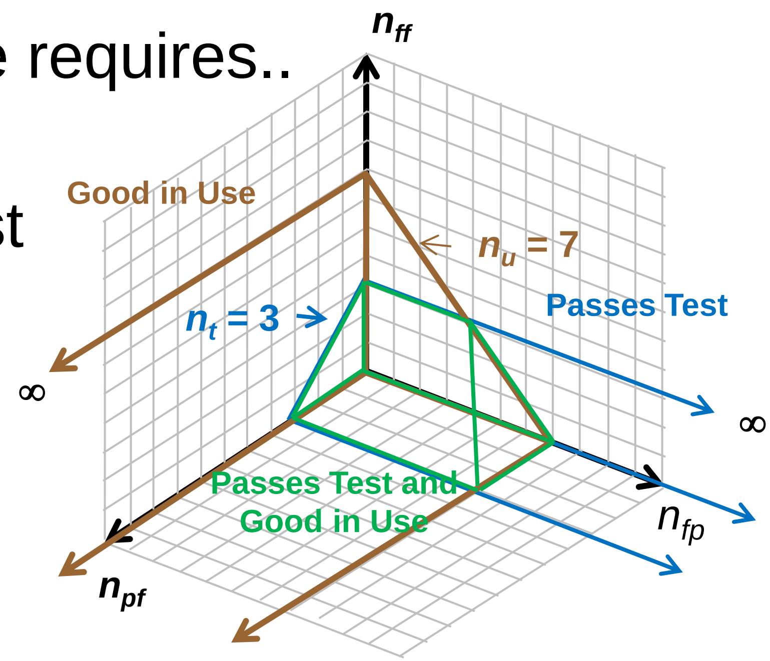
$$n_{pf} + n_{ff} \le n_t \qquad 0 \le n_{fp} \le \infty$$

- Good in Use, irrespective of Test requires..

$$n_{fp} + n_{ff} \le n_u \qquad 0 \le n_{pf} \le \infty$$

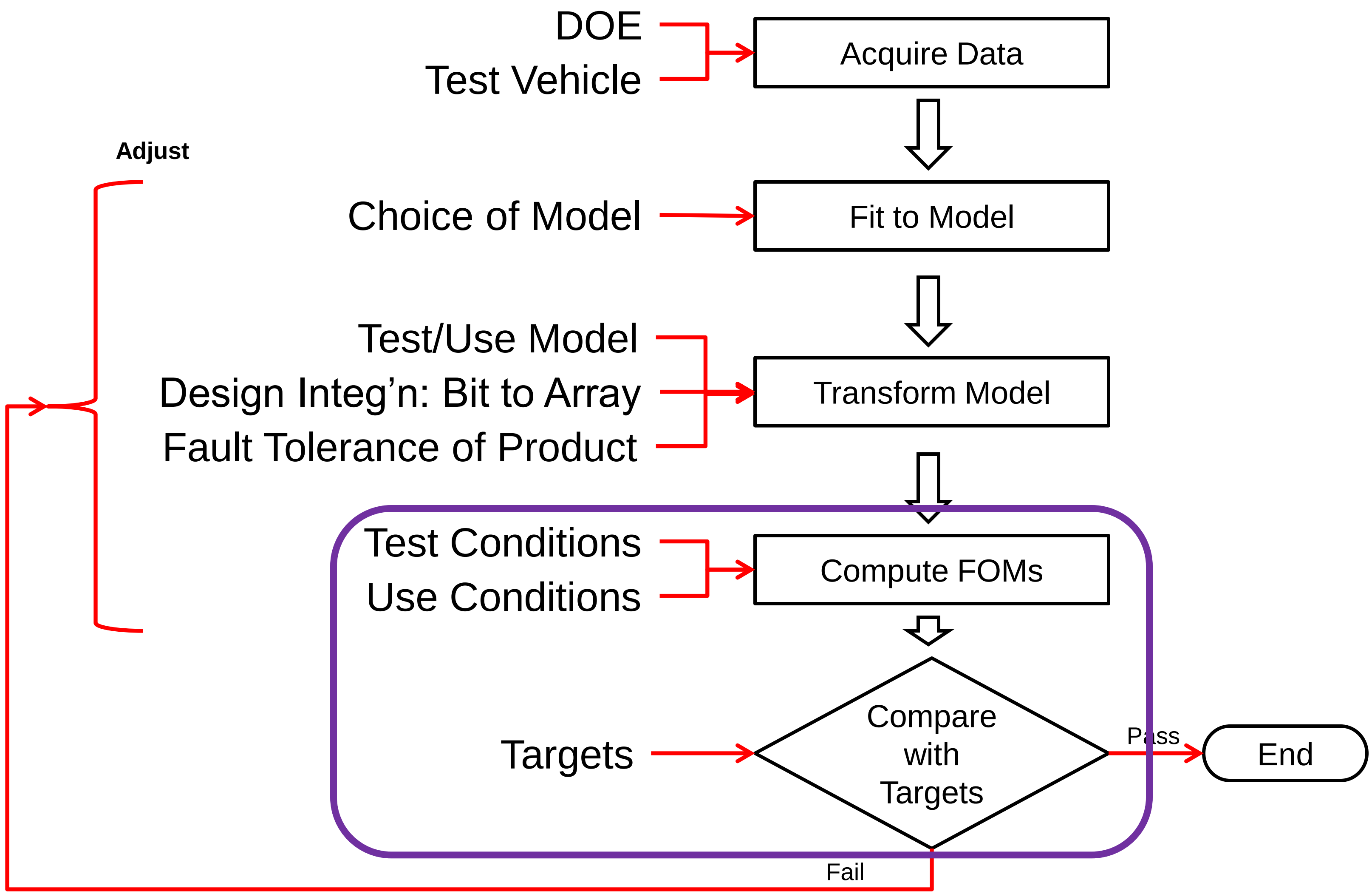
- Passes Test and Good in Use requires..

$$n_{pf} + n_{ff} \le n_t \qquad n_{fp} + n_{ff} \le n_u$$



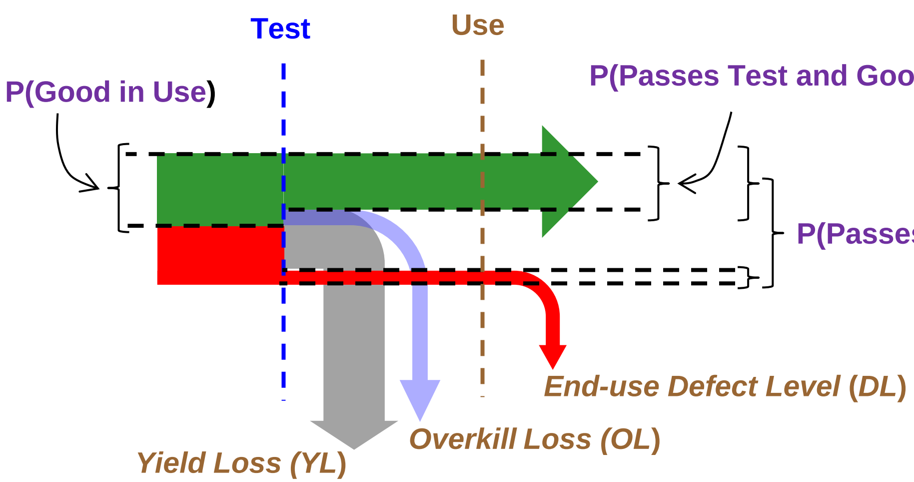
- Sum terms over index space to get *array* probabilities
  - *P*(Good in Use), *P*(Passes Test), *P*(Passes Test and Good in Use)

Modeling Miscorrelation



Calculate FOMs from Probabilities

- Figures of Merit are *designed*
  - With identified stakeholder in mind.
  - So that values *all* lie in the range [0,1].
  - So that more is worse for *all* FOMs.
  - To compare with do-not-exceed targets.



$$YL = 1 - P(\text{Passes Test})$$

$$OL = P(\text{Good in Use}) - P(\text{Passes Test and Good in Use})$$

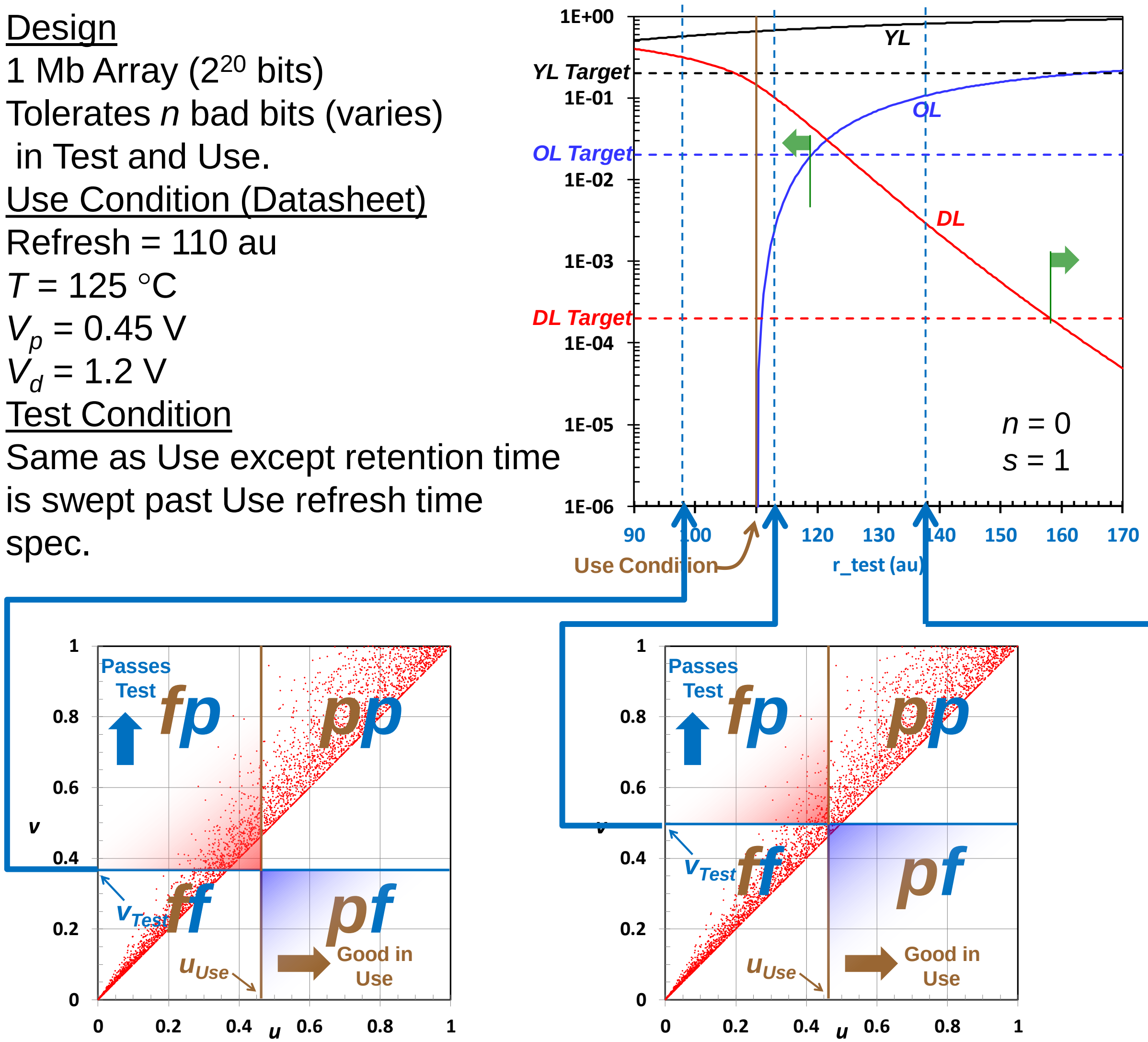
$$DL = 1 - P(\text{Passes Test and Good in Use}) / P(\text{Passes Test})$$

Plot FOMs as Test Sweeps Past Use

Design
1 Mb Array (2^{20} bits)
Tolerates n bad bits (varies) in Test and Use.

