Copula Methods in Manufacturing Test

A DRAM Case Study

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Outline

- Integrated Circuit Design and Test Laboratory at PSU.
- Background
  - Motivation
  - Multinormal vs copula-based multivariate modeling
  - Survey of copulas
- DRAM Case Study
  - VRT mechanism
  - Data acquisition
  - Fitting a model
  - Application of the model
- Final Thoughts

ICD&T Mission

- Semiconductor Research Corporation 2008
  - National Technical Merit Award
  - Burn-in reduction collaboration with Texas Instruments
- Founded 1998

  The mission of the Integrated Circuits Design and Test Laboratory is to become the local, regional, and national focal point for innovative research and education in device characterization and integrated circuit design and test.

ICD&T Laboratory Goals

- PSU Spire of Excellence regional relevance
  - Establish research and educational setting for IC test providers and users.
- PSU Spire of Excellence national prominence
  - Establish and retain state-of-the-art facilities to support research outcomes and train next-generation leaders in IC test.

ICD&T History

- Graduates placed
  - LTX-Credence, IBM, Intel, Lattice, LSI, Maxim, Mentor Graphics, Micron, Sandia and Los Alamos NL, Tektronix, Texas Instruments.
- Research Partnerships
  - Cisco, IBM, LSI, Oregon Metals Initiative, Texas Instruments, Semiconductor Research Corporation (SRC).
- Research Cooperation
  - Defect Products Consortium, Oregon Nanoscience and Microtechnologies Institute.

ICD&T Laboratory Results

- Advanced chip screening (Daasch)
  - Design and evaluate new tests for integrated circuits
  - Defect screening
  - Yield and Reliability
  - E-test data screening and trend analysis
- Cost of test
  - New test methods, statistical test methods
- Analog design and test
  - “Digital CMOS”
- Device modeling and characterization (Pejcinovic)
  - Semiconductor Process

IC D&T Recent Research

Sponsor	Project Description	Award	Years
Semiconductor Research Corp.	High-Frequency, High-Q, Self-Correcting Analog Filters	$320K	3
Semiconductor Research Corp.	Outlier Screening to Relax Burn-in Requirements	$750K	6
Semiconductor Research Corp	Statistical Adaptive Test	$320K	3
Cisco	Virtual Integrated Device Manufacture	$280K	2
LSI	Statistical Post-Processing Data Mining Statistical Process Control	$770K	10

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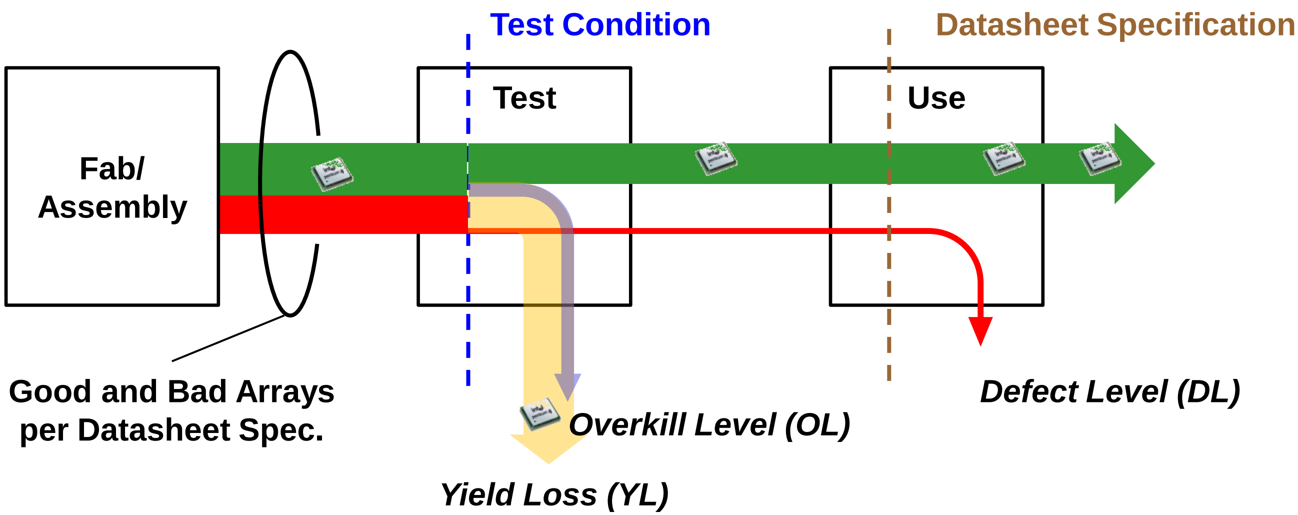
Motivation

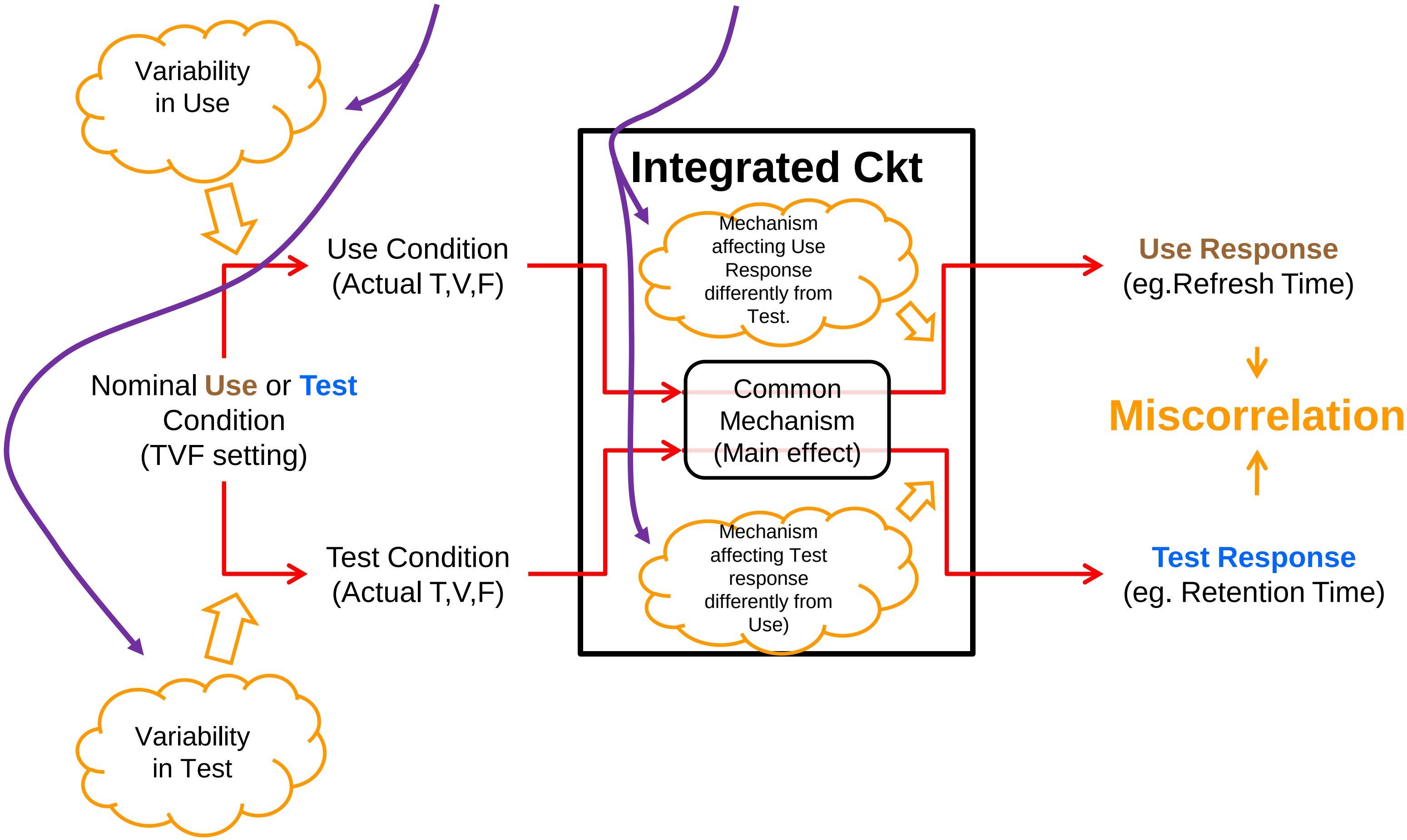
- An IC product is designed, manufactured and used.



- Statistical models are needed throughout the product lifecycle from product definition to manufacturing.
- Key Figures of Merit (FOMs) are: Yield Loss (*YL*), Overkill Level (*OL*) and Defect Level (*DL*) in end use.

Causes of Test/Use Miscorrelation

- Causes are external and internal to the IC.



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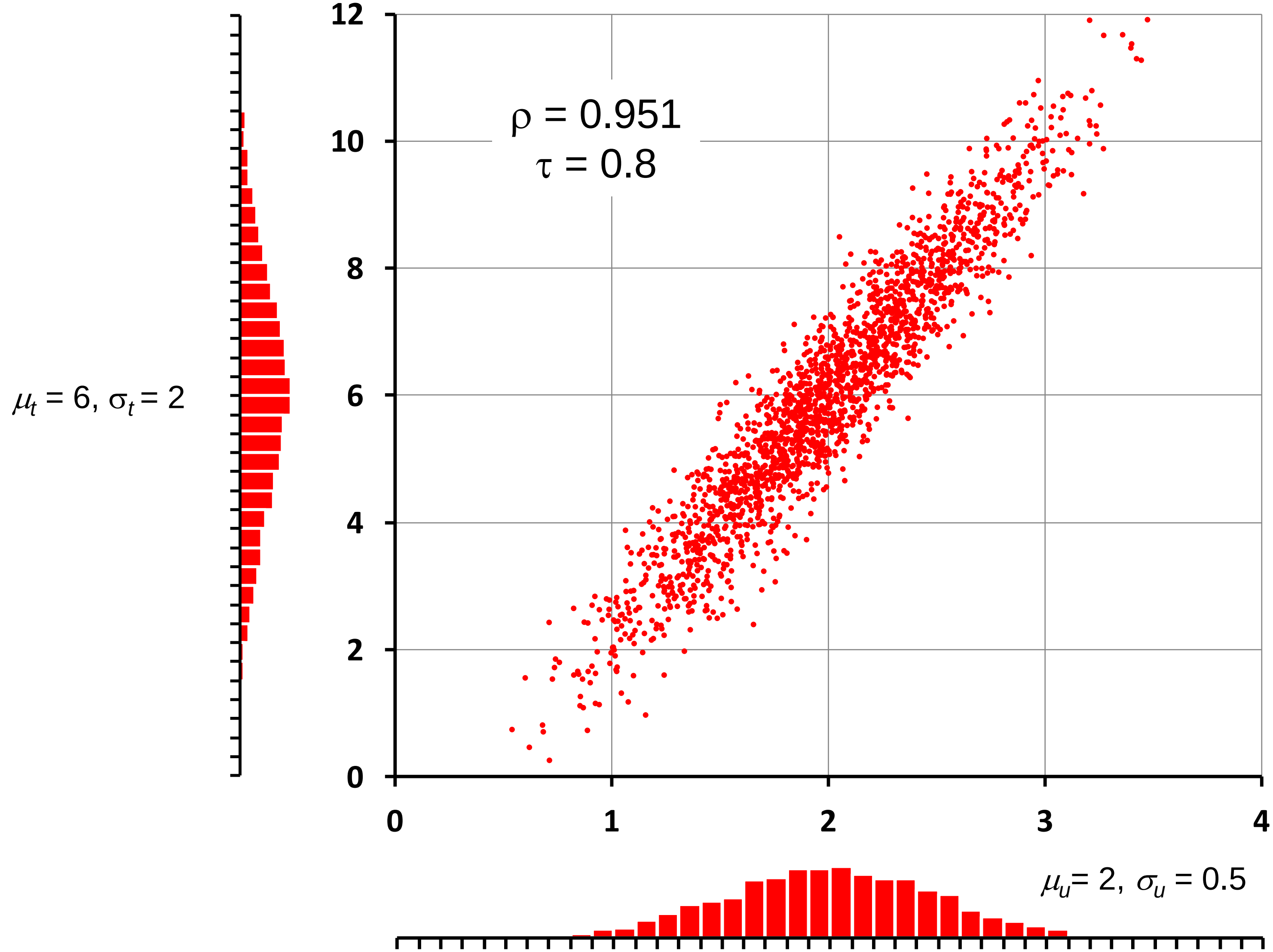
Standard Multinormal Modeling

- Measure “Test” attribute and “Use”-like attribute of each unit.
- Transform variables so that they have normal distributions.
- Fit (normal) “marginal” distributions to each attribute.
  - Extract Means (μ_u, μ_t) and SDs (σ_u, σ_t) for “Test” and “Use”.
- Calculate correlation coefficient, ρ.