Use Condition (Datasheet)
Refresh = 110 au
T = 125 °C
V_p = 0.45 V
V_d = 1.2 V

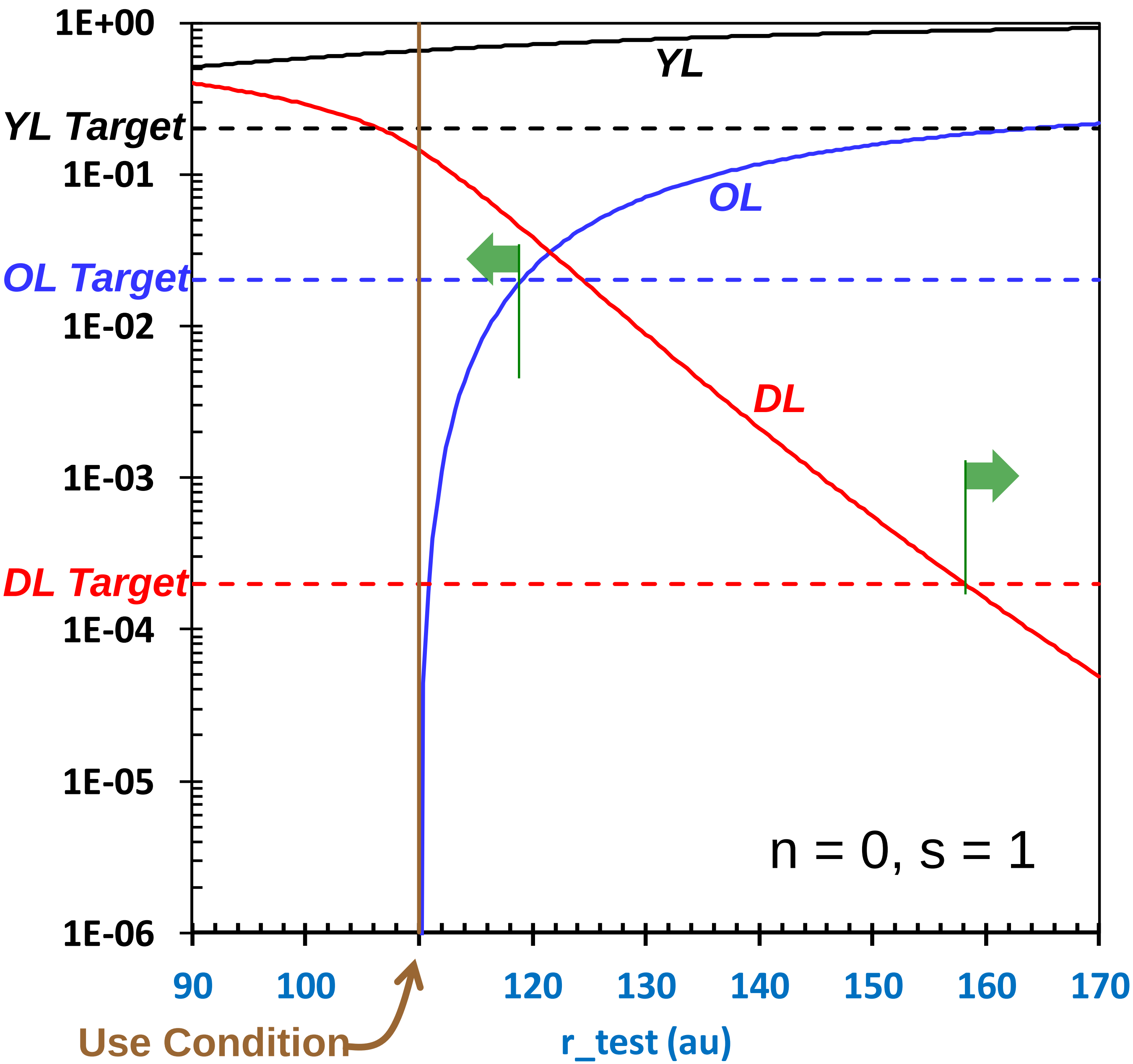
Test Condition
Same as Use except retention time is swept past Use refresh time spec.

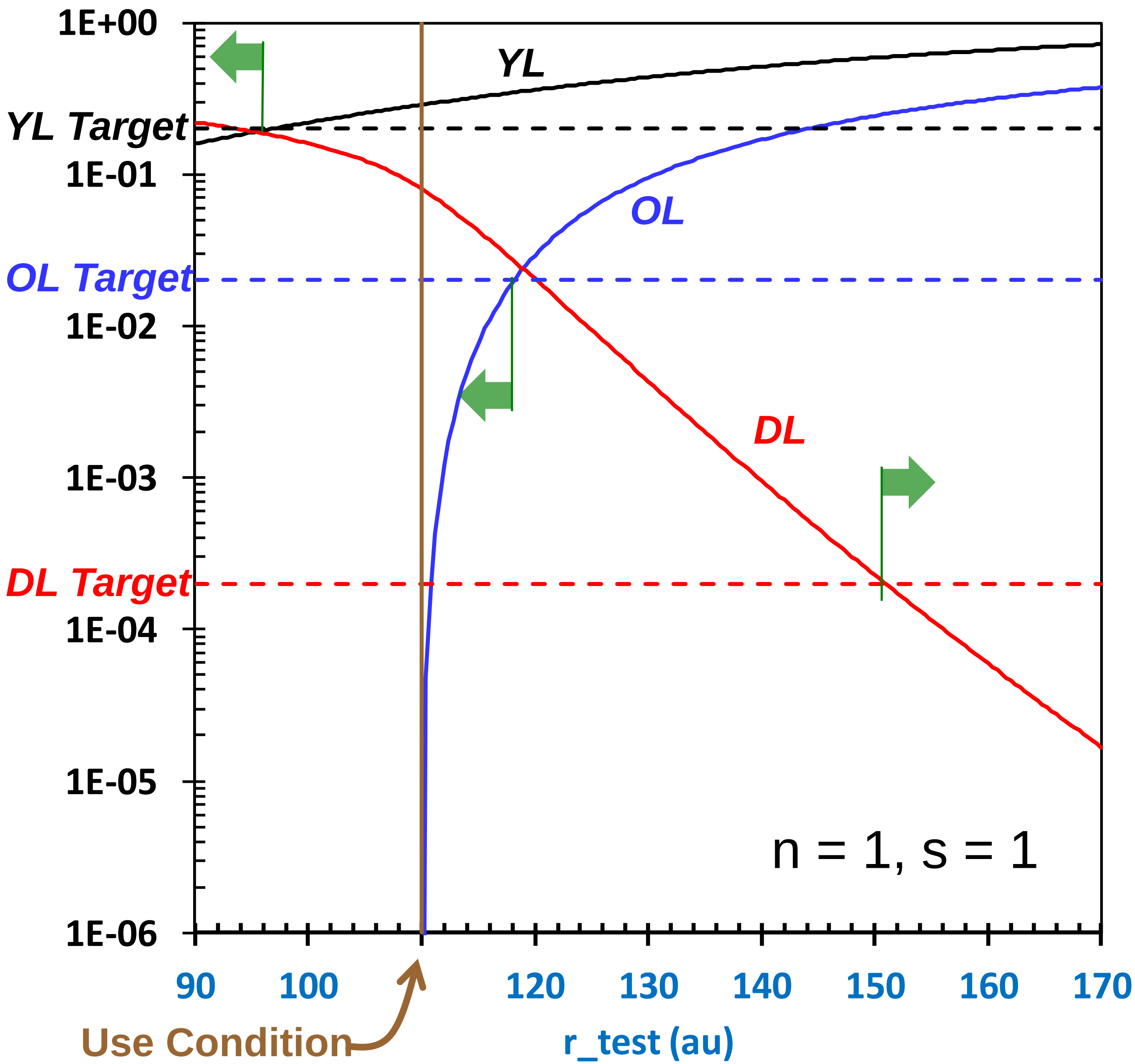


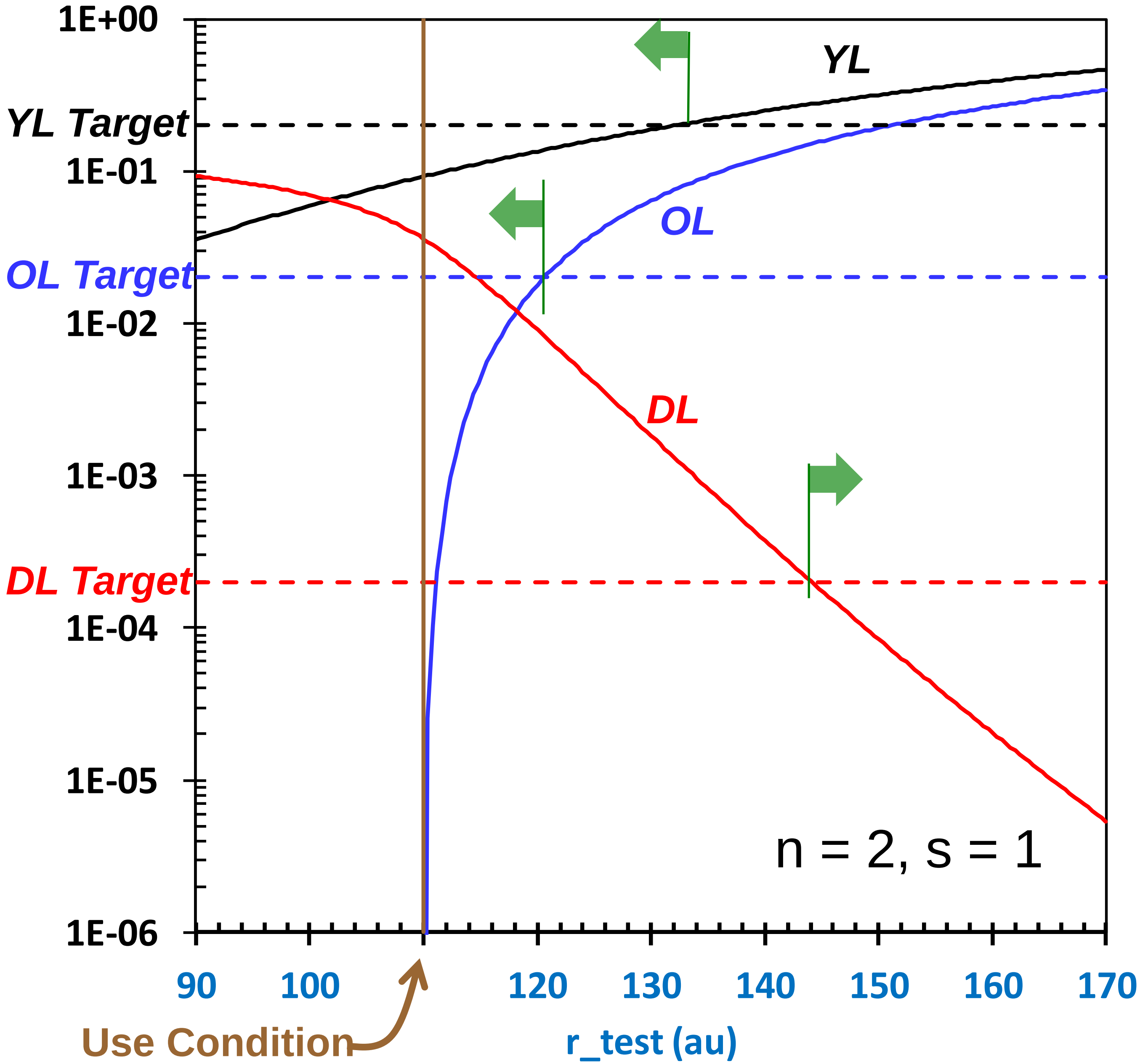
Targets
$YL \le 20\%$
$OL \le 2\%$
$DL \le 200$ PPM

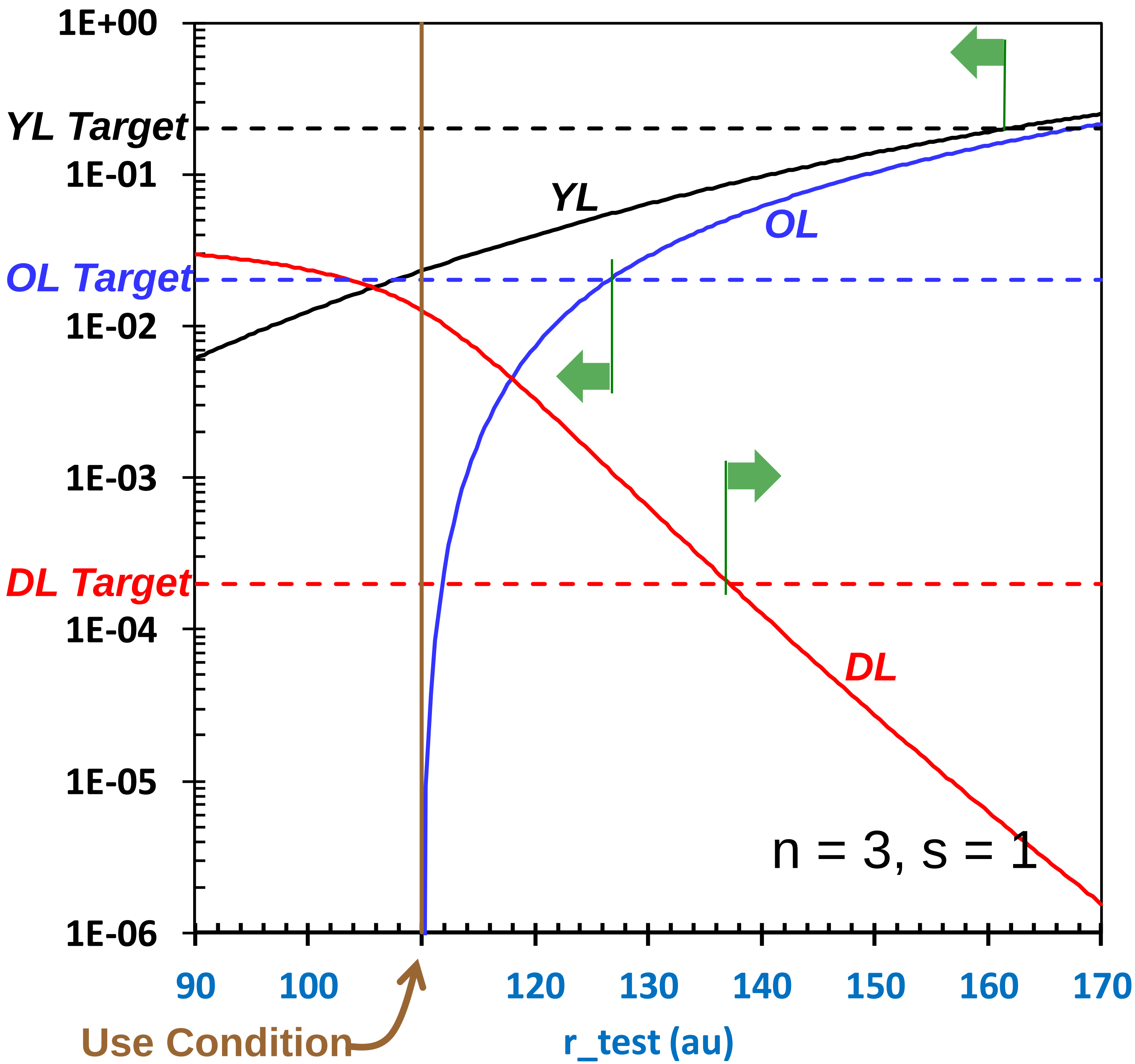
Test Model
Conservative (s = 1)
Less Conservative (s < 1)

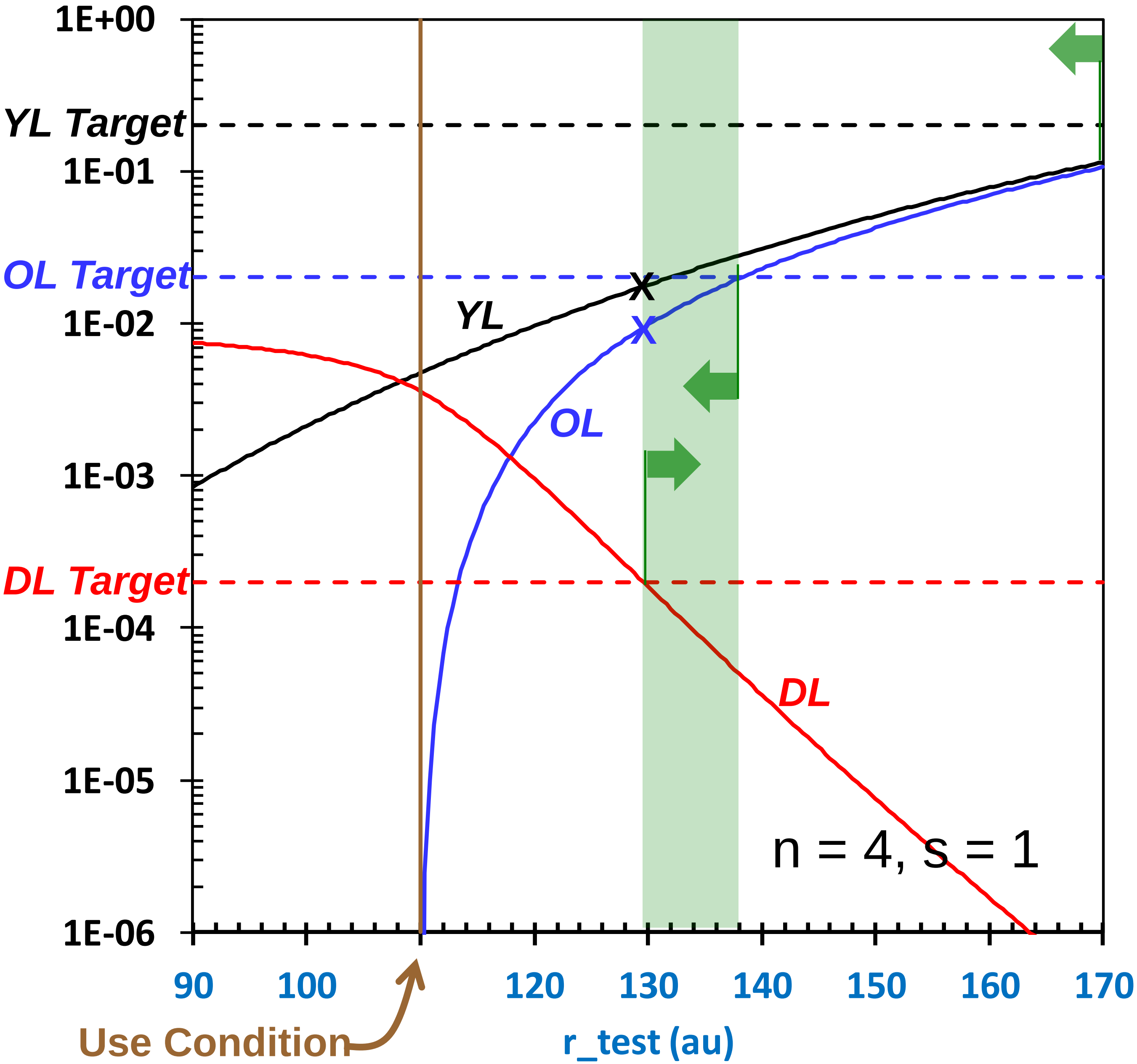
(s is the fraction of time Test finds a bit in the maximum retention time state.)


1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL
YL Target
OL
OL Target
DL
DL Target
n = 0, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL
YL Target
OL
OL Target
DL
DL Target
n = 1, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL Target
OL Target
DL Target
YL
OL
DL
n = 2, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
YL
OL
OL Target
1E-02
1E-03
DL Target
1E-04
DL
1E-05
n = 3, s = 1
1E-06
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
OL Target
1E-02
1E-03
DL Target
1E-04
1E-05
1E-06
YL
OL
DL
n = 4, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)

Outline

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Final Thoughts

- Copula methods are *necessary*..
  - To capture the phenomenon of dependent extreme values.
  - Eg. The DRAM dependency *cannot* be described by a multinormal distribution.
- Copula methods have great *convenience and flexibility*.
  - *Any* marginal models may be coupled using *any* copula.
  - Marginal and copula models may be fitted independently.
  - Efficiencies in Monte-Carlo synthesis are often available.
- Flexibility leads to the question of *copula choice*.
  - Tail dependences help choose a mathematical form, but...
  - What are the underlying stochastic mechanisms that would enable construction of a copula from first principles?

Backup

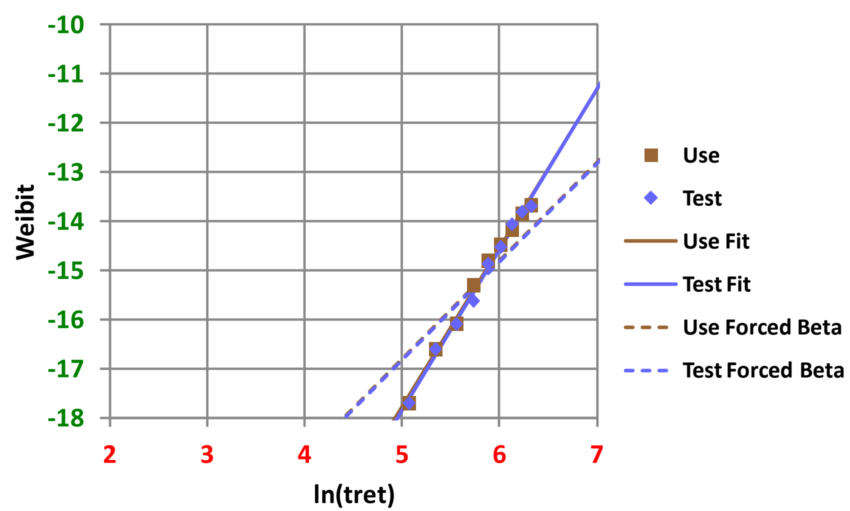
Data Record

Identity					Environmental Condition			Results of Test			
skew	chip	macro	PX	PY	VP	VDD	temp	IRetMin	IRetMax	IRetDelta	LoopGroups
4	1	0	1	238	0.4	0.85	125	10	11	1	00000000001 00000000011 00000000001 00000000011 00000000001
4	1	0	1	238	0.4	1	125	10	11	1	00000000011 00000000011 00000000011 00000000011 00000000001
4	1	0	1	238	0.4	1.2	125	10	11	1	00000000001 00000000011 00000000011 00000000001 00000000001
4	1	0	1	238	0.45	0.85	125	8	9	1	00000000111 00000001111 00000000111 00000000111 00000001111
4	1	0	1	238	0.45	1	125	9	9	0	00000000111 00000000111 00000000111 00000000111 00000000111
4	1	0	1	238	0.45	1.2	125	10	11	1	00000000011 00000000001 00000000001 00000000011 00000000001
4	1	0	16	520	0.4	0.85	105	4	4	0	00001111111 00001111111 00001111111 00001111111 00001111111
4	1	0	16	520	0.4	0.85	115	3	3	0	00011111111 00011111111 00011111111 00011111111 00011111111
4	1	0	16	520	0.4	0.85	125	2	2	0	00111111111 00111111111 00111111111 00111111111 00111111111
4	1	0	16	520	0.4	1	105	3	3	0	00011111111 00011111111 00011111111 00011111111 00011111111
4	1	0	16	520	0.4	1	115	2	2	0	00111111111 00111111111 00111111111 00111111111 00111111111
4	1	0	16	520	0.4	1	125	1	1	0	01111111111 01111111111 01111111111 01111111111 01111111111
4	1	0	16	520	0.4	1.2	105	2	2	0	00111111111 00111111111 00111111111 00111111111 00111111111
4	1	0	16	520	0.4	1.2	115	1	1	0	01111111111 01111111111 01111111111 01111111111 01111111111
4	1	0	16	520	0.4	1.2	125	1	1	0	01111111111 01111111111 01111111111 01111111111 01111111111
4	1	0	16	520	0.45	0.85	105	3	3	0	00011111111 00011111111 00011111111 00011111111 00011111111
4	1	0	16	520	0.45	0.85	115	2	2	0	00111111111 00111111111 00111111111 00111111111 00111111111
4	1	0	16	520	0.45	0.85	125	1	1	0	01111111111 01111111111 01111111111 01111111111 01111111111
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

T/Vp/Vd = 105/.4/.85

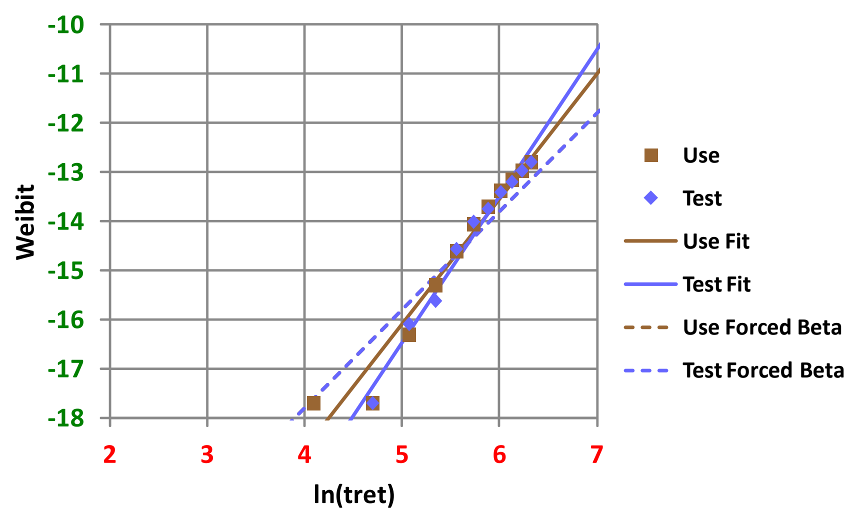
ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	0	0	0	0	0	0	0	0	3	0
6.32	555	-13.69	1.13	55	6	11	0	0	0	0	0	0	0	0	0	1	3	1	1
6.23	505	-13.81	1.01	49	11	10	0	0	0	0	0	0	0	0	1	1	3	5	1
6.12	456	-14.06	0.78	38	14	9	0	0	0	0	0	0	0	0	3	4	7	0	0
6.01	406	-14.52	0.49	24	7	8	0	0	0	0	0	0	0	3	2	2	0	0	0
5.88	357	-14.87	0.35	17	9	7	0	0	0	0	0	0	4	3	1	1	0	0	0
5.73	307	-15.62	0.16	8	3	6	0	0	0	0	0	0	2	1	0	0	0	0	0
5.55	258	-16.09	0.10	5	2	5	0	0	0	0	0	2	0	0	0	0	0	0	0
5.34	208	-16.60	0.06	3	2	4	0	0	0	0	2	0	0	0	0	0	0	0	0
5.07	159	-17.70	0.02	1	1	3	0	0	0	1	0	0	0	0	0	0	0	0	0
4.69	109	#N/A	0.00	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0

Use	0	1	2	3	4	5	6	7	8	9	10	11	12
N		0	0	1	2	2	6	7	7	9	13	9	
CumN		0	0	1	3	5	11	18	25	34	47	56	
F	0.00	0.00	0.02	0.06	0.10	0.23	0.37	0.51	0.70	0.96	1.15		
Weibit	#N/A	#N/A	-17.70	-16.60	-16.09	-15.30	-14.81	-14.48	-14.18	-13.85	-13.68		
tRet	60	109	159	208	258	307	357	406	456	505	555		
ln(tRet)	4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		



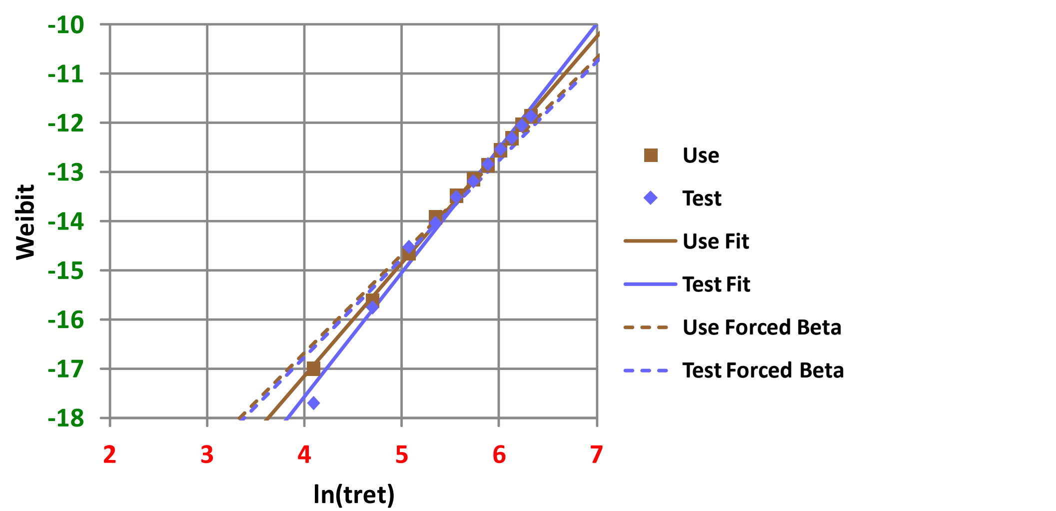
T/Vp/Vd = 115/.4/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	0	0	0	0	1	1	0	2	5	0
6.32	555	-12.80	2.77	135	22	11	0	0	0	0	0	0	0	0	0	1	5	8	8
6.23	505	-12.97	2.32	113	23	10	0	0	0	0	0	0	0	0	1	7	5	8	2
6.12	456	-13.20	1.85	90	17	9	0	0	0	0	0	0	0	1	7	3	6	0	0
6.01	406	-13.41	1.50	73	21	8	0	0	0	0	0	0	0	6	8	7	0	0	0
5.88	357	-13.75	1.07	52	12	7	0	0	0	0	0	0	6	2	2	1	1	0	0
5.73	307	-14.01	0.82	40	17	6	0	0	0	0	0	2	7	6	2	0	0	0	0
5.55	258	-14.57	0.47	23	15	5	0	0	0	0	4	8	3	0	0	0	0	0	0
5.34	208	-15.62	0.16	8	3	4	0	0	0	0	2	1	0	0	0	0	0	0	0
5.07	159	-16.09	0.10	5	4	3	0	0	0	3	1	0	0	0	0	0	0	0	0
4.69	109	-17.70	0.02	1	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								1	0	3	7	11	16	16	21	19	19	21	N
								1	1	4	11	22	38	54	75	94	113	134	CumN
								0.02	0.02	0.08	0.23	0.45	0.78	1.11	1.54	1.93	2.32	2.75	F
								-17.70	-17.70	-16.32	-15.30	-14.61	-14.06	-13.71	-13.38	-13.16	-12.97	-12.80	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