Unit Number	Use(Value)	Test(Value)
1	1.290	3.549
2	2.007	5.805
3	1.753	4.856
4	2.696	8.160
5	2.420	7.352
6	1.808	6.085
7	2.071	6.484
8	1.672	4.877
9	2.365	6.862
10	1.913	6.540
11	2.090	6.160
12	1.242	2.223
13	1.824	5.865
14	2.180	5.289
15	1.512	4.670
16	1.930	4.873
17	0.975	2.795
18	2.250	7.010
19	2.251	7.023

...
(2000 units total)

Acquire Data



Unit Number	Use(Value)	Test(Value)
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15	1.512	4.670
16	1.930	4.873
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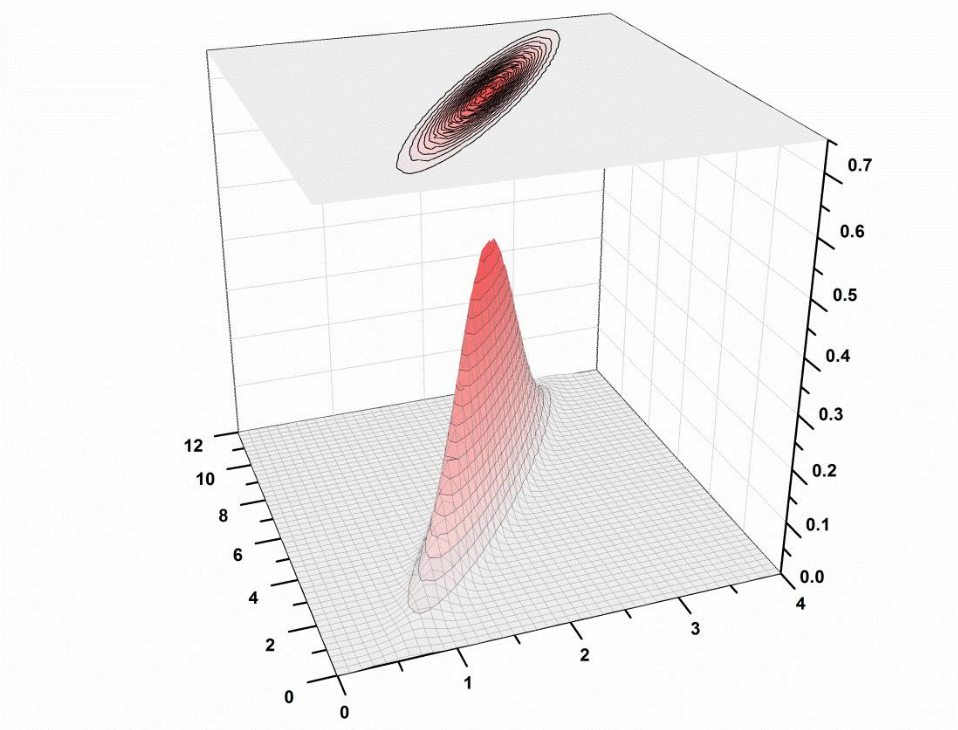
...
(2000 units total)

Fit to Model

- This determines the 2-dimensional Gaussian pdf:

$$f(x,y)=\frac{1}{2\pi\sigma_u\sigma_t\sqrt{1-\rho^2}}\exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_u)^2}{\sigma_u^2}+\frac{(x-\mu_t)^2}{\sigma_t^2}-\frac{2\rho(x-\mu_u)(x-\mu_t)}{\sigma_u\sigma_t}\right]\right)$$



Calculate Figures of Merit

- Yield Loss (*YL*). Fraction rejected by Test, irrespective of Use.
- Overkill (*OL*). Rejected by Test and Good in Use.
- End Use Defect Level (*DL*). Bad in Use as fraction of Passes Test

$$h = (u - \mu_u)/\sigma_u \quad k = (t - \mu_t)/\sigma_t$$

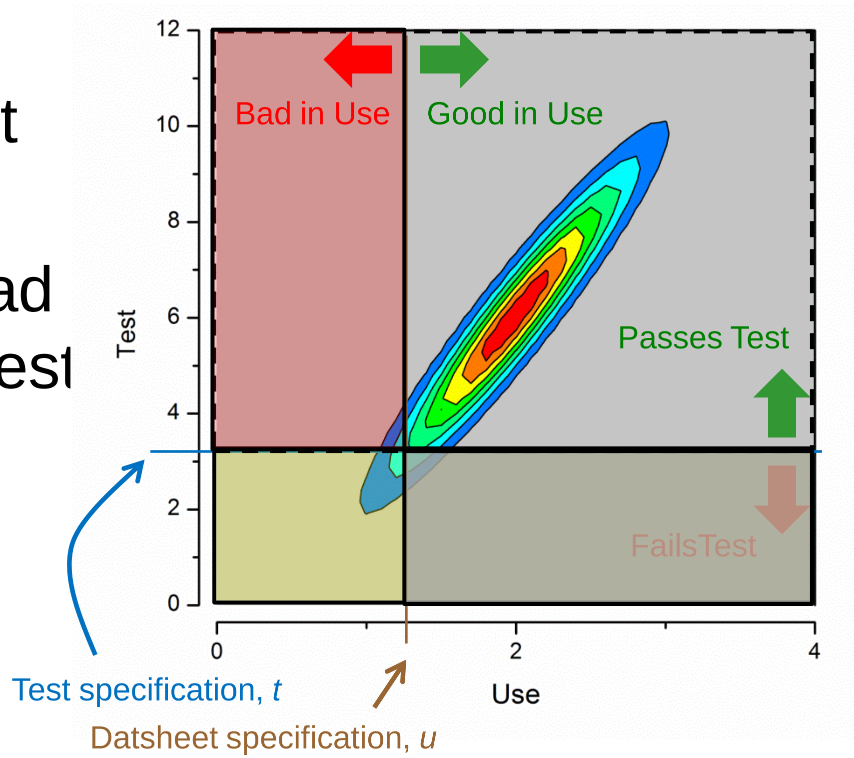
$$YL = \phi(k)$$

$$OL = \phi(k) - \Phi(h,k,\rho)$$

$$DL = \frac{\phi(h) - \Phi(h,k,\rho)}{1 - \phi(k)}$$

$$\phi(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} \exp\left(-\frac{x^2}{2}\right) dx$$

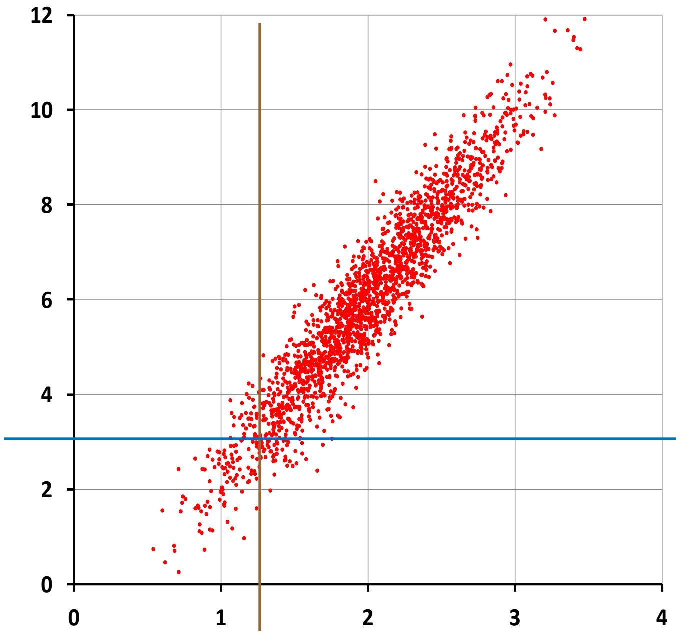
$$\Phi(h,k,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{h} \int_{-\infty}^{k} \exp\left[-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right] dxdy$$



Drezner, Wesolowsky, On the computation of the bivariate normal integral, Journal of Statistical Computation and Simulation, Vol. 35, pp101 - 107 (1990).
Genz, Numerical computation of rectangular bivariate and trivariate normal and t probabilities Statistics and Computing, Vol. 14 pp 251-260 (2004).

Monte-Carlo Simulation..

- ..may also be used to estimate FOMs.
- But for Gaussian distributions synthesized points can't be confined to the tail regions.
- Many synthesized points must be rejected leading to inefficiency.
- We will see ways to sidestep this problem.



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \mu_u \\ \mu_t \end{bmatrix} + \begin{bmatrix} \sigma_u & 0 \\ \sigma_t \rho & \sigma_t \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} \Phi^{-1}(U_u) \\ \Phi^{-1}(U_t) \end{bmatrix}$$

U_u and U_t are iid, uniformly distributed on [0,1].

Copula Approach to Modeling

- Measure “Test” attribute and “Use”-like attribute of each unit.
- Fit 1-D distributions to each margin (Use column, Test column).
  - Use any distribution (eg. Weibull, Beta, Gamma, sums of dist’ns,..)
  - Determine parameters of marginal dist’ns.
- Get *ranks* and *normalized ranks*.
- Fit a copula to the correlation of *ranks*.
  - Determine copula parameter(s).

Ranks normalized to [0,1].