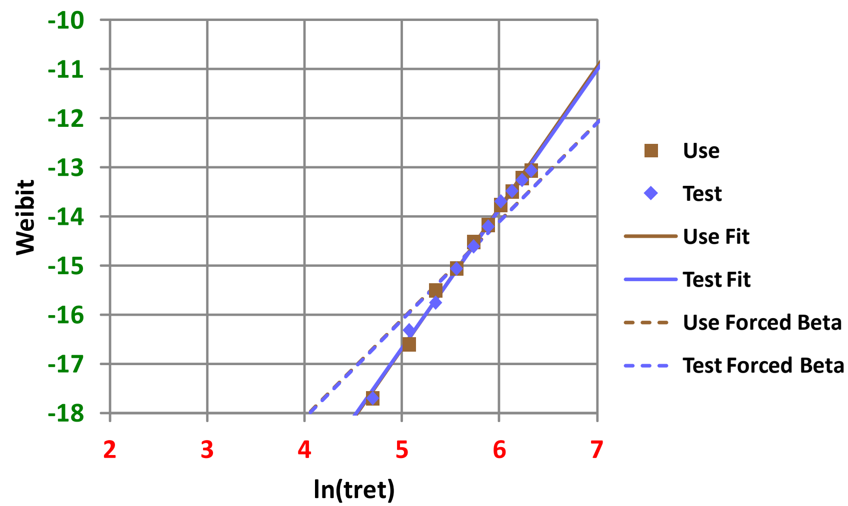
T/Vp/Vd = 125/.4/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	0	0	0	0	0	3	12	0	
6.32	555	-11.86	7.04	343	59	11	0	0	0	0	0	0	0	0	0	0	28	15	16	
6.23	505	-12.05	5.83	284	64	10	0	0	0	0	0	0	0	0	1	12	23	27	1	
6.12	456	-12.31	4.51	220	45	9	0	0	0	0	0	0	0	1	10	19	14	1	0	
6.01	406	-12.54	3.59	175	46	8	0	0	0	0	0	0	0	8	22	15	1	0	0	
5.88	357	-12.84	2.65	129	38	7	0	0	0	0	0	1	7	17	12	1	0	0	0	
5.73	307	-13.19	1.87	91	25	6	0	0	0	0	0	5	15	5	0	0	0	0	0	
5.55	258	-13.51	1.35	66	27	5	0	0	0	1	5	16	5	0	0	0	0	0	0	
5.34	208	-14.04	0.80	39	15	4	0	0	0	0	13	2	0	0	0	0	0	0	0	
5.07	159	-14.52	0.49	24	17	3	0	0	2	11	4	0	0	0	0	0	0	0	0	
4.69	109	-15.76	0.14	7	6	2	0	1	4	1	0	0	0	0	0	0	0	0	0	
4.09	60	-17.70	0.02	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								2	6	13	23	24	27	31	45	47	69	55		N
								2	8	21	44	68	95	126	171	218	287	342		CumN
								0.04	0.16	0.43	0.90	1.39	1.95	2.58	3.51	4.47	5.89	7.02		F
								-17.01	-15.62	-14.66	-13.92	-13.48	-13.15	-12.87	-12.56	-12.32	-12.04	-11.87		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



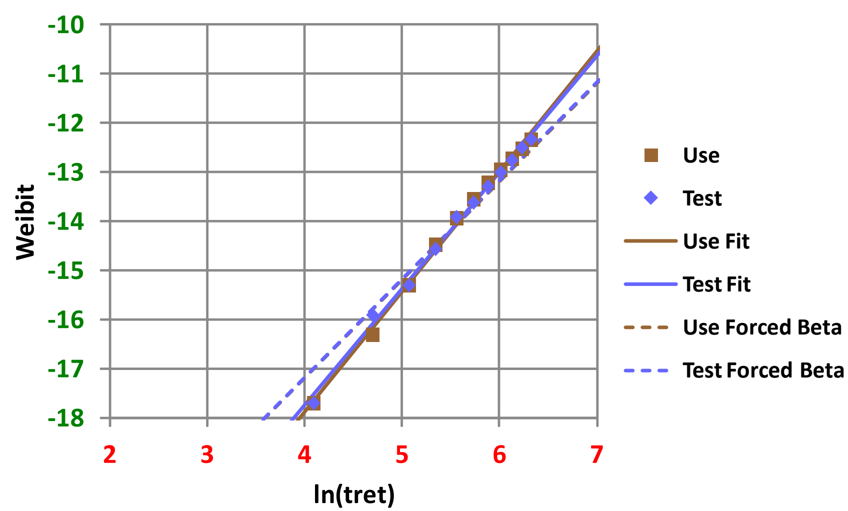
T/Vp/Vd = 105/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	0	0	0	0	0	5	0
6.32	555	-13.07	2.11	103	18	11	0	0	0	0	0	0	0	0	0	0	9	6	3
6.23	505	-13.26	1.74	85	17	10	0	0	0	0	0	0	0	0	1	3	8	4	1
6.12	456	-13.48	1.39	68	13	9	0	0	0	0	0	0	0	0	1	7	4	0	1
6.01	406	-13.69	1.13	55	22	8	0	0	0	0	0	0	1	3	12	6	0	0	0
5.88	357	-14.21	0.68	33	11	7	0	0	0	0	0	0	1	7	3	0	0	0	0
5.73	307	-14.61	0.45	22	8	6	0	0	0	0	0	0	8	0	0	0	0	0	0
5.55	258	-15.06	0.29	14	7	5	0	0	0	0	2	5	0	0	0	0	0	0	0
5.34	208	-15.76	0.14	7	3	4	0	0	0	0	3	0	0	0	0	0	0	0	0
5.07	159	-16.32	0.08	4	3	3	0	0	0	2	1	0	0	0	0	0	0	0	0
4.69	109	-17.70	0.02	1	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								0	1	2	6	5	10	10	17	16	21	15	N
								0	1	3	9	14	24	34	51	67	88	103	CumN
								0.00	0.02	0.06	0.18	0.29	0.49	0.70	1.05	1.37	1.81	2.11	F
								#N/A	-17.70	-16.60	-15.50	-15.06	-14.52	-14.18	-13.77	-13.50	-13.22	-13.07	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



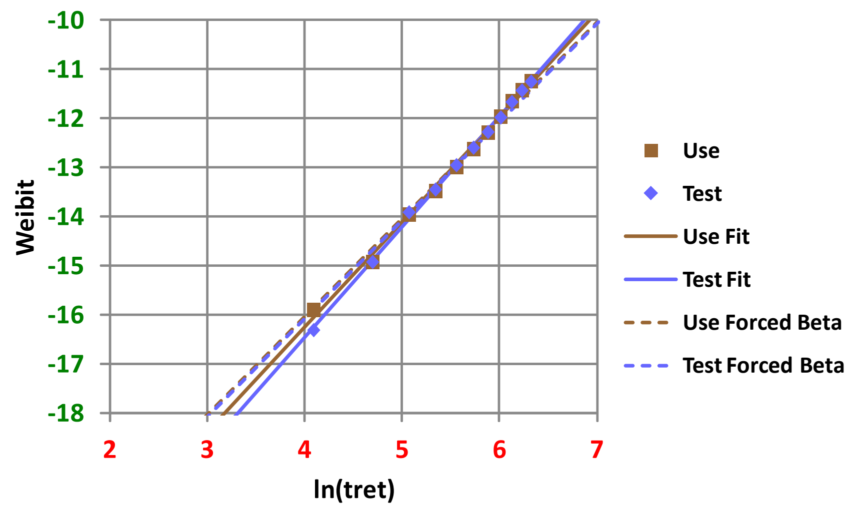
T/Vp/Vd = 115/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	1	0	1	0	0	0	2	0
6.32	555	-12.34	4.39	214	33	11	0	0	0	0	0	0	0	0	0	2	6	21	4
6.23	505	-12.50	3.71	181	41	10	0	0	0	0	0	0	0	0	2	7	19	12	1
6.12	456	-12.76	2.87	140	31	9	0	0	0	0	0	0	0	1	10	11	7	2	0
6.01	406	-13.01	2.24	109	27	8	0	0	0	0	0	0	0	10	8	9	0	0	0
5.88	357	-13.30	1.68	82	23	7	0	0	0	0	0	0	3	13	7	0	0	0	0
5.73	307	-13.62	1.21	59	15	6	0	0	0	0	0	3	12	0	0	0	0	0	0
5.55	258	-13.92	0.90	44	21	5	0	0	0	0	4	12	5	0	0	0	0	0	0
5.34	208	-14.57	0.47	23	12	4	0	0	0	1	9	2	0	0	0	0	0	0	0
5.07	159	-15.30	0.23	11	5	3	0	0	0	4	1	0	0	0	0	0	0	0	0
4.69	109	-15.91	0.12	6	5	2	0	0	3	2	0	0	0	0	0	0	0	0	0
4.09	60	-17.70	0.02	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								1	3	7	14	18	20	25	27	29	32	37	N
								1	4	11	25	43	63	88	115	144	176	213	CumN
								0.02	0.08	0.23	0.51	0.88	1.29	1.81	2.36	2.95	3.61	4.37	F
								-17.70	-16.32	-15.30	-14.48	-13.94	-13.56	-13.22	-12.96	-12.73	-12.53	-12.34	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



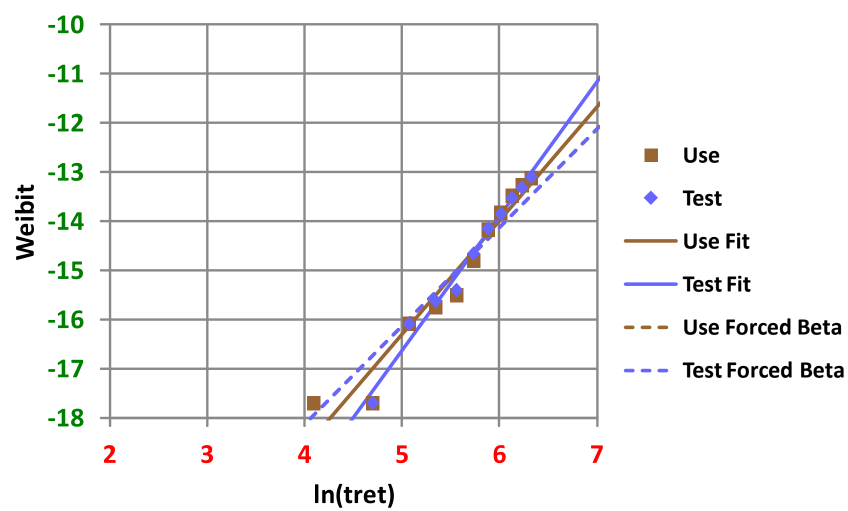
T/Vp/Vd = 125/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	1	0	0	0	0	3	27	0	
6.32	555	-11.26	12.92	630	104	**11**	0	0	0	0	0	0	0	0	1	5	36	39	23	
6.23	505	-11.44	10.79	526	110	**10**	0	0	0	0	0	0	0	0	3	33	39	35	0	
6.12	456	-11.67	8.53	416	112	**9**	0	0	0	0	0	0	0	2	27	48	28	6	1	
6.01	406	-11.99	6.24	304	78	**8**	0	0	0	0	0	0	0	17	36	24	0	1	0	
5.88	357	-12.28	4.64	226	62	**7**	0	0	0	0	0	0	8	32	19	2	1	0	0	
5.73	307	-12.60	3.36	164	49	**6**	0	0	0	0	0	8	28	12	0	0	0	0	1	
5.55	258	-12.96	2.36	115	45	**5**	0	0	0	0	4	28	13	0	0	0	0	0	0	
5.34	208	-13.45	1.44	70	26	**4**	0	0	0	1	19	6	0	0	0	0	0	0	0	
5.07	159	-13.92	0.90	44	28	**3**	0	0	2	23	2	0	0	0	0	0	0	0	1	
4.69	109	-14.93	0.33	16	12	**2**	0	2	8	2	0	0	0	0	0	0	0	0	0	
4.09	60	-16.32	0.08	4	4	**1**	0	4	0	0	0	0	0	0	0	0	0	0	0	
						0	1	0	0	0	0	0	0	0	0	0	0	0	0	
							0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Use**
								6	10	26	26	43	49	63	86	112	107	108		**N**
								6	16	42	68	111	160	223	309	421	528	636		**CumN**
								0.12	0.33	0.86	1.39	2.28	3.28	4.57	6.34	8.64	10.83	13.05		**F**
								-15.91	-14.93	-13.96	-13.48	-12.99	-12.63	-12.30	-11.97	-11.66	-11.43	-11.25		**Weibit**
								60	109	159	208	258	307	357	406	456	505	555		**tRet**
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		**ln(tRet)**