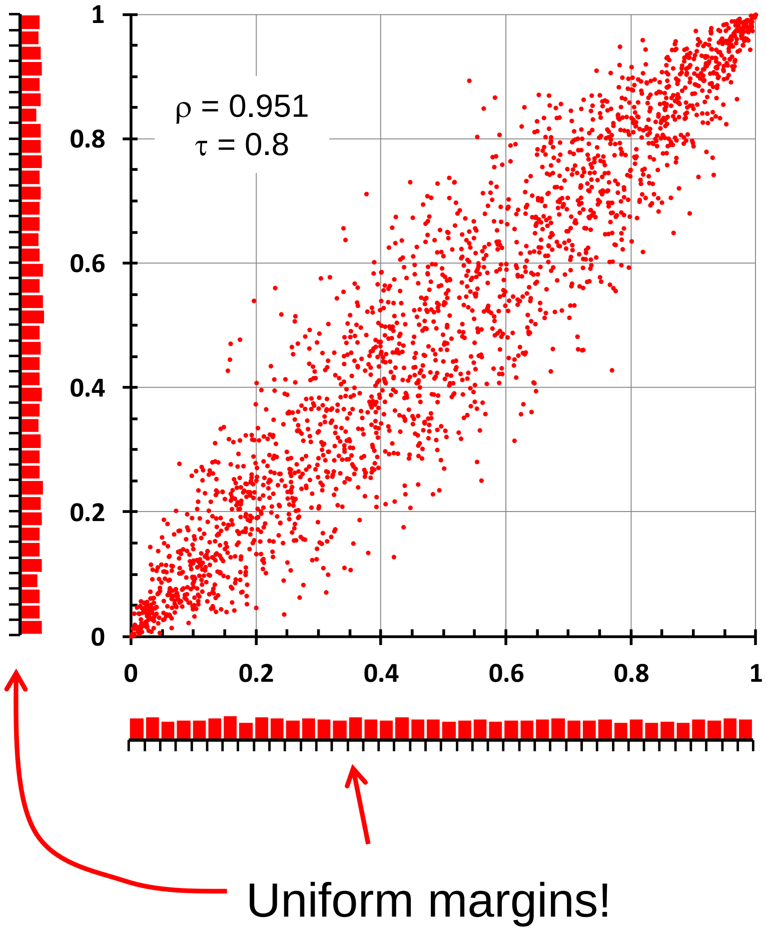
Ranks

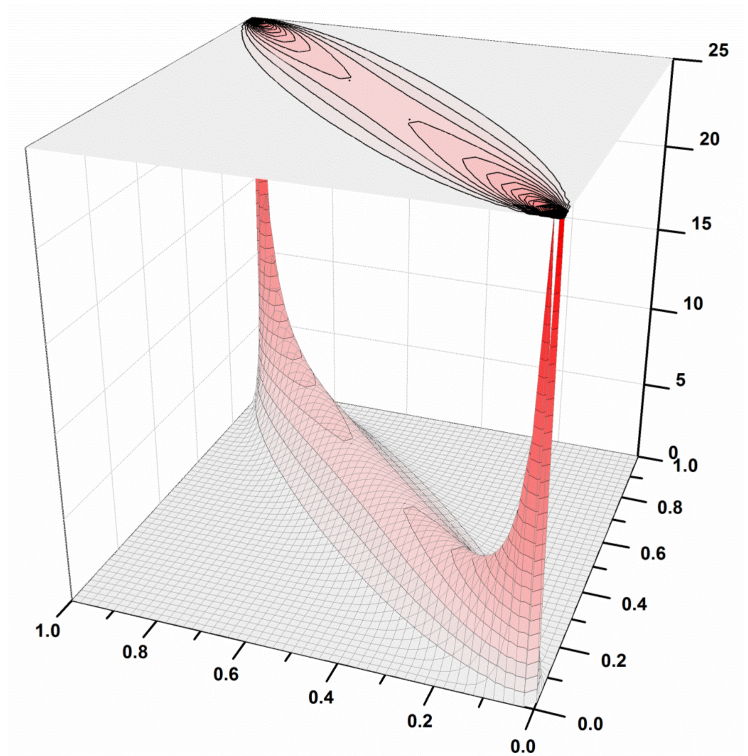
Unit Number	Use(Value)	Test(Value)	Use (Rank)	Test (Rank)	Use (F)	Use (F)
1	1.290	3.549	1843	1771	0.921	0.885
2	2.007	5.805	973	1068	0.486	0.534
3	1.753	4.856	1382	1431	0.691	0.715
4	2.696	8.160	172	282	0.086	0.141
5	2.420	7.352	407	511	0.203	0.255
6	1.808	6.085	1306	967	0.653	0.483
7	2.071	6.484	870	807	0.435	0.403
8	1.672	4.877	1491	1420	0.745	0.710
9	2.365	6.862	473	677	0.236	0.338
10	1.913	6.540	1132	787	0.566	0.393
11	2.090	6.160	845	937	0.422	0.468
12	1.242	2.223	1871	1951	0.935	0.975
13	1.824	5.865	1283	1044	0.641	0.522
14	2.180	5.289	731	1277	0.365	0.638
15	1.512	4.670	1666	1500	0.833	0.750
16	1.930	4.873	1107	1422	0.553	0.711
17	0.975	2.795	1964	1889	0.982	0.944
18	2.250	7.010	627	621	0.313	0.310
19	2.251	7.023	626	615	0.313	0.307

...

(2000 units total)

Copula Density (Gaussian Copula)





3 ways to visualize a copula pdf:
- Synthesized points.
- Contour plot.
- 3D plot.

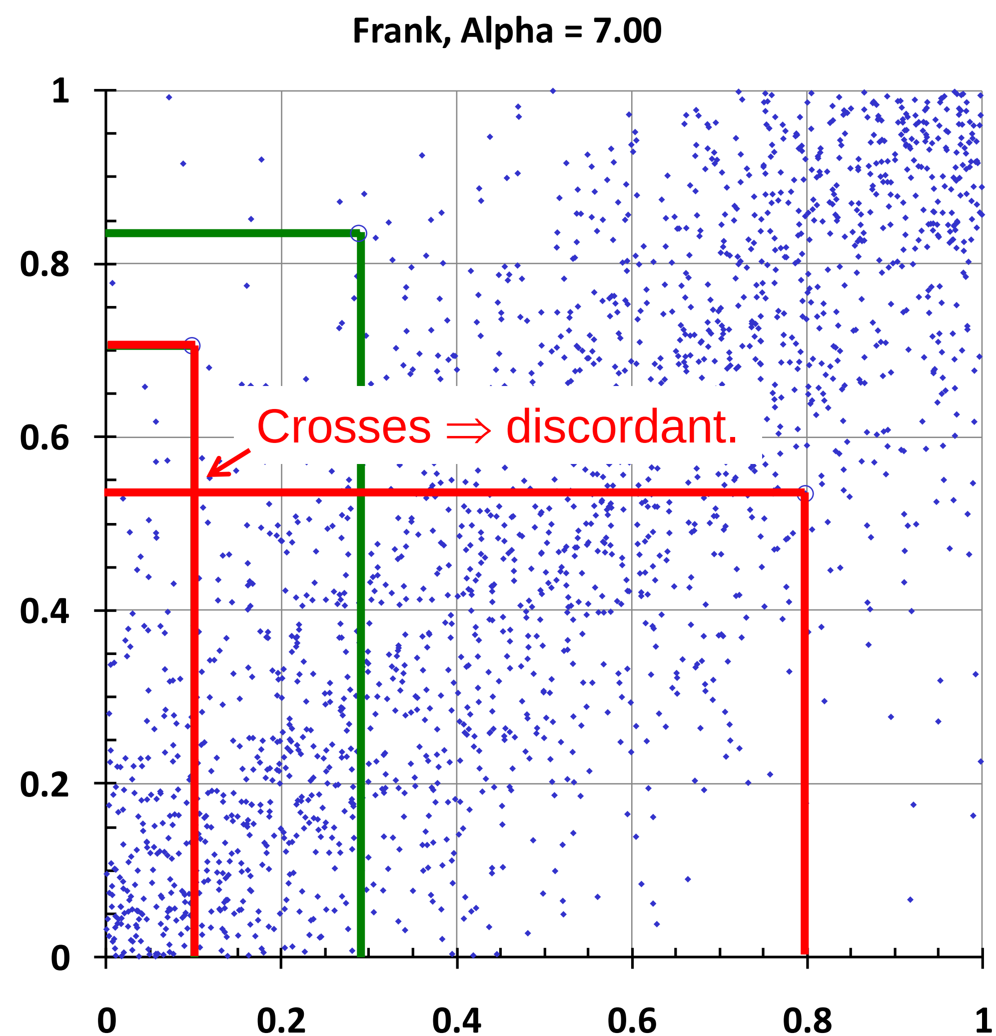
Measures of Dependence

- A single number that characterizes the “scatter” of data:
  - Perfect correlation: 1
  - Independence: 0
  - Perfect anti-correlation: -1
- Pearson’s correlation coefficient. $\rho = \dfrac{\sum_i (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_i (x_i - \mu_x)^2 (y_i - \mu_y)^2}}$
  - Correlation coefficient of *data*.
- Spearman’s Rho
  - Correlation coefficient of *ranks* of data.
  - Independent of marginal distributions.
- Kendall’s Tau
  - Next slide.
  - Independent of marginal distributions.

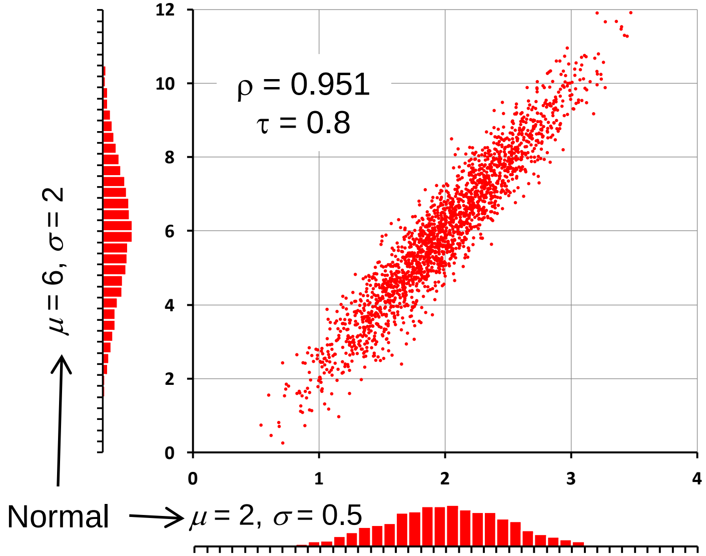
Kendall's Tau

- If there are *n* points in a plot like this, there are *n*(*n*-1)/2 pairs of points.
- Every pair may be classified as "concordant", or "discordant".
  - *c* is the number of concordant pairs.
  - *d* is the number of discordant pairs.

$$\tau = \frac{c-d}{c+d} = \frac{2(c-d)}{n(n-1)}$$

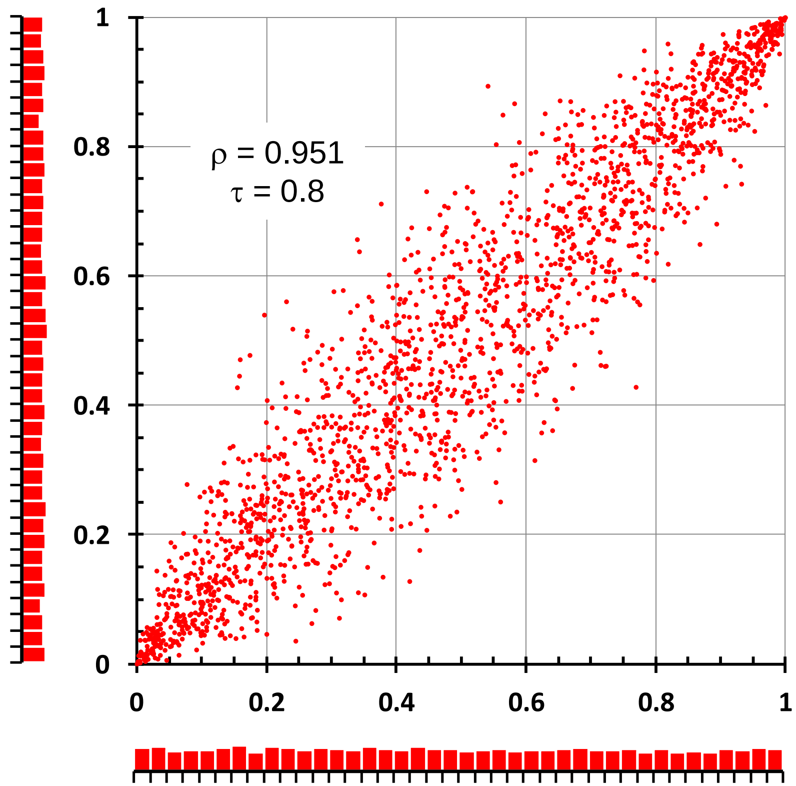


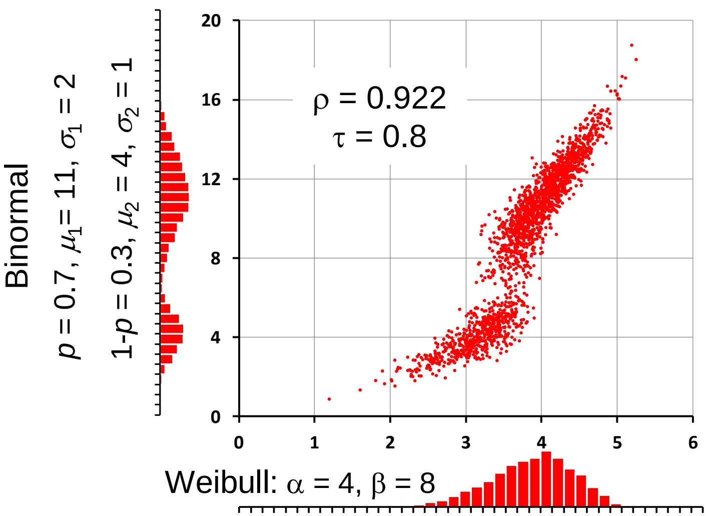
Different Margins, Same Copula..



τ = 0.8

Synthesized **Gaussian** Copula

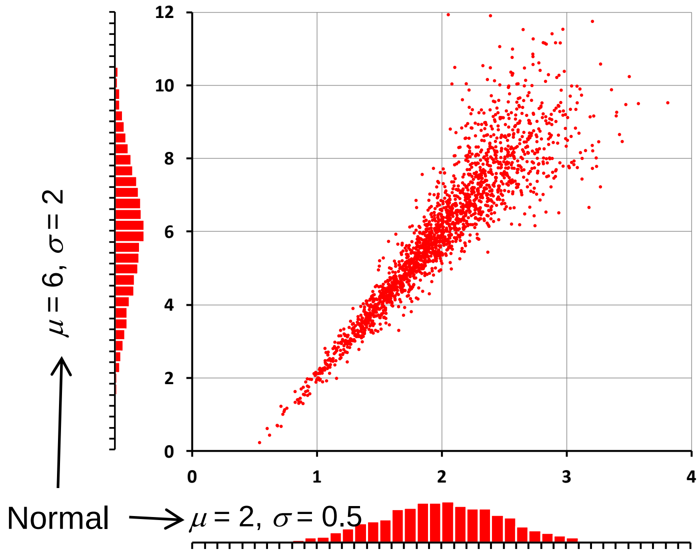


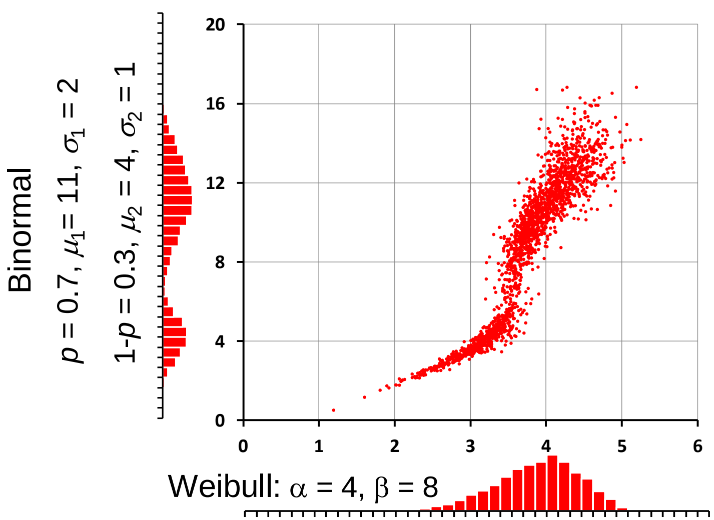


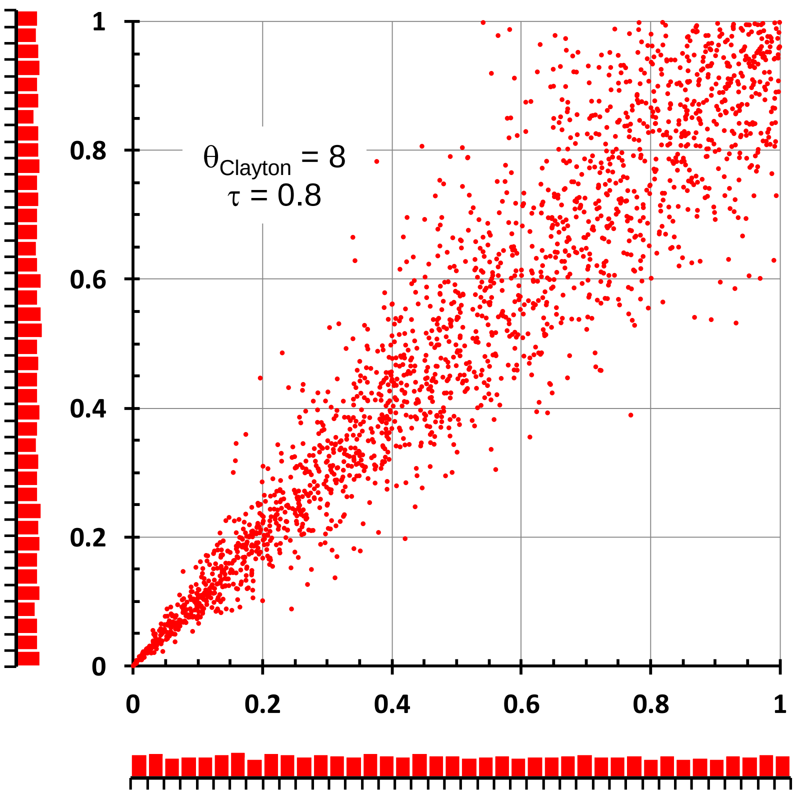
Another Copula..