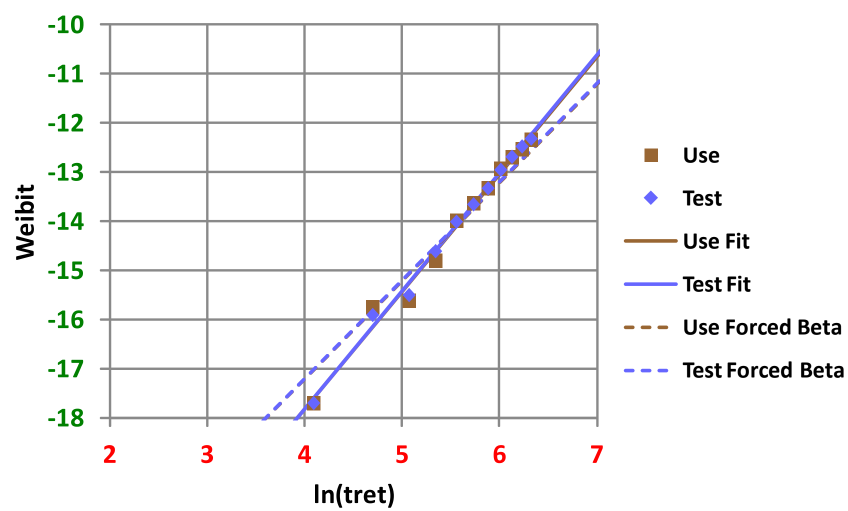
T/Vp/Vd = 105/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	0	0	0	4	0	
6.32	555	-13.10	2.05	100	20	11	0	0	0	0	0	0	0	0	0	1	10	4	5	
6.23	505	-13.32	1.64	80	15	10	0	0	0	0	0	0	0	0	2	5	2	4	2	
6.12	456	-13.53	1.33	65	18	9	0	0	0	0	0	0	0	1	5	7	4	1	0	
6.01	406	-13.85	0.96	47	12	8	0	0	0	0	0	0	0	3	3	6	0	0	0	
5.88	357	-14.15	0.72	35	14	7	0	0	0	0	0	0	2	7	4	1	0	0	0	
5.73	307	-14.66	0.43	21	11	6	0	0	0	0	0	1	5	5	0	0	0	0	0	
5.55	258	-15.40	0.21	10	2	5	0	0	0	0	0	0	2	0	0	0	0	0	0	
5.34	208	-15.62	0.16	8	3	4	0	0	0	1	1	1	0	0	0	0	0	0	0	
5.07	159	-16.09	0.10	5	4	3	0	0	0	3	1	0	0	0	0	0	0	0	0	
4.69	109	-17.70	0.02	1	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0	
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								1	0	4	2	2	9	16	14	20	16	13		N
								1	1	5	7	9	18	34	48	68	84	97		CumN
								0.02	0.02	0.10	0.14	0.18	0.37	0.70	0.98	1.39	1.72	1.99		F
								-17.70	-17.70	-16.09	-15.76	-15.50	-14.81	-14.18	-13.83	-13.48	-13.27	-13.13		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



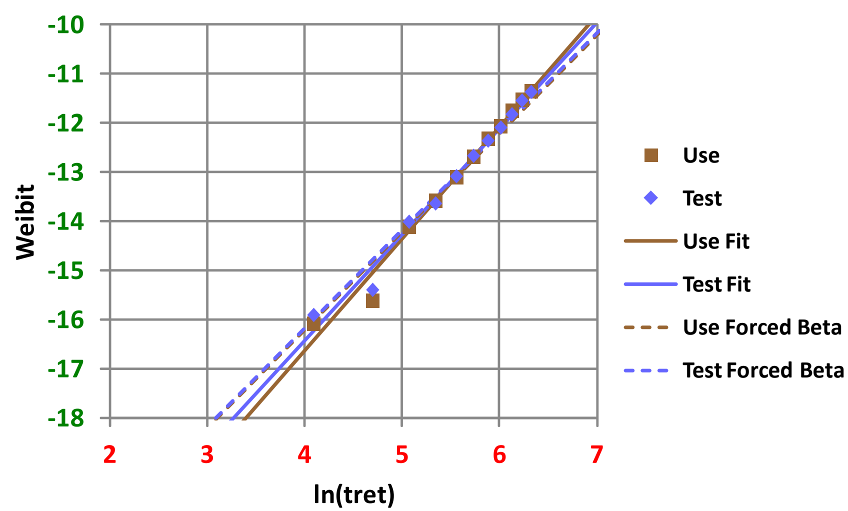
T/Vp/Vd = 115/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	1	0	0	0	2	4	0	
6.32	555	-12.32	4.45	217	32	**11**	0	0	0	0	0	0	0	1	0	1	7	13	10	
6.23	505	-12.48	3.79	185	35	**10**	0	0	0	0	0	0	0	0	1	9	7	16	2	
6.12	456	-12.69	3.08	150	35	**9**	0	0	0	0	0	0	0	2	12	9	9	3	0	
6.01	406	-12.96	2.36	115	36	**8**	0	0	0	0	0	0	0	6	17	11	1	1	0	
5.88	357	-13.33	1.62	79	22	**7**	0	0	0	0	0	0	8	5	9	0	0	0	0	
5.73	307	-13.66	1.17	57	17	**6**	0	0	0	0	0	4	5	7	0	0	0	0	1	
5.55	258	-14.01	0.82	40	18	**5**	0	0	0	0	2	13	3	0	0	0	0	0	0	
5.34	208	-14.61	0.45	22	13	**4**	0	0	0	0	7	6	0	0	0	0	0	0	0	
5.07	159	-15.50	0.18	9	3	**3**	0	0	1	1	1	0	0	0	0	0	0	0	0	
4.69	109	-15.91	0.12	6	5	**2**	0	0	5	0	0	0	0	0	0	0	0	0	0	
4.09	60	-17.70	0.02	1	1	**1**	0	1	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Use**
								1	6	1	10	23	17	21	39	30	26	37		**N**
								1	7	8	18	41	58	79	118	148	174	211		**CumN**
								0.02	0.14	0.16	0.37	0.84	1.19	1.62	2.42	3.04	3.57	4.33		**F**
								-17.70	-15.76	-15.62	-14.81	-13.99	-13.64	-13.33	-12.93	-12.71	-12.54	-12.35		**Weibit**
								60	109	159	208	258	307	357	406	456	505	555		**tRet**
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		**ln(tRet)**



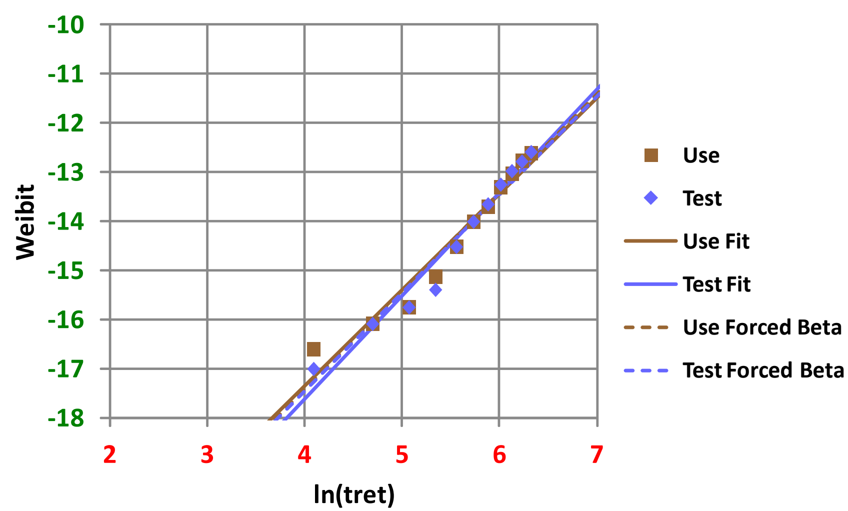
T/Vp/Vd = 125/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	1	0	1	0	7	23	0	
6.32	555	-11.37	11.49	560	89	11	0	0	0	0	1	0	0	0	0	2	31	**38**	17	
6.23	505	-11.55	9.66	471	115	10	0	0	0	0	0	0	0	1	3	38	**37**	29	7	
6.12	456	-11.83	7.30	356	84	9	0	0	0	0	0	0	0	3	22	**42**	17	0	0	
6.01	406	-12.10	5.58	272	64	8	0	0	0	0	0	0	0	19	**24**	19	2	0	0	
5.88	357	-12.36	4.27	208	55	7	0	0	0	0	0	2	12	**25**	14	2	0	0	0	
5.73	307	-12.67	3.14	153	52	6	0	0	0	0	0	8	**26**	18	0	0	0	0	0	
5.55	258	-13.09	2.07	101	43	5	0	0	0	0	6	**24**	12	0	0	0	1	0	0	
5.34	208	-13.64	1.19	58	18	4	0	0	0	3	**11**	4	0	0	0	0	0	0	0	
5.07	159	-14.01	0.82	40	30	3	0	0	0	**23**	7	0	0	0	0	0	0	0	0	
4.69	109	-15.40	0.21	10	4	2	0	0	**2**	2	0	0	0	0	0	0	0	0	0	
4.09	60	-15.91	0.12	6	6	1	0	**5**	1	0	0	0	0	0	0	0	0	0	0	
						0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								5	3	28	25	38	51	66	64	103	95	90		N
								5	8	36	61	99	150	216	280	383	478	568		CumN
								0.10	0.16	0.74	1.25	2.03	3.08	4.43	5.74	7.86	9.81	11.65		F
								-16.09	-15.62	-14.12	-13.59	-13.11	-12.69	-12.33	-12.07	-11.75	-11.53	-11.36		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



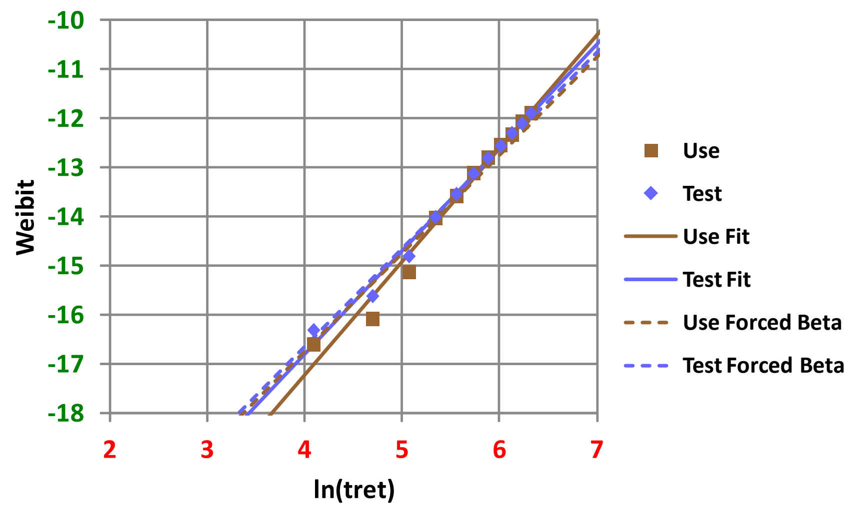
T/Vp/Vd = 105/.45/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	0	0	0	4	0	
6.32	555	-12.60	3.38	165	29	11	0	0	0	0	0	0	0	1	0	0	13	9	6	
6.23	505	-12.79	2.79	136	24	10	0	0	0	0	0	0	0	0	0	4	10	9	1	
6.12	456	-12.98	2.30	112	27	9	0	0	0	0	0	0	0	0	4	13	9	1	0	
6.01	406	-13.26	1.74	85	28	8	0	0	0	0	0	0	0	8	12	7	0	0	1	
5.88	357	-13.66	1.17	57	17	7	0	0	0	0	0	0	1	4	11	1	0	0	0	
5.73	307	-14.01	0.82	40	16	6	0	0	0	0	0	4	11	1	0	0	0	0	0	
5.55	258	-14.52	0.49	24	14	5	0	0	0	0	3	7	4	0	0	0	0	0	0	
5.34	208	-15.40	0.21	10	3	4	0	0	0	0	3	0	0	0	0	0	0	0	0	
5.07	159	-15.76	0.14	7	2	3	0	0	0	2	0	0	0	0	0	0	0	0	0	
4.69	109	-16.09	0.10	5	3	2	0	1	2	0	0	0	0	0	0	0	0	0	0	
4.09	60	-17.01	0.04	2	2	1	0	2	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	2	2	6	11	16	14	27	25	32	23		N
								3	5	7	13	24	40	54	81	106	138	161		CumN
								0.06	0.10	0.14	0.27	0.49	0.82	1.11	1.66	2.17	2.83	3.30		F
								-16.60	-16.09	-15.76	-15.14	-14.52	-14.01	-13.71	-13.31	-13.04	-12.77	-12.62		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