$\tau = 0.8$

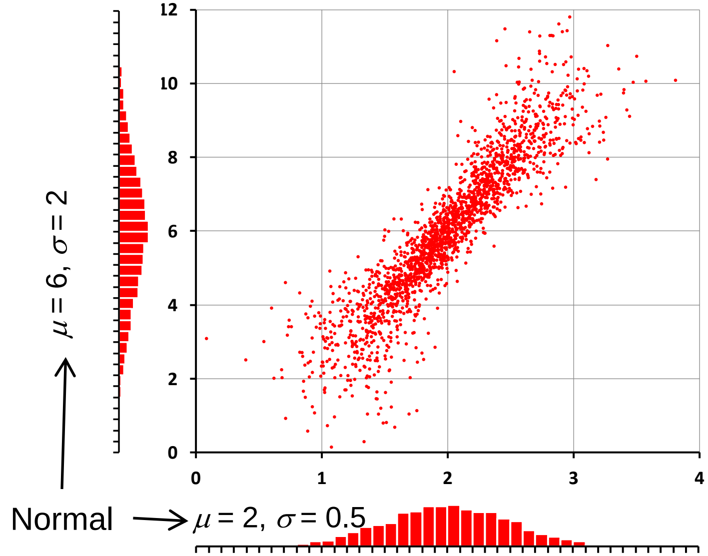
Synthesized **Clayton** Copula

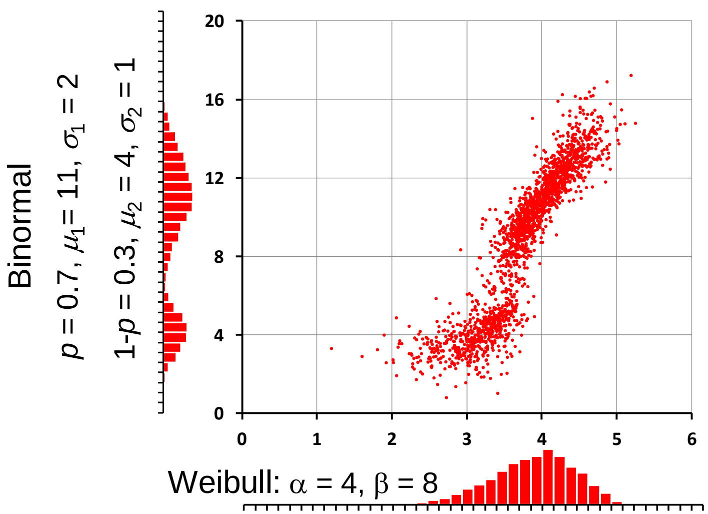






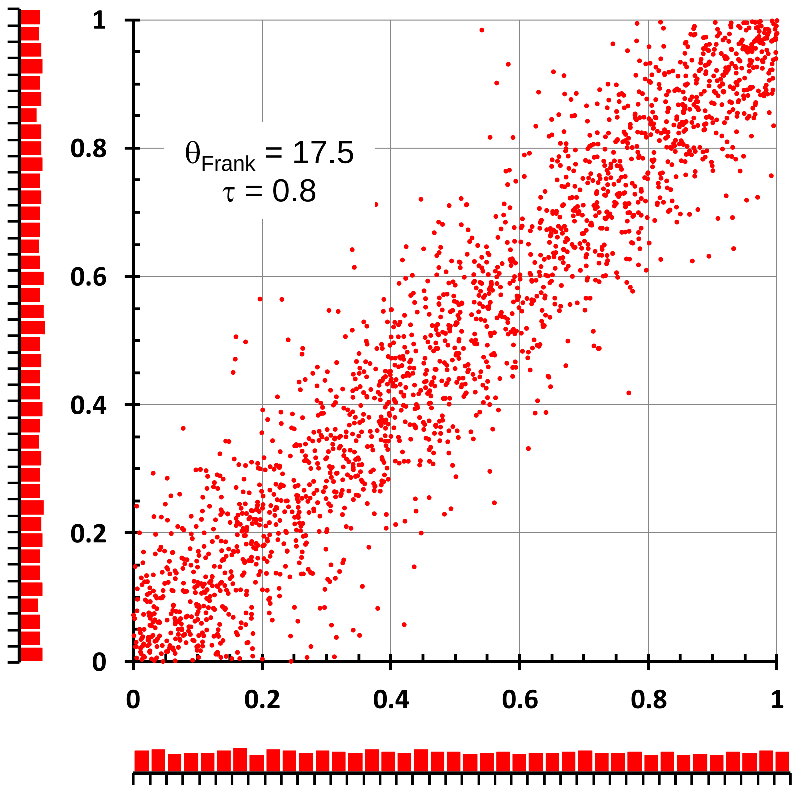
Yet Another Copula



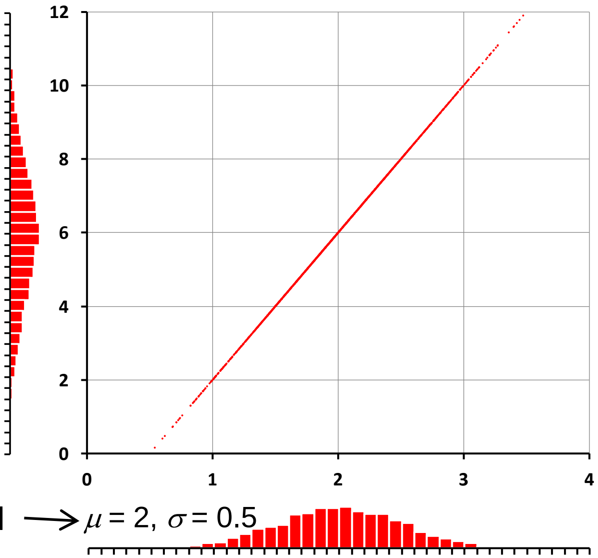


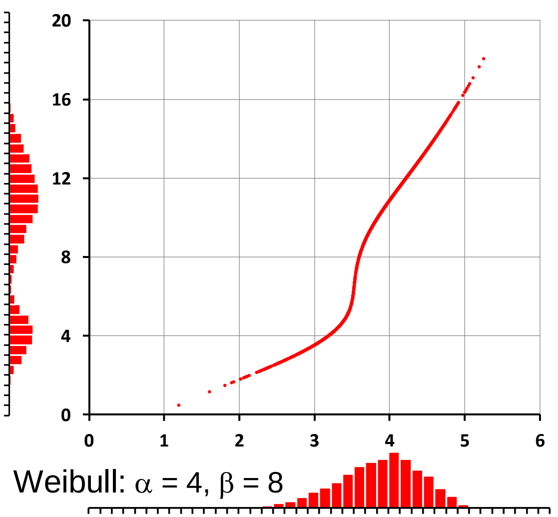
$\tau = 0.8$

Synthesized
Frank Copula



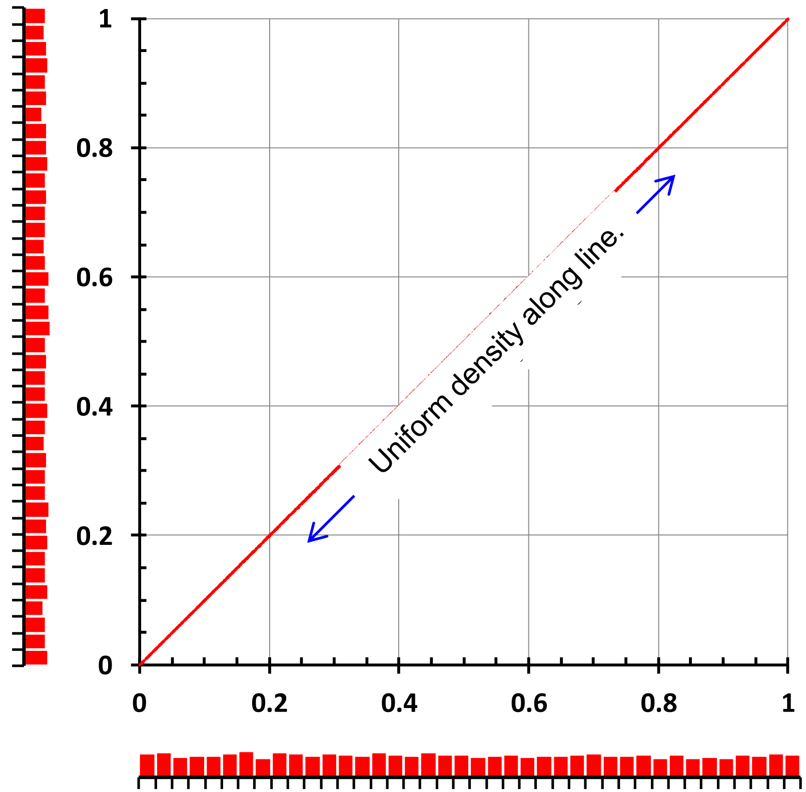
Perfect Correlation



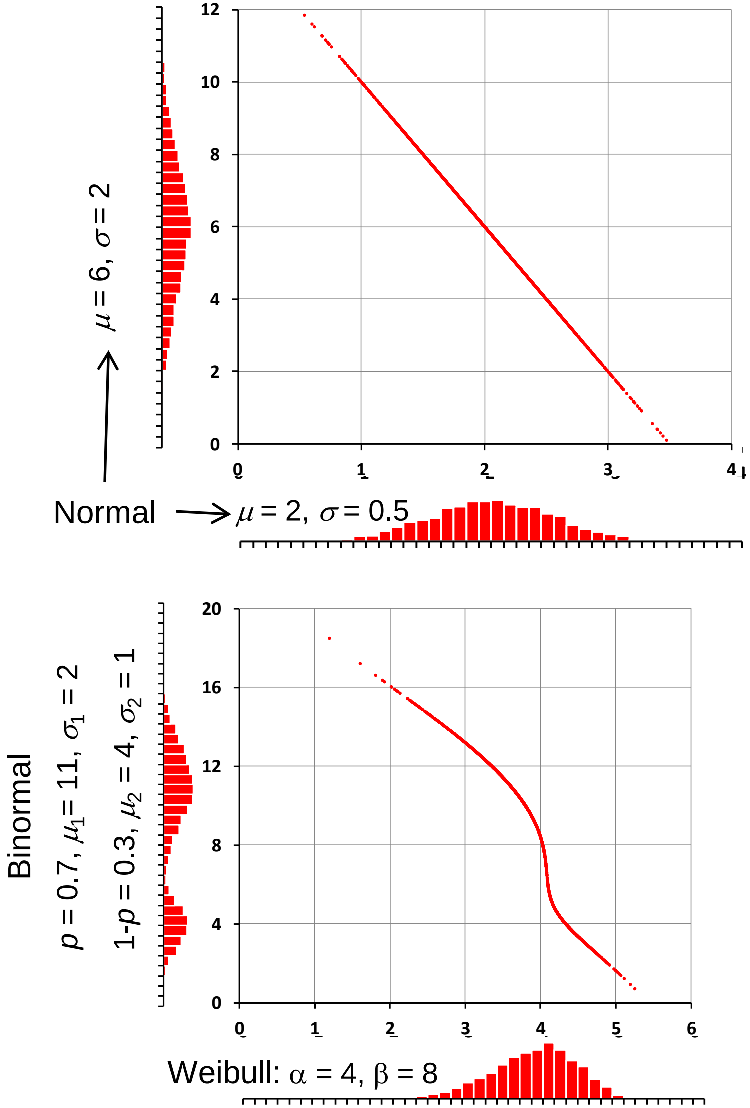


$\tau = 1$

Synthesized
Perfect Correlation Copula

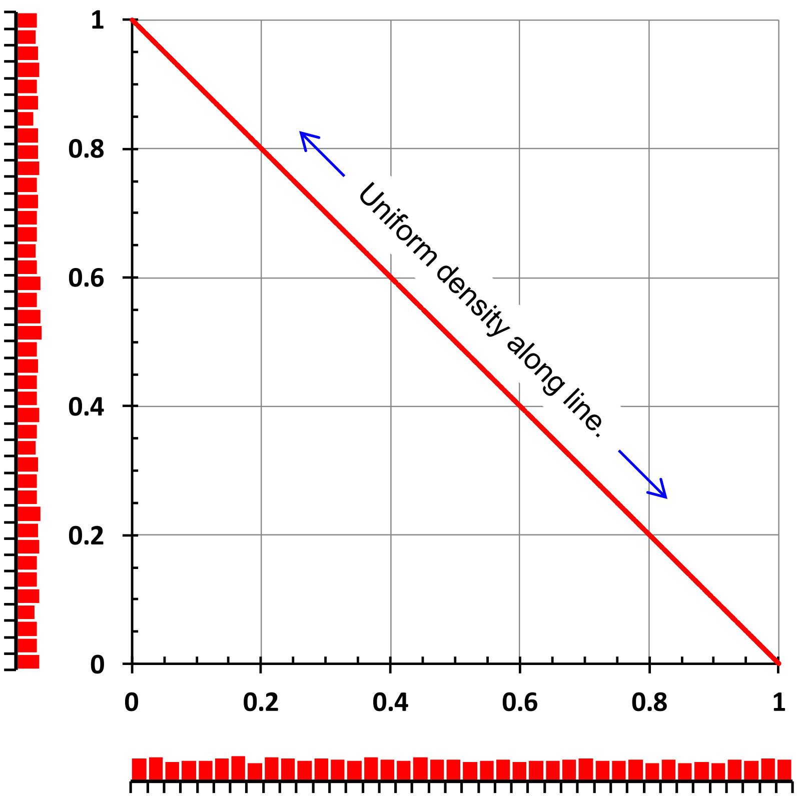


Perfect Anti-Correlation

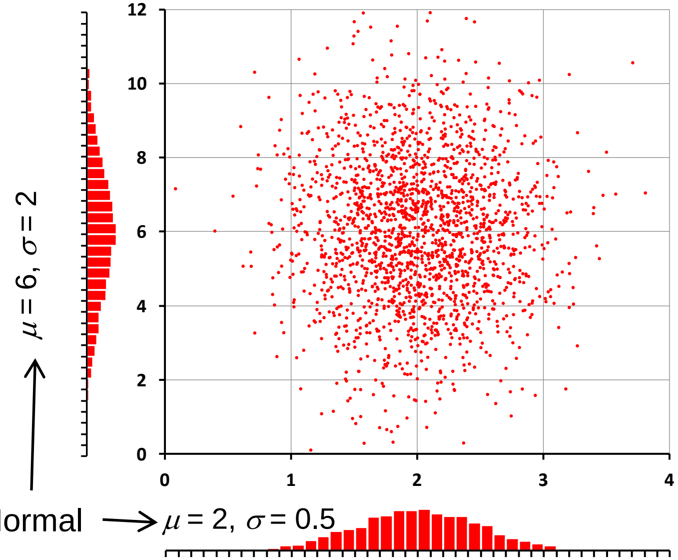


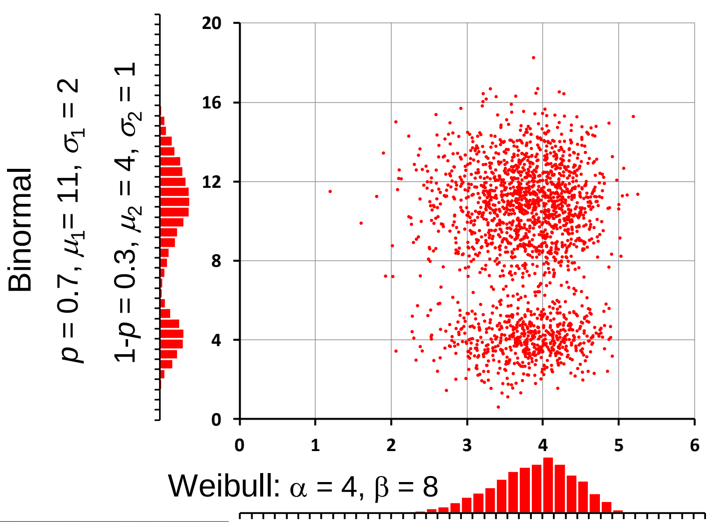
$\tau = -1$

Synthesized **Perfect Anti-Correlation** Copula



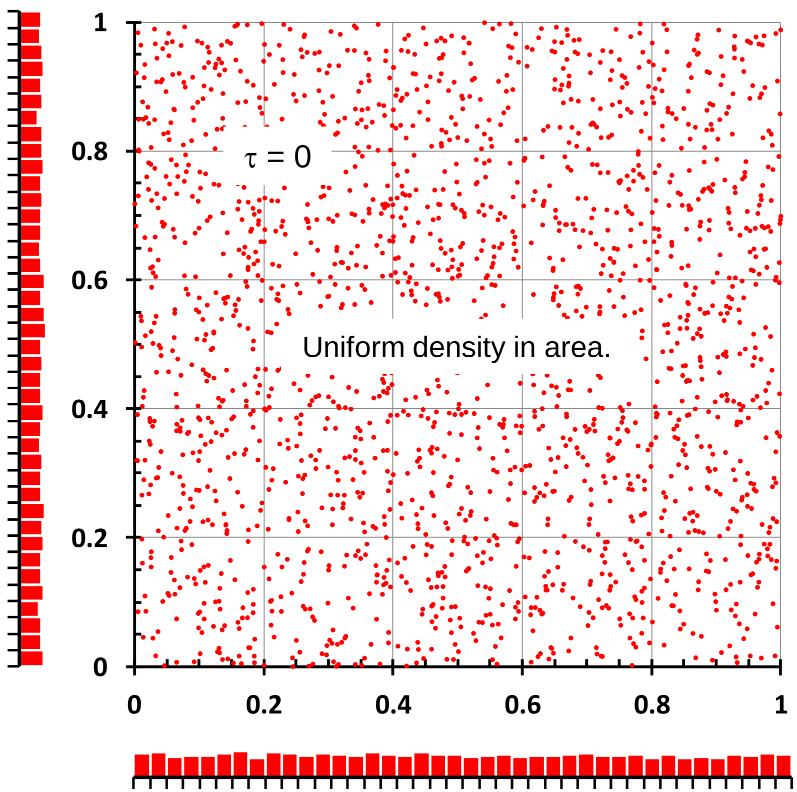
Independence





$\tau = 0$

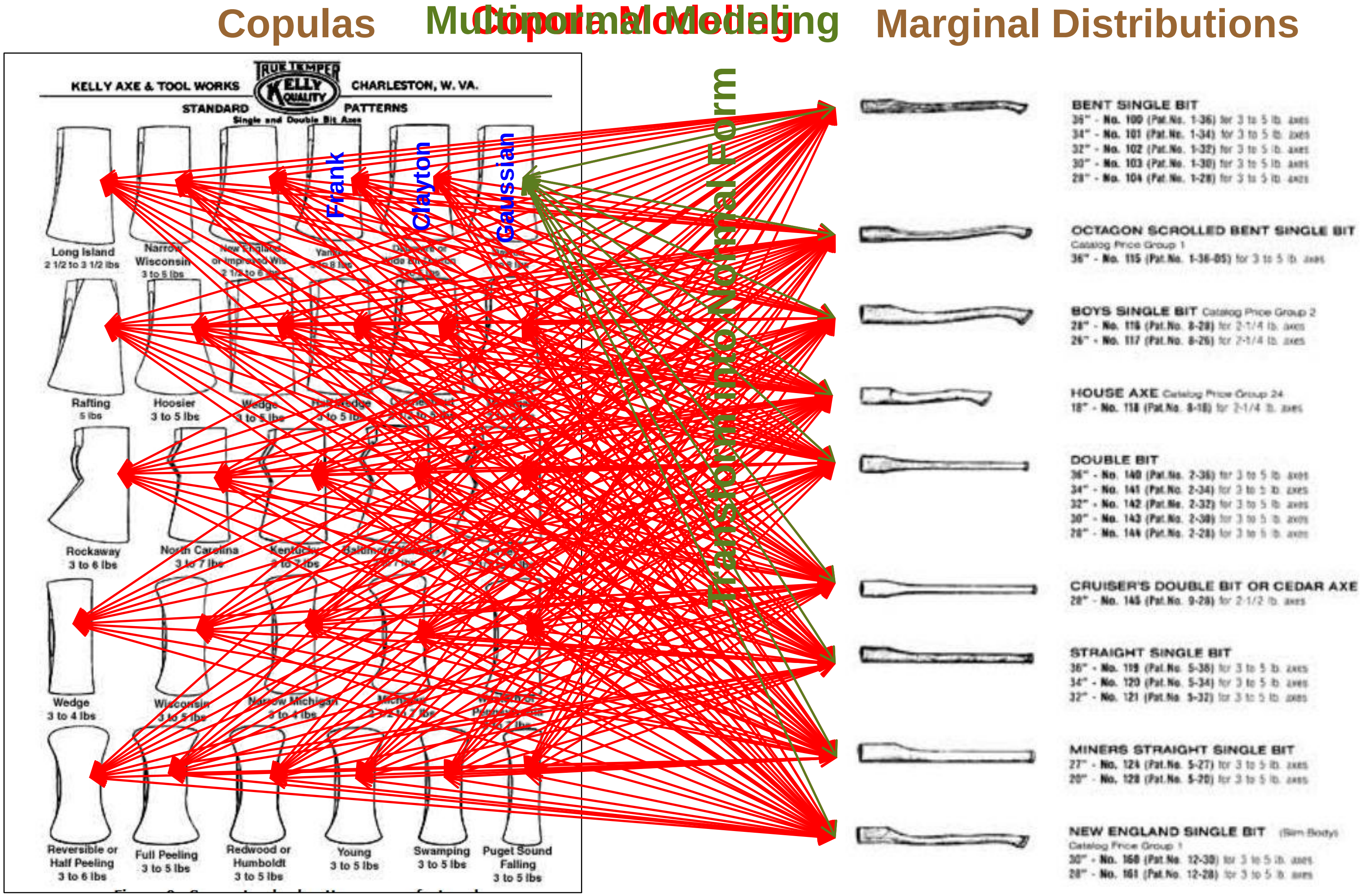
Synthesized
Independence Copula



Multinormal vs Copula Modeling

- Multinormal modeling.
  - Can choose any 1D distribution for the margins, but..
  - Margins must be transformed into normal distributions.