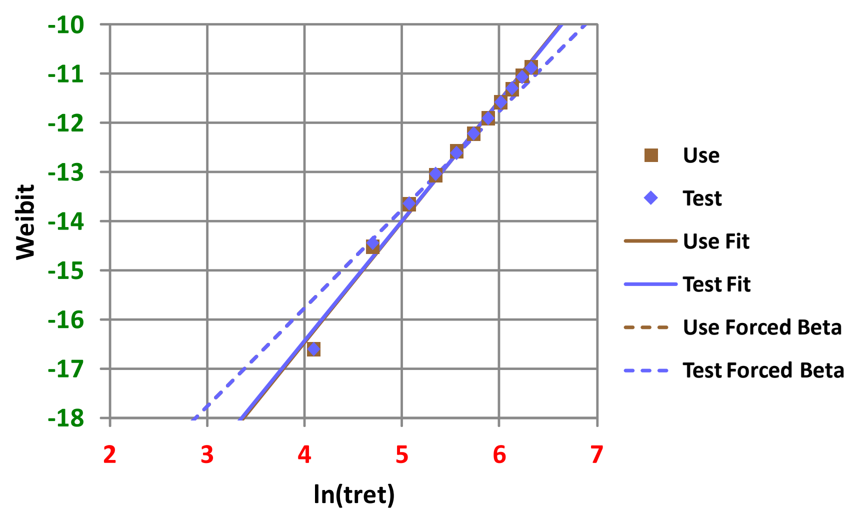
T/Vp/Vd = 115/.45/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	1	0	0	0	0	4	9	0
6.32	555	-11.90	6.81	332	63	11	0	0	0	0	0	0	0	0	0	0	26	27	10
6.23	505	-12.11	5.52	269	48	10	0	0	0	0	0	0	0	0	0	15	19	12	2
6.12	456	-12.30	4.53	221	50	9	0	0	0	0	0	0	0	2	9	18	16	4	1
6.01	406	-12.56	3.51	171	38	8	0	0	0	0	0	0	0	11	18	8	1	0	0
5.88	357	-12.81	2.73	133	36	7	0	0	0	0	0	0	9	15	12	0	0	0	0
5.73	307	-13.13	1.99	97	33	6	0	0	0	0	1	4	20	8	0	0	0	0	0
5.55	258	-13.54	1.31	64	24	5	0	0	0	0	2	14	8	0	0	0	0	0	0
5.34	208	-14.01	0.82	40	22	4	0	0	0	2	17	3	0	0	0	0	0	0	0
5.07	159	-14.81	0.37	18	10	3	0	0	0	4	6	0	0	0	0	0	0	0	0
4.69	109	-15.62	0.16	8	4	2	0	0	2	2	0	0	0	0	0	0	0	0	0
4.09	60	-16.32	0.08	4	4	1	1	3	0	0	0	0	0	0	0	0	0	0	0
						0	1	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								3	2	8	26	22	37	36	39	41	66	52	N
								3	5	13	39	61	98	134	173	214	280	332	CumN
								0.06	0.10	0.27	0.80	1.25	2.01	2.75	3.55	4.39	5.74	6.81	F
								-16.60	-16.09	-15.14	-14.04	-13.59	-13.12	-12.80	-12.55	-12.34	-12.07	-11.90	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



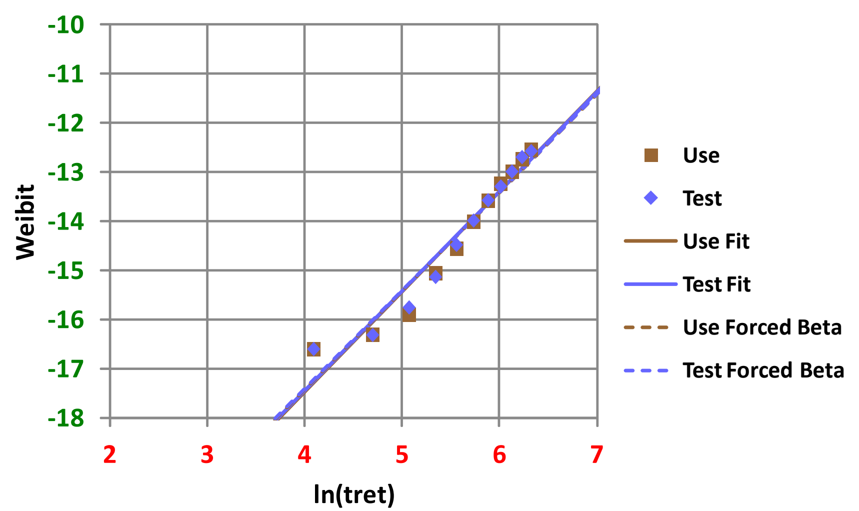
T/Vp/Vd = 125/.45/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	1	0	0	1	0	1	0	10	42	0
6.32	555	-10.88	18.75	914	154	11	0	0	0	0	0	0	0	0	0	3	66	49	36
6.23	505	-11.07	15.59	760	155	10	0	0	0	0	0	0	0	0	3	39	58	48	7
6.12	456	-11.30	12.41	605	149	9	0	0	0	0	0	0	1	1	41	53	47	6	0
6.01	406	-11.58	9.35	456	127	8	0	0	0	0	0	0	0	28	53	39	7	0	0
5.88	357	-11.91	6.75	329	88	7	0	0	0	0	0	0	21	37	27	1	0	0	2
5.73	307	-12.22	4.94	241	79	6	0	0	0	0	0	20	34	21	4	0	0	0	0
5.55	258	-12.61	3.32	162	56	5	0	0	0	0	9	31	15	1	0	0	0	0	0
5.34	208	-13.04	2.17	106	48	4	0	0	0	5	30	13	0	0	0	0	0	0	0
5.07	159	-13.64	1.19	58	32	3	0	0	3	22	7	0	0	0	0	0	0	0	0
4.69	109	-14.44	0.53	26	23	2	0	0	18	5	0	0	0	0	0	0	0	0	0
4.09	60	-16.60	0.06	3	3	1	0	3	0	0	0	0	0	0	0	0	0	0	0
						0	4	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	21	33	46	64	72	88	129	135	188	145		N
								3	24	57	103	167	239	327	456	591	779	924		CumN
								0.06	0.49	1.17	2.11	3.43	4.90	6.71	9.35	12.12	15.98	18.95		F
								-16.60	-14.52	-13.66	-13.07	-12.58	-12.23	-11.91	-11.58	-11.32	-11.04	-10.87		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



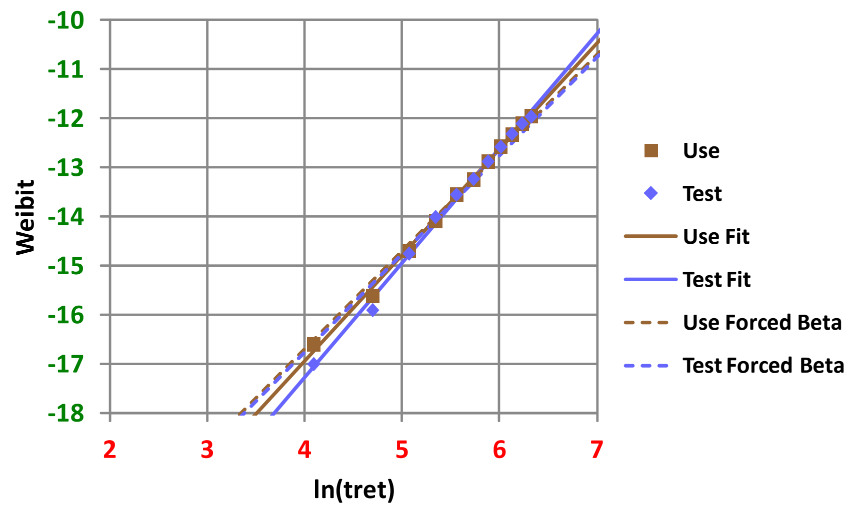
T/Vp/Vd = 105/.4/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	0	0	0	0	0	8	0
6.32	555	-12.58	3.45	168	19	11	0	0	0	0	0	0	0	0	0	0	3	13	3
6.23	505	-12.70	3.06	149	38	10	0	0	0	0	0	0	0	0	0	10	18	10	0
6.12	456	-12.99	2.28	111	29	9	0	0	0	0	0	0	0	1	9	10	9	0	0
6.01	406	-13.30	1.68	82	20	8	0	0	0	0	0	0	0	5	10	4	1	0	0
5.88	357	-13.58	1.27	62	21	7	0	0	0	0	0	0	5	10	6	0	0	0	0
5.73	307	-13.99	0.84	41	16	6	0	0	0	0	0	4	6	5	1	0	0	0	0
5.55	258	-14.48	0.51	25	12	5	0	0	0	0	2	4	6	0	0	0	0	0	0
5.34	208	-15.14	0.27	13	6	4	0	0	0	0	5	1	0	0	0	0	0	0	0
5.07	159	-15.76	0.14	7	3	3	0	0	0	2	1	0	0	0	0	0	0	0	0
4.69	109	-16.32	0.08	4	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0
4.09	60	-16.60	0.06	3	3	1	0	3	0	0	0	0	0	0	0	0	0	0	0
						0	1	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	1	2	8	9	17	21	26	24	31	31		N
								3	4	6	14	23	40	61	87	111	142	173		CumN
								0.06	0.08	0.12	0.29	0.47	0.82	1.25	1.78	2.28	2.91	3.55		F
								-16.60	-16.32	-15.91	-15.06	-14.57	-14.01	-13.59	-13.24	-12.99	-12.75	-12.55		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



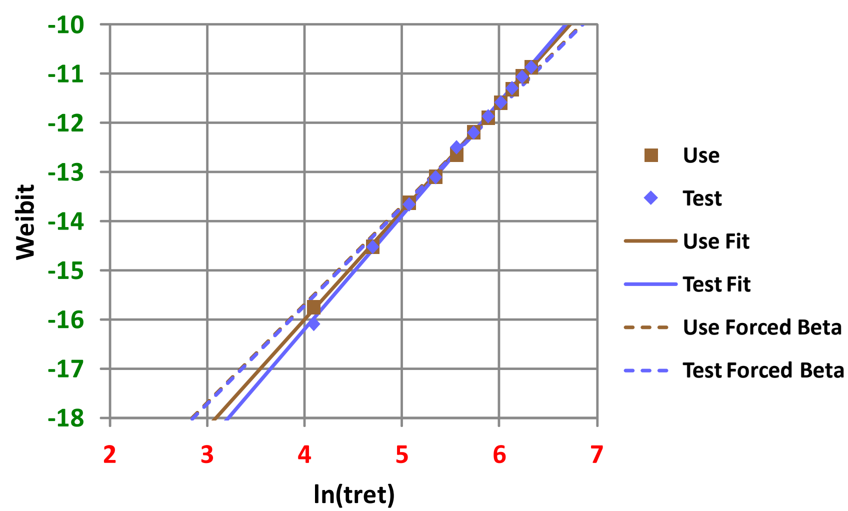
T/Vp/Vd = 115/.4/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	1	0	0	0	1	2	16	0	
6.32	555	-11.97	6.34	309	41	11	0	0	0	0	0	0	0	0	1	2	15	15	8	
6.23	505	-12.11	5.50	268	50	10	0	0	0	0	0	0	0	0	0	15	15	13	7	
6.12	456	-12.32	4.47	218	51	9	0	0	0	0	0	0	0	0	15	16	17	2	1	
6.01	406	-12.58	3.43	167	43	8	0	0	0	0	0	0	0	14	16	10	3	0	0	
5.88	357	-12.88	2.54	124	37	7	0	0	0	0	0	0	8	13	12	3	0	0	1	
5.73	307	-13.24	1.78	87	24	6	0	0	0	0	0	4	9	11	0	0	0	0	0	
5.55	258	-13.56	1.29	63	23	5	0	0	0	0	2	15	6	0	0	0	0	0	0	
5.34	208	-14.01	0.82	40	21	4	0	0	0	2	13	6	0	0	0	0	0	0	0	
5.07	159	-14.76	0.39	19	13	3	0	0	1	10	2	0	0	0	0	0	0	0	0	
4.69	109	-15.91	0.12	6	4	2	0	0	4	0	0	0	0	0	0	0	0	0	0	
4.09	60	-17.01	0.04	2	2	1	0	2	0	0	0	0	0	0	0	0	0	0	0	
						0	2	1	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	5	12	17	26	23	38	44	47	52	46		N
								3	8	20	37	63	86	124	168	215	267	313		CumN
								0.06	0.16	0.41	0.76	1.29	1.76	2.54	3.45	4.41	5.48	6.42		F
								-16.60	-15.62	-14.71	-14.09	-13.56	-13.25	-12.88	-12.58	-12.33	-12.11	-11.96		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



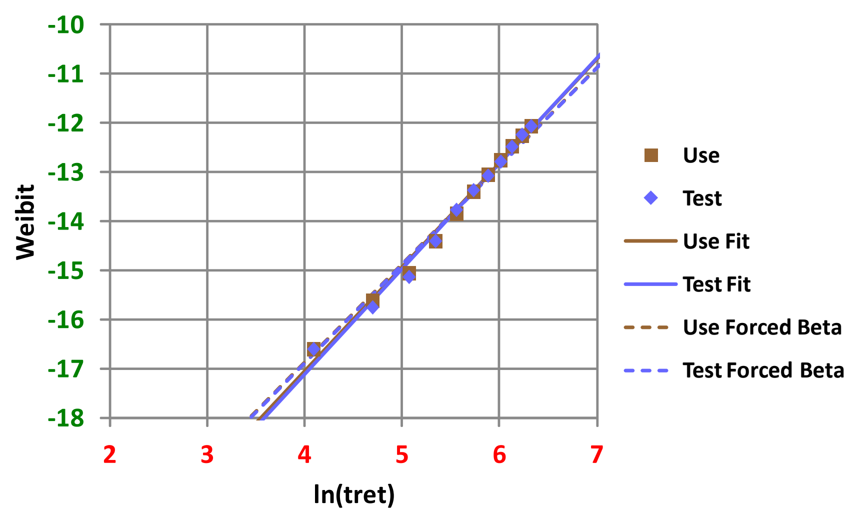
T/Vp/Vd = 125/.4/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	1	0	0	1	0	2	1	6	38	0
6.32	555	-10.87	19.06	929	167	**11**	0	0	0	0	0	0	0	0	2	7	61	**61**	36
6.23	505	-11.07	15.63	762	156	**10**	0	0	0	0	0	0	0	0	5	36	**59**	49	7
6.12	456	-11.30	12.43	606	152	**9**	0	0	0	0	0	0	0	3	38	**54**	48	4	5
6.01	406	-11.58	9.31	454	112	**8**	0	0	0	0	0	0	2	24	**38**	39	8	0	1
5.88	357	-11.87	7.02	342	97	**7**	0	0	0	0	0	0	22	**39**	34	1	0	0	1
5.73	307	-12.20	5.03	245	63	**6**	0	0	0	0	0	10	**32**	19	0	0	0	0	2
5.55	258	-12.50	3.73	182	83	**5**	0	0	0	1	6	**40**	33	1	1	0	0	0	1
5.34	208	-13.11	2.03	99	42	**4**	0	0	0	7	**28**	6	0	0	0	0	0	0	1
5.07	159	-13.66	1.17	57	33	**3**	0	0	5	**21**	7	0	0	0	0	0	0	0	0
4.69	109	-14.52	0.49	24	19	**2**	0	4	**10**	5	0	0	0	0	0	0	0	0	0
4.09	60	-16.09	0.10	5	5	**1**	0	**3**	2	0	0	0	0	0	0	0	0	0	0
						0	**4**	0	0	0	0	0	0	0	0	0	0	0	0
							0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12** Use
								7	17	35	41	56	90	86	120	138	182	152	N
								7	24	59	100	156	246	332	452	590	772	924	CumN
								0.14	0.49	1.21	2.05	3.20	5.05	6.81	9.27	12.10	15.84	18.95	F
								-15.76	-14.52	-13.62	-13.10	-12.65	-12.20	-11.90	-11.59	-11.32	-11.05	-10.87	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



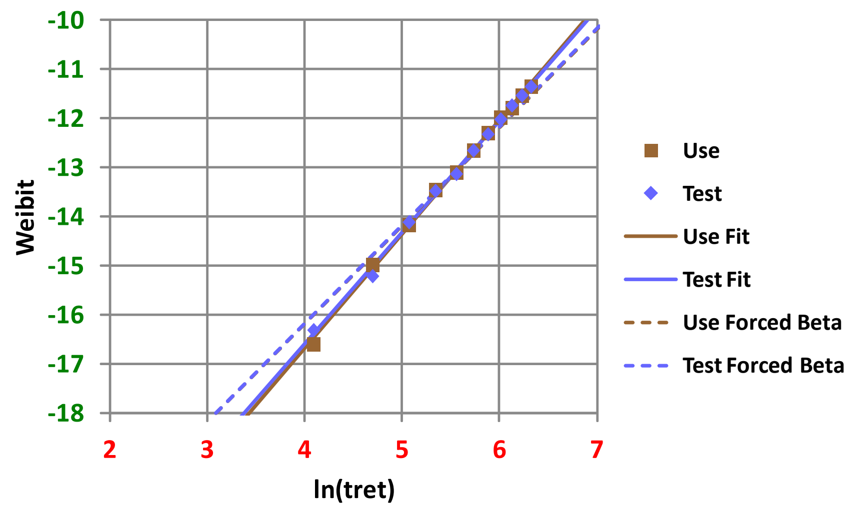
T/Vp/Vd = 105/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	1	0	0	17	0	
6.32	555	-12.07	5.74	280	45	11	0	0	0	0	0	0	0	0	0	5	8	17	15	
6.23	505	-12.24	4.82	235	51	10	0	0	0	0	0	0	0	1	1	9	23	14	3	
6.12	456	-12.49	3.77	184	48	9	0	0	0	0	0	0	0	1	12	19	13	3	0	
6.01	406	-12.79	2.79	136	34	8	0	0	0	0	0	0	0	8	15	11	0	0	0	
5.88	357	-13.08	2.09	102	26	7	0	0	0	0	0	0	6	12	7	1	0	0	0	
5.73	307	-13.37	1.56	76	25	6	0	0	0	0	0	3	15	6	0	0	0	0	1	
5.55	258	-13.77	1.05	51	24	5	0	0	0	0	2	14	6	2	0	0	0	0	0	
5.34	208	-14.41	0.55	27	14	4	0	0	0	0	11	3	0	0	0	0	0	0	0	
5.07	159	-15.14	0.27	13	6	3	0	0	1	5	0	0	0	0	0	0	0	0	0	
4.69	109	-15.76	0.14	7	4	2	0	0	3	1	0	0	0	0	0	0	0	0	0	
4.09	60	-16.60	0.06	3	3	1	0	2	1	0	0	0	0	0	0	0	0	0	0	
						0	1	1	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	5	6	13	20	27	30	36	45	44	51		N
								3	8	14	27	47	74	104	140	185	229	280		CumN
								0.06	0.16	0.29	0.55	0.96	1.52	2.13	2.87	3.79	4.70	5.74		F
								-16.60	-15.62	-15.06	-14.41	-13.85	-13.40	-13.06	-12.76	-12.48	-12.27	-12.07		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



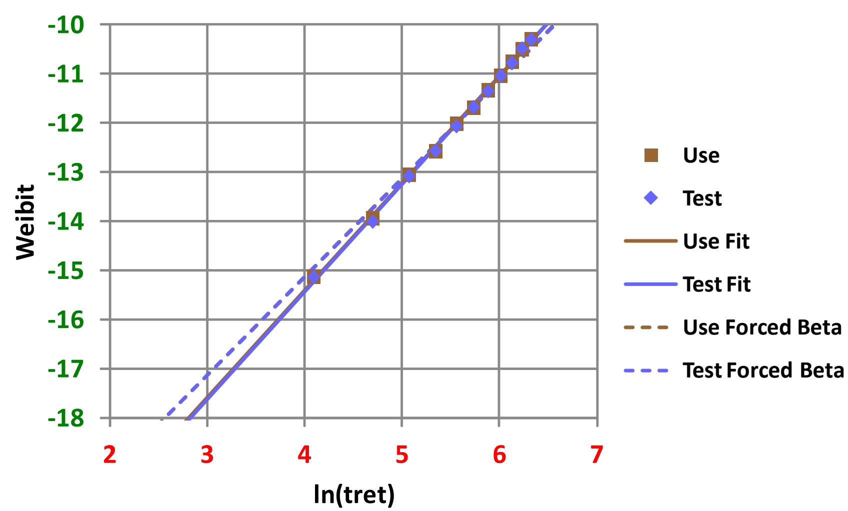
T/Vp/Vd = 115/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	1	0	0	0	0	0	1	10	23	0	
6.32	555	-11.36	11.67	569	94	11	0	0	0	0	0	0	0	1	1	1	32	37	22	
6.23	505	-11.54	9.74	475	87	10	0	0	0	0	0	0	0	0	2	20	25	30	10	
6.12	456	-11.74	7.96	388	96	9	0	0	0	0	0	0	0	1	29	26	32	8	0	
6.01	406	-12.03	5.99	292	76	8	0	0	0	0	0	0	2	18	35	15	6	0	0	
5.88	357	-12.33	4.43	216	61	7	0	0	0	0	0	0	12	32	16	0	1	0	0	
5.73	307	-12.66	3.18	155	59	6	0	0	0	0	1	10	34	13	0	0	0	0	1	
5.55	258	-13.14	1.97	96	28	5	0	0	0	0	6	15	7	0	0	0	0	0	0	
5.34	208	-13.48	1.39	68	32	4	0	0	0	2	25	5	0	0	0	0	0	0	0	
5.07	159	-14.12	0.74	36	24	3	0	0	4	16	3	0	0	1	0	0	0	0	0	
4.69	109	-15.22	0.25	12	8	2	0	1	7	0	0	0	0	0	0	0	0	0	0	
4.09	60	-16.32	0.08	4	4	1	1	2	1	0	0	0	0	0	0	0	0	0	0	
						0	4	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	12	19	35	30	55	66	83	63	106	98		N
								3	15	34	69	99	154	220	303	366	472	570		CumN
								0.06	0.31	0.70	1.42	2.03	3.16	4.51	6.22	7.51	9.68	11.69		F
								-16.60	-14.99	-14.18	-13.47	-13.11	-12.67	-12.31	-11.99	-11.80	-11.55	-11.36		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



T/Vp/Vd = 125/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	0	1	1	1	1	18	69	0	
6.32	555	-10.30	33.50	1633	273	11	0	0	0	0	0	0	0	0	1	14	104	95	59	
6.23	505	-10.49	27.90	1360	343	10	0	0	1	0	0	0	1	2	10	97	92	120	20	
6.12	456	-10.78	20.86	1017	237	9	0	0	0	0	0	0	1	5	46	83	86	14	2	
6.01	406	-11.04	16.00	780	211	8	0	0	0	0	0	1	4	59	80	56	6	2	3	
5.88	357	-11.36	11.67	569	156	7	0	0	0	0	0	1	29	68	53	2	1	2	0	
5.73	307	-11.68	8.47	413	134	6	0	0	0	0	0	36	56	38	2	1	1	0	0	
5.55	258	-12.07	5.72	279	109	5	0	0	0	0	15	71	20	2	1	0	0	0	0	
5.34	208	-12.57	3.49	170	69	4	0	0	0	12	38	18	0	0	0	1	0	0	0	
5.07	159	-13.09	2.07	101	61	3	0	0	8	43	9	0	0	0	0	0	0	0	1	
4.69	109	-14.01	0.82	40	27	2	0	1	19	6	0	0	0	0	1	0	0	0	0	
4.09	60	-15.14	0.27	13	13	1	0	11	2	0	0	0	0	0	0	0	0	0	0	
						0	5	1	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								13	30	61	63	127	112	175	195	255	308	302		N
								13	43	104	167	294	406	581	776	1031	1339	1641		CumN
								0.27	0.88	2.13	3.43	6.03	8.33	11.92	15.92	21.15	27.47	33.66		F
								-15.14	-13.94	-13.06	-12.58	-12.02	-11.70	-11.34	-11.05	-10.76	-10.50	-10.30		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



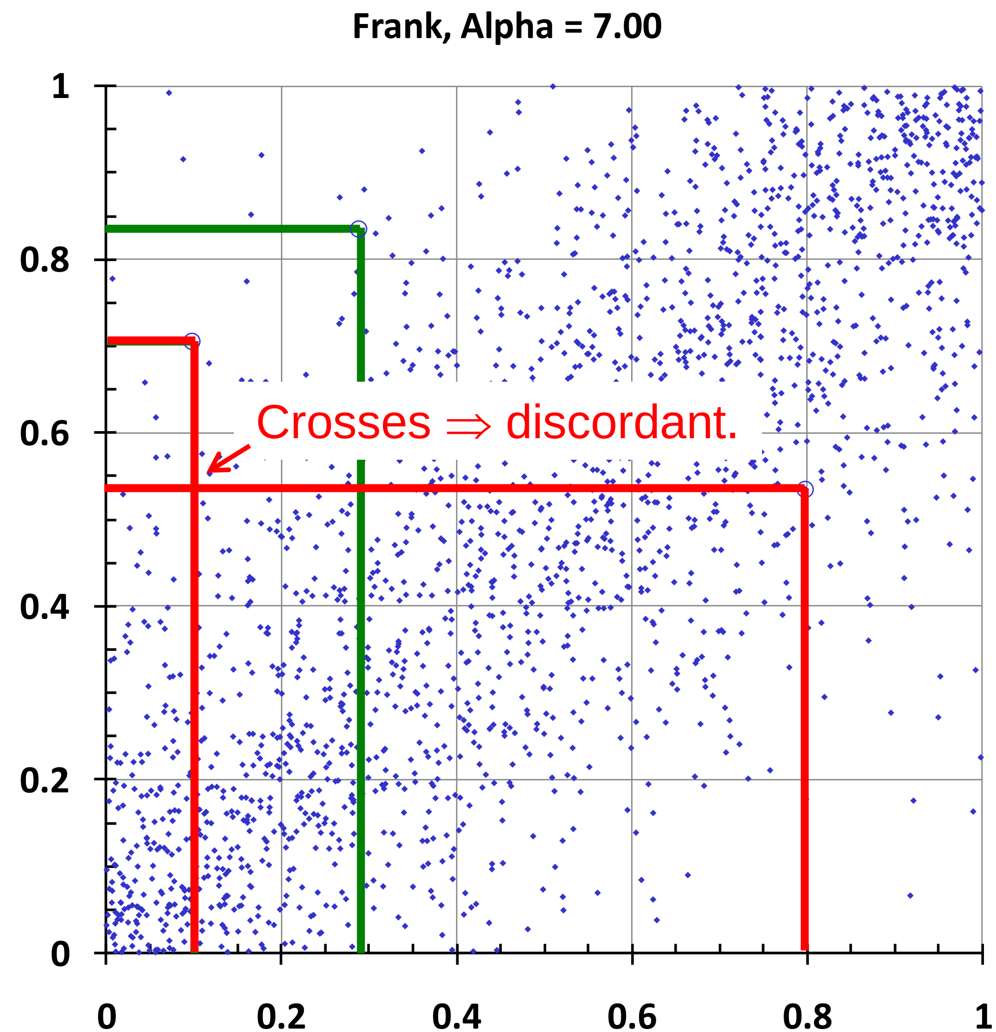
Measures of Dependence

- A single number that characterizes the “scatter” of data:
  - Perfect correlation: 1
  - Independence: 0
  - Perfect anti-correlation: -1
- Pearson’s correlation coefficient. $\rho = \frac{\sum_i (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_i (x_i - \mu_x)^2 (y_i - \mu_y)^2}}$
  - Correlation coefficient of *data*.
- Spearman’s Rho
  - Correlation coefficient of *ranks* of data.
  - Independent of marginal distributions.
- Kendall’s Tau
  - Next slide.
  - Independent of marginal distributions.

Kendall's Tau

- If there are *n* points in a plot like this, there are *n*(*n*-1)/2 pairs of points.
- Every pair may be classified as "concordant", or "discordant".
  - *c* is the number of concordant pairs.
  - *d* is the number of discordant pairs.