  - Assumes dependency is modeled by Gaussian copula.
  - Gaussian copula has a tail shape which may not be realistic.
- Copula –based modeling.
  - Can choose any 1D distribution for the margins.
  - Freedom to choose any mix of marginal and copula models.
  - Vast number of copula families to choose from, hence..
  - Flexibility to choose copula with desired tail dependence.

Many Heads, Many Handles



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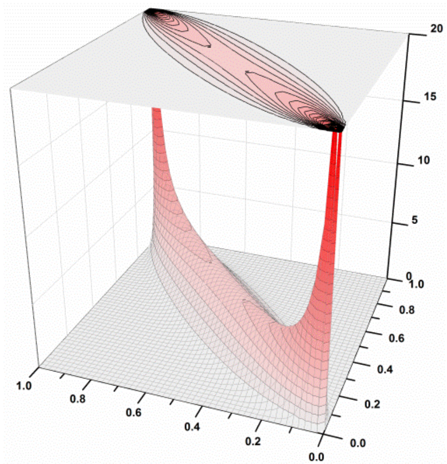
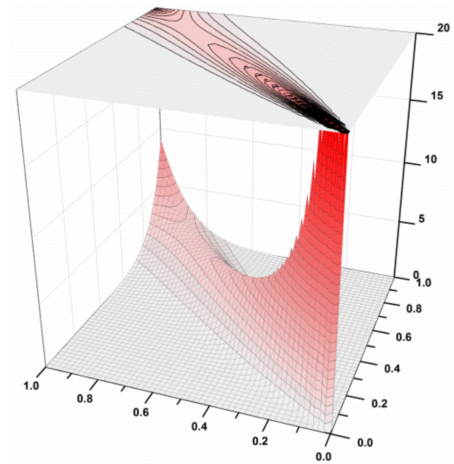
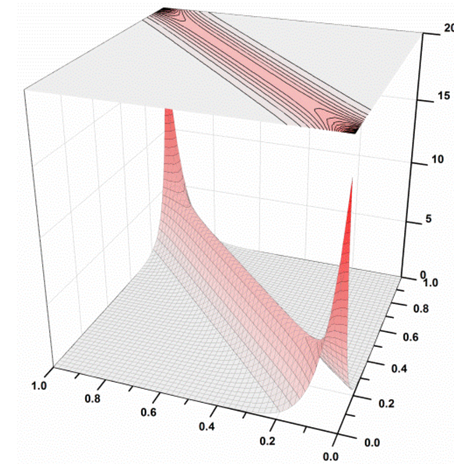
Examples of Copula PDFs



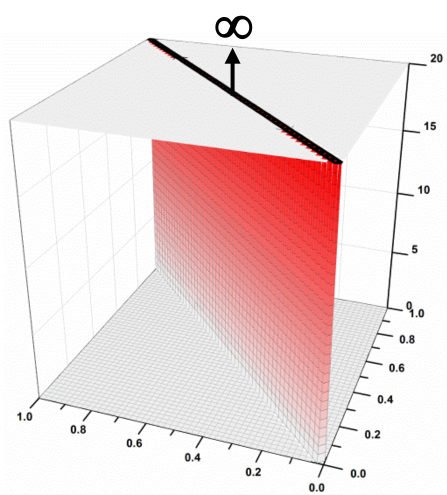
Gaussian, $\tau = 0.8$, $\rho = 0.951$



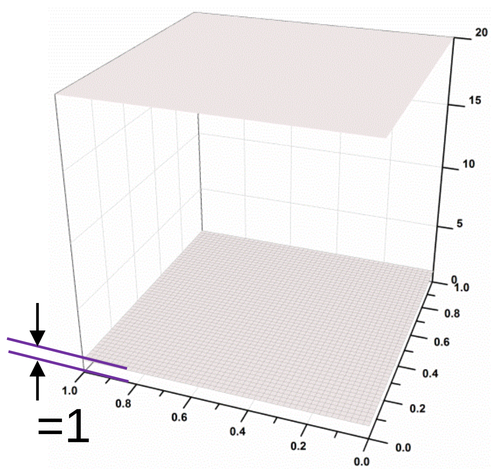
Clayton, $\tau = 0.8$, $\theta = 8$



Frank, $\tau = 0.8$, $\theta = 17.5$



Perfect Correlation, $\tau = 1$

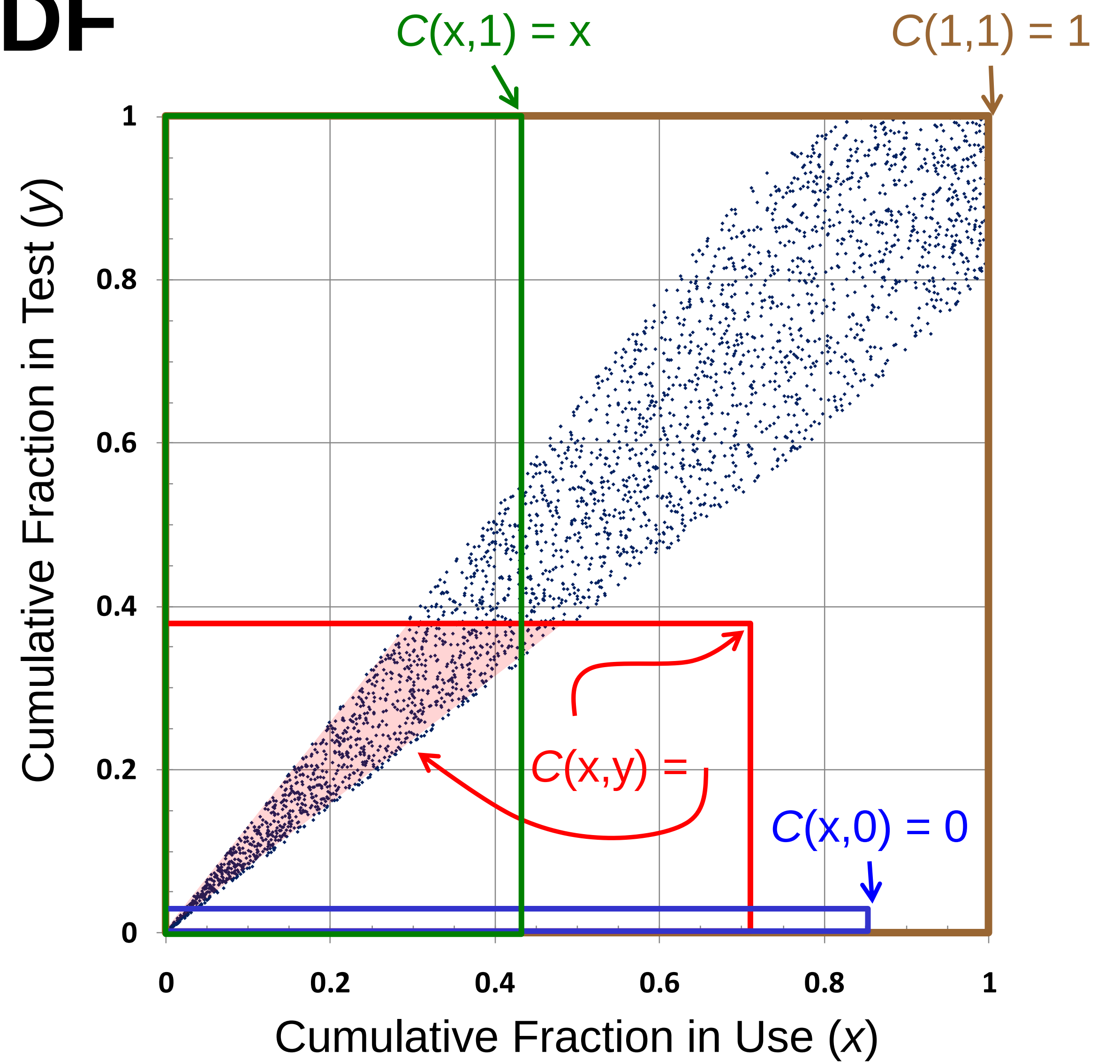


Independence, $\tau = 0$

A copula pdf is a probability density function which integrates to unity on the unit square.