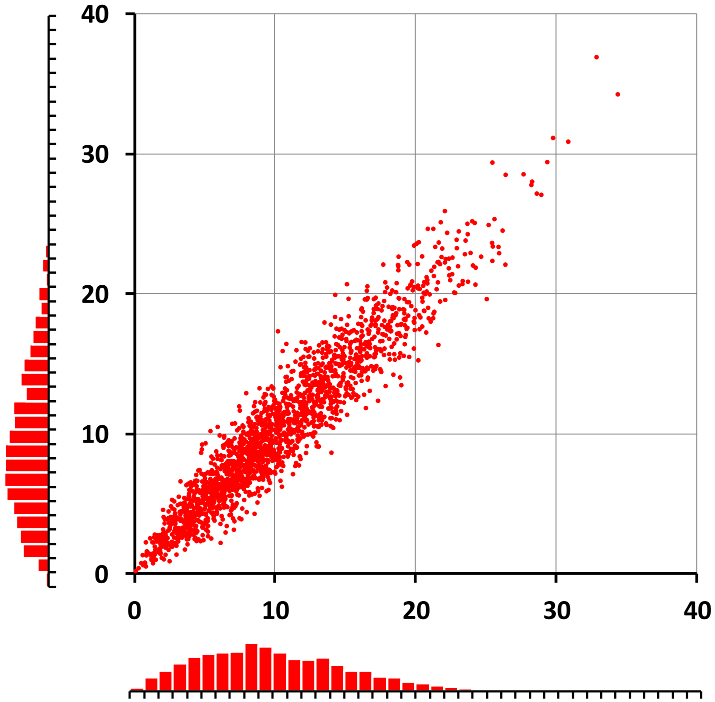
$$\tau = \frac{c-d}{c+d} = \frac{2(c-d)}{n(n-1)}$$

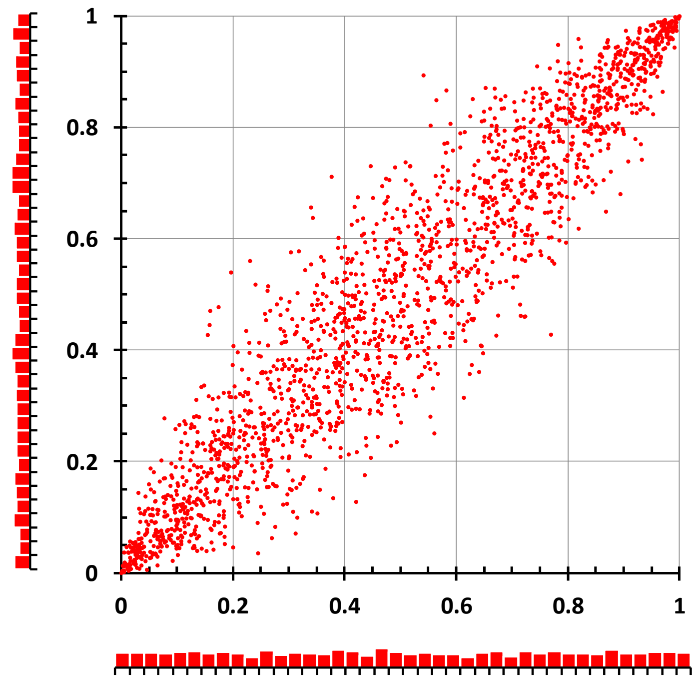


Rank Correlation from Correlation

- Correlation plot of ranks is the empirical copula.
- Kendall's Tau is a measure of miscorrelation (scatter).
- Tau depends only on ranks of data.

Margins
Weibull
α= 11.7
β = 2





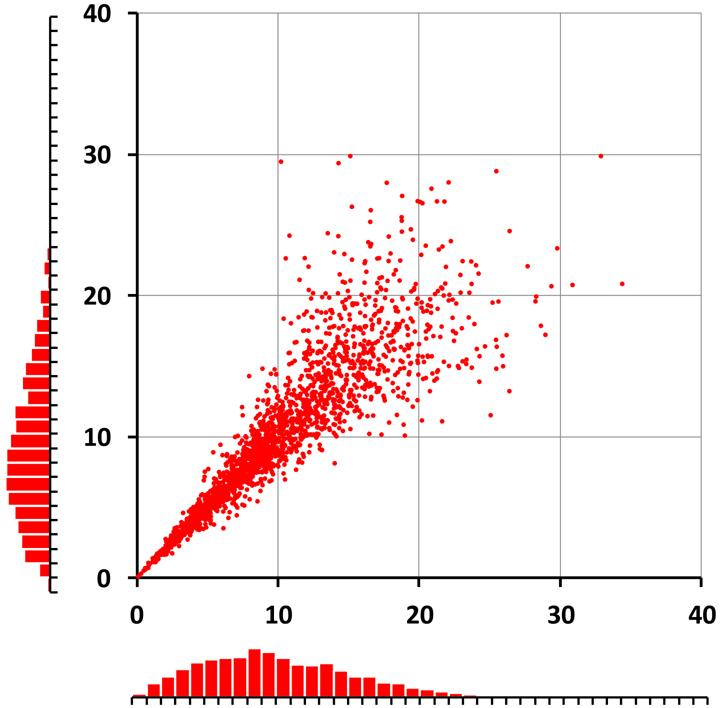
Copula
Gaussian
τ = 0.8
ρ = 0.951

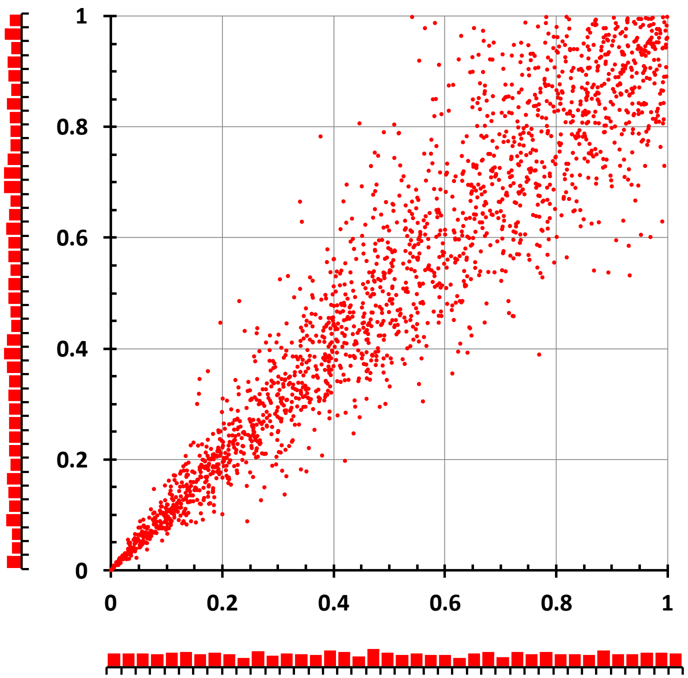
Synthesized data.

Rank Correlation from Correlation

- Another example with the same margins, but different copula (Clayton copula).
- Very similar to DRAM model (incl. parameter values).

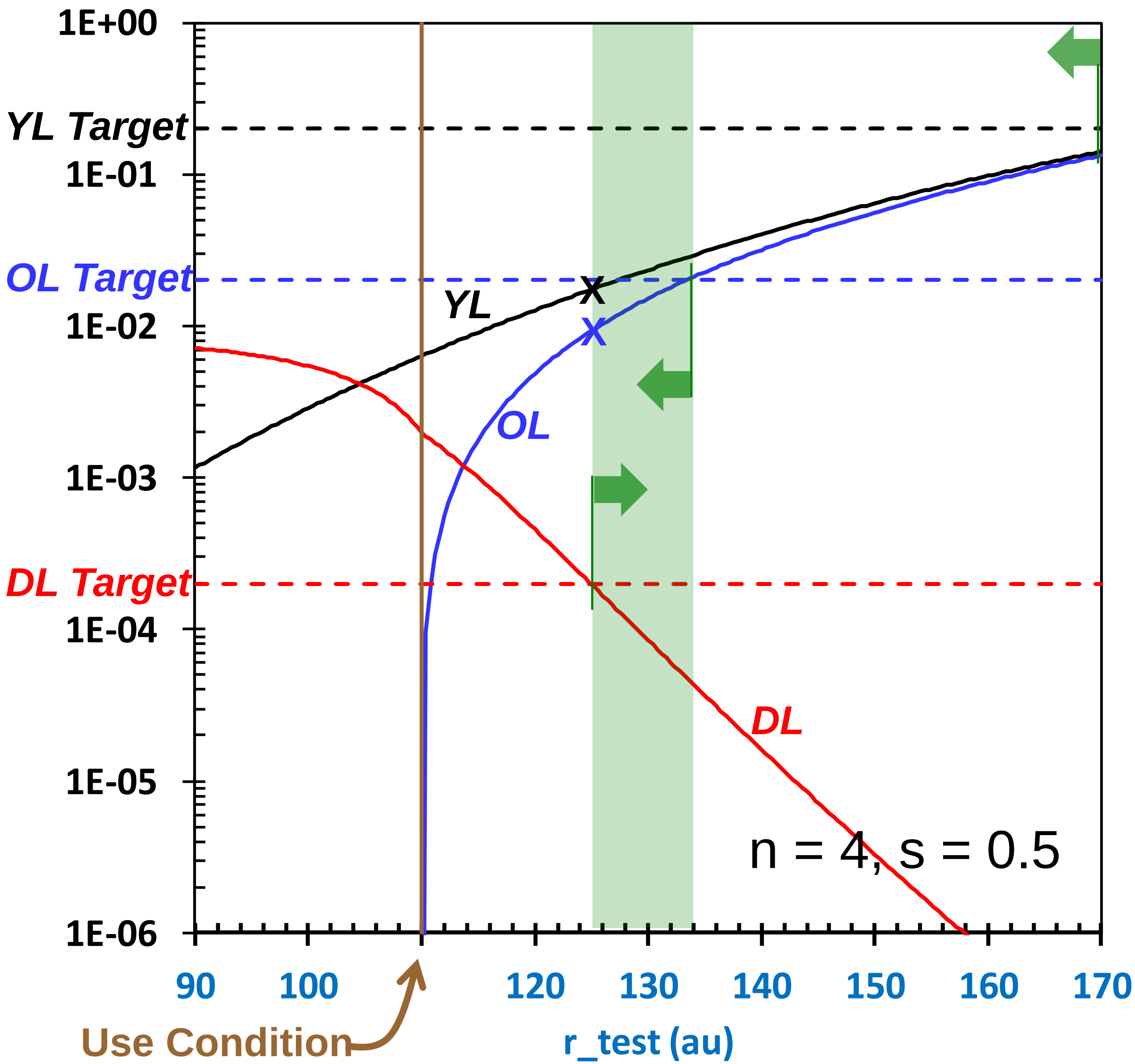
Margins
Weibull
α= 11.7
β = 2

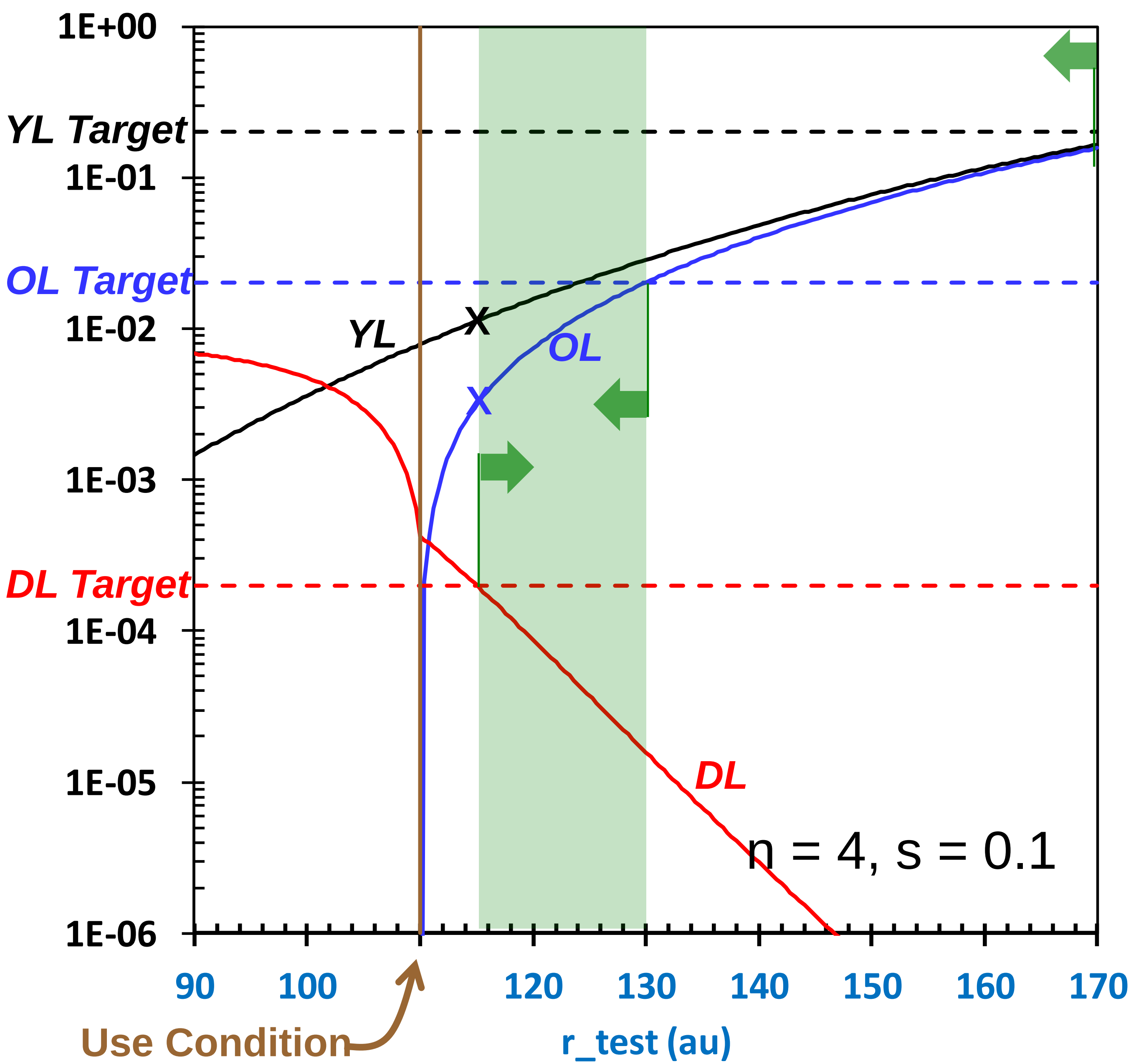




Copula
Clayton
τ = 0.8
θ = 8

Synthesized data.


1E+00
YL Target
1E-01
OL Target
1E-02
YL
X
X
OL
1E-03
DL Target
1E-04
DL
1E-05
n = 4, s = 0.5
1E-06
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
OL Target
1E-02
1E-03
DL Target
1E-04
1E-05
1E-06
YL
X
OL
X
DL
n = 4, s = 0.1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)