Copula is a CDF

- To get the copula from the copula PDF:
  - The copula is the probability mass in rectangles $[0,x]\times[0,y]$ as a function of (x,y).
- Definition of a copula
  - $C(1,1) = 1$
  - $C(x,0)=C(0,y)=0$
  - $C(x,1)=x$, $C(1,y)=y$
  - 2-increasing



Copula Examples

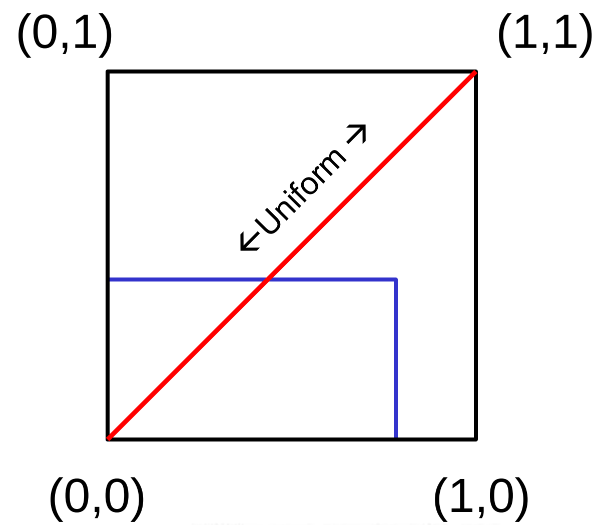
- Perfect correlation

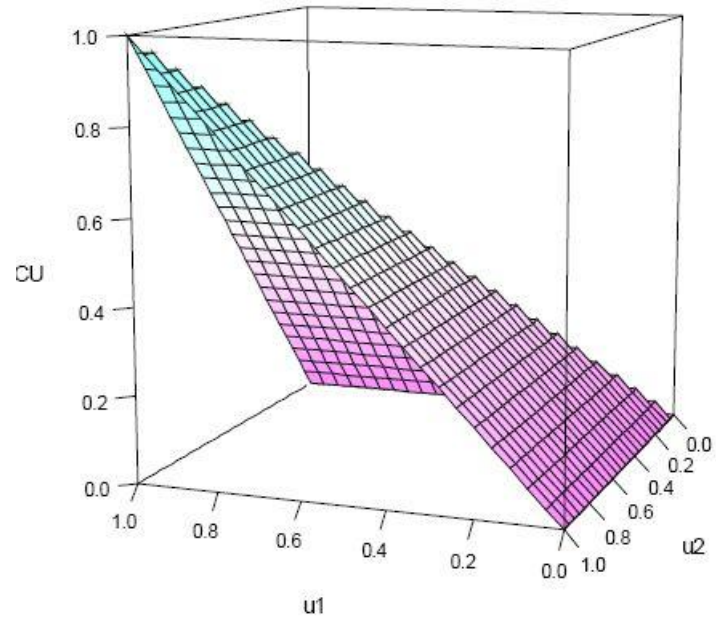
Dirac delta function

$$\frac{\partial^2 C(x,y)}{\partial x \partial y} = \delta\left(x-y\right)$$

$$C(x,y) = \min\left[x,y\right]$$

- This is called the Frechet Upper Bound (FUB).





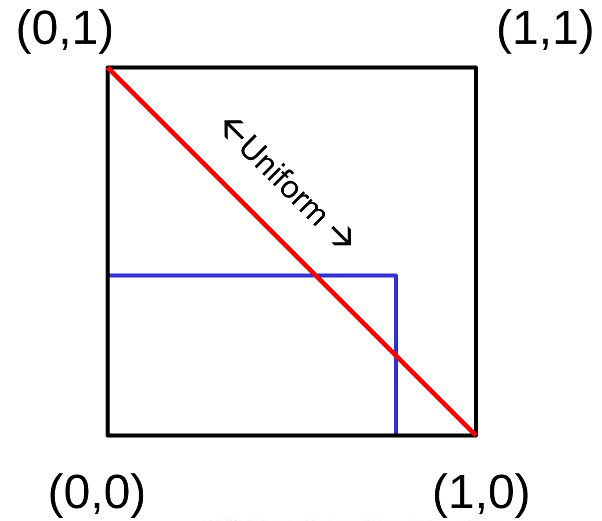
Copula Examples, ct'd

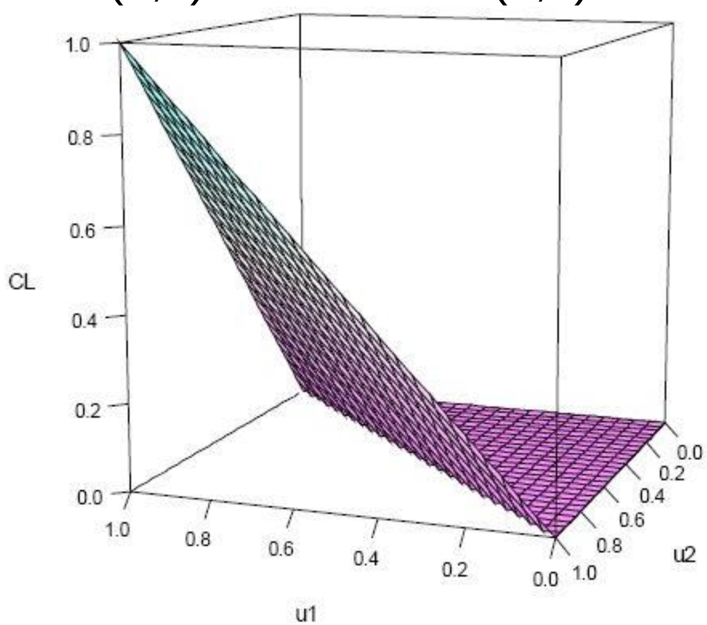
- Perfect anti-correlation

$$\frac{\partial^2 C(x,y)}{\partial x \partial y} = \delta\left(x+y-1\right)$$

$$C(x,y) = \max\left[x+y-1,0\right]$$

- This is called the Frechet Lower Bound (FLB).



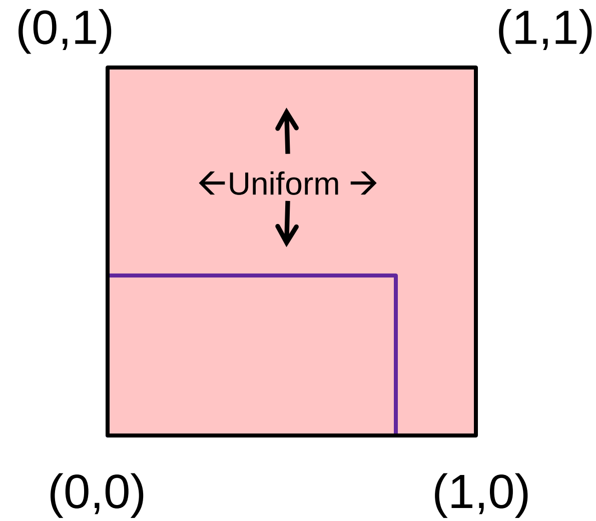


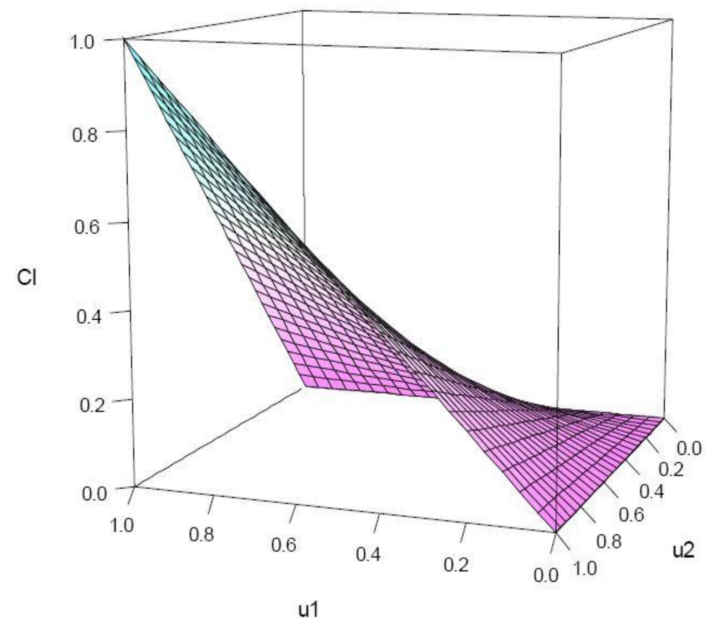
Copula Examples, ct'd

- Independence

$$\frac{\partial^2 C(x,y)}{\partial x \partial y} = 1$$

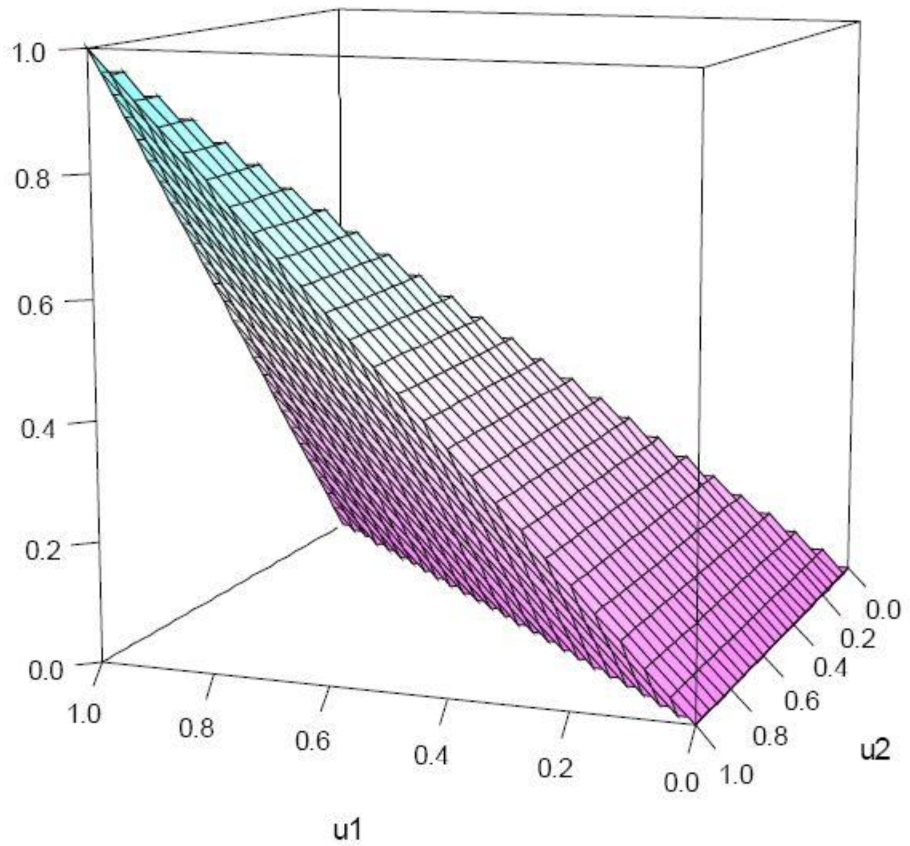
$$C(x,y) = xy$$





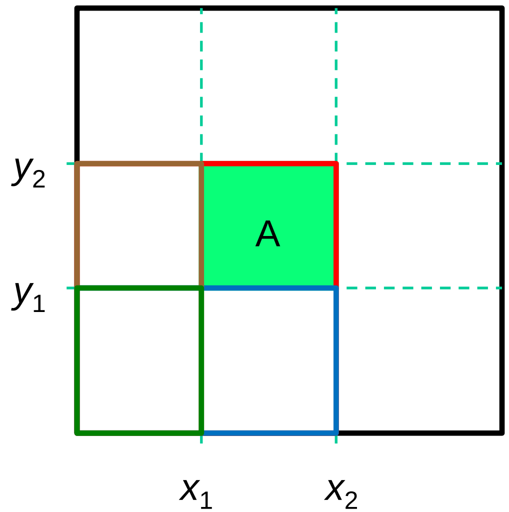
FUB and FLB Bound All Copulas

- All copulas will lie in this volume:



Probability Calculation

- The probability mass in any rectangle, A, can be calculated from the copula.
  - Probability Mass = $C(x_2,y_2)$-$C(x_2,y_1)$-$C(x_1,y_2)$ +$C(x_1,y_1)$

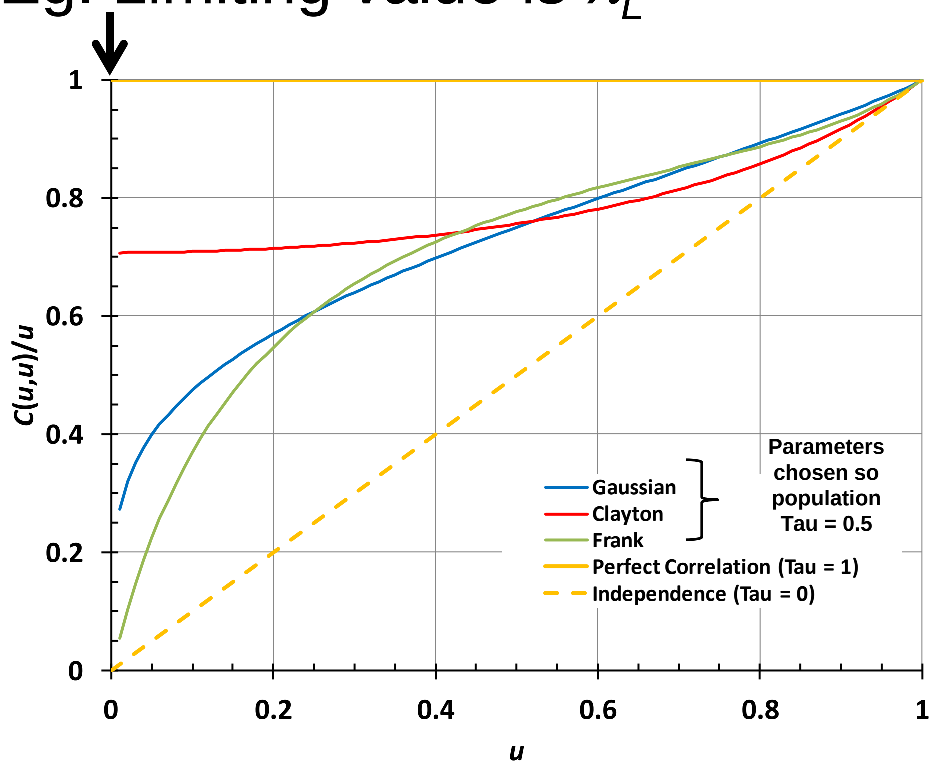


This calculation is done frequently.

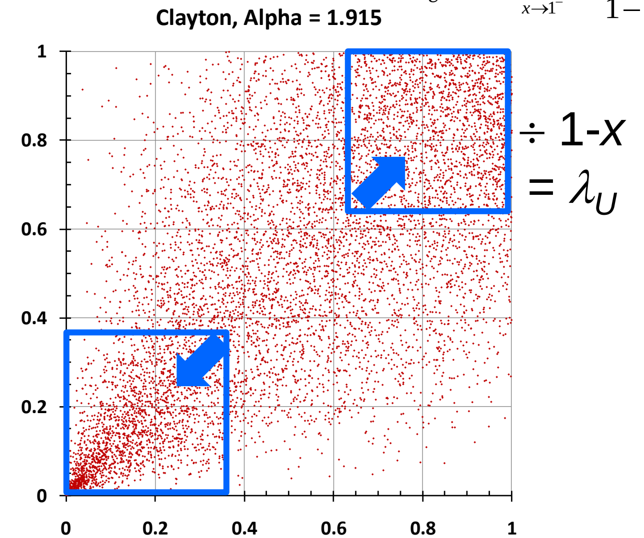
- The area of A is
  - Area = $(x_2 - x_1)\times(y_2 - y_1) = x_2y_2-x_2y_1-x_1y_2+x_1y_1$
- 2-Increasing means, for a rectangle:
  - When Area A ≥ 0, then Probability Mass in A ≥ 0

Measures of Tail Dependence

- Probability mass in corner squares, divided by size of square, as size $\to$ 0.
- Eg. Limiting value is λ_L



$$\lambda_U = 2 - \lim_{x \to 1^-} \frac{1 - C(x,x)}{1 - x}$$



$$\lambda_L = \lim_{x \to 0+} \frac{C(x,x)}{x}$$

Copula	λ_L	λ_U
Independence	0	0
Perfect Correlation	1	1
Gaussian (ρ = 1)	1	1
Gaussian (ρ < 1)	0	0
Clayton	$2^{-1/\theta}$	0
Frank	0	0
Gumbell	0	$2 - 2^{1/\theta}$

Families of Copulas

- Elliptical copulas.
  - Gaussian, t-copula, ..
- Archimedian copulas.
  - Clayton, Frank, Gumbel, and many more.
  - Construction by a particular method.
- Geometrical copulas.
  - Constructed by geometrical methods.

Elliptical Copulas

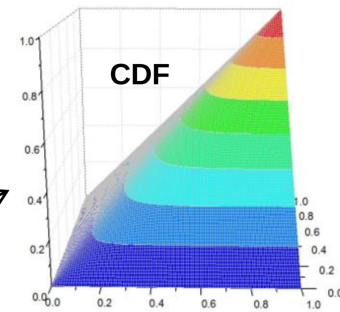
- t-Copula, based on Student's t distribution. "Elliptical" contours..

$$C_{\nu,\rho}(x,y)=\int_{-\infty}^{t_\nu^{-1}(x)}\int_{-\infty}^{t_\nu^{-1}(y)}\frac{1}{\sqrt{2\pi(1-\rho)}}\left\{1+\frac{x^2-2\rho xy+y^2}{\nu(1-\rho^2)}\right\}^{\frac{\nu+2}{2}}dxdy$$

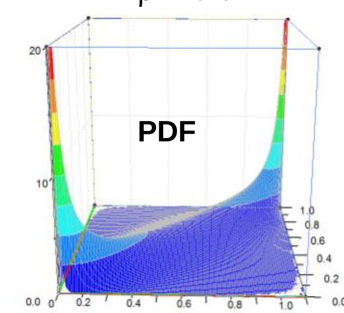
  - t^{-1}_{ν} is the inverse of the standard univariate *t* dist'n.
- Gaussian copula. The $\nu\to\infty$ limit of the t-copula.

$$C_\rho(x,y)=\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{\Phi_1^{-1}(x)}\int_{-\infty}^{\Phi_1^{-1}(y)}\exp\left[-\frac{s^2-2\rho st+t^2}{2\left(1-\rho^2\right)}\right]dsdt$$

$$\Phi_1(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\tfrac{1}{2}s^2\right)ds$$



ρ = 0.9



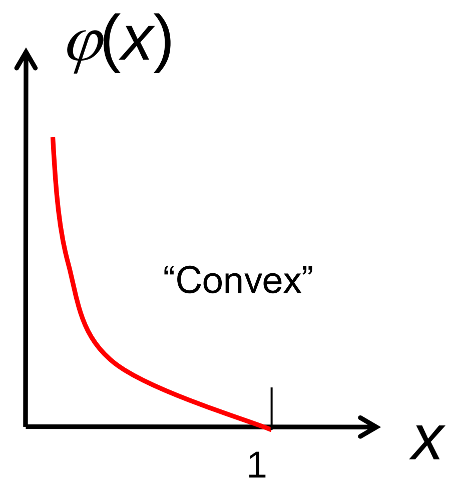
- Easily generalized to many dimensions.

Achimedian Copulas

- Archimedian copulas are a large class with..
  - Convenient, pretty, mathematical properties.
  - Easily generalized to > 2 dimensions, but with only one parameter.
  - Large literature.
- Defined by

$$C(x,y) = \varphi^{-1}\left[\varphi(x) + \varphi(y)\right]$$

where φ is a *generating function* (GF):



Frank Copula

- Archimedian GF

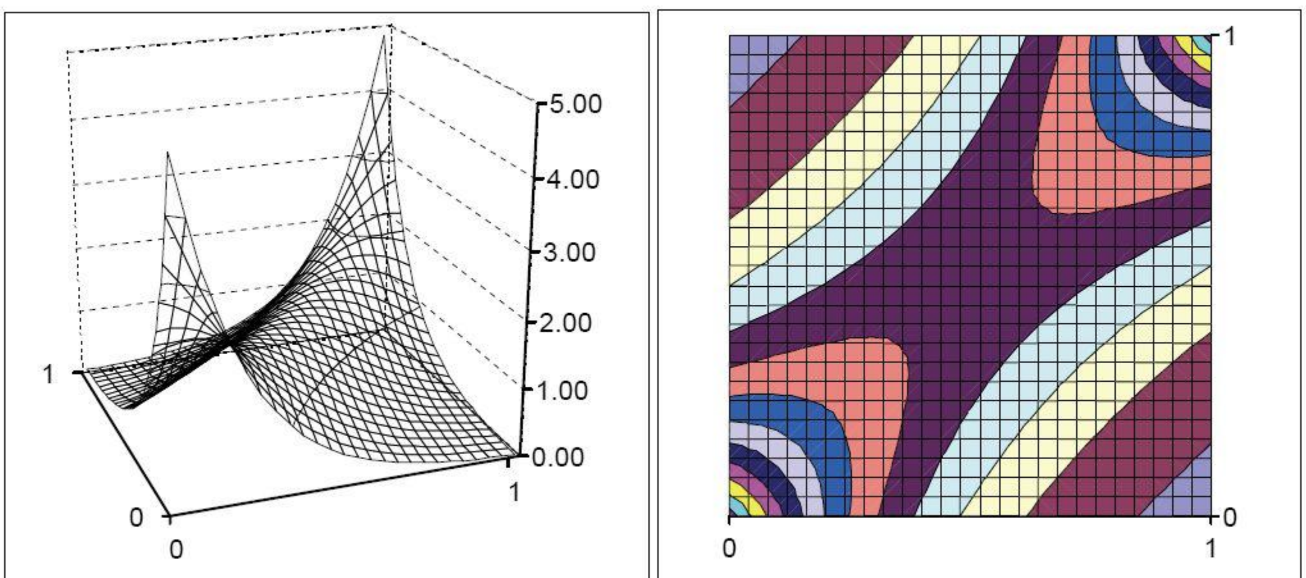
$$\varphi(t) = \ln \frac{e^{\alpha t} - 1}{e^{\alpha} - 1}$$

- CDF

$$C(x, y \mid \alpha) = -\frac{1}{\alpha} \ln\left\{1 + \frac{\left(e^{-\alpha x} - 1\right)\left(e^{-\alpha y} - 1\right)}{e^{-\alpha} - 1}\right\}, \quad \alpha \in \mathbb{R} \setminus \{0\}$$

- PDF

$\alpha = 4.875$



Images: J. René van Dorp

Clayton Copula

- Archimedian GF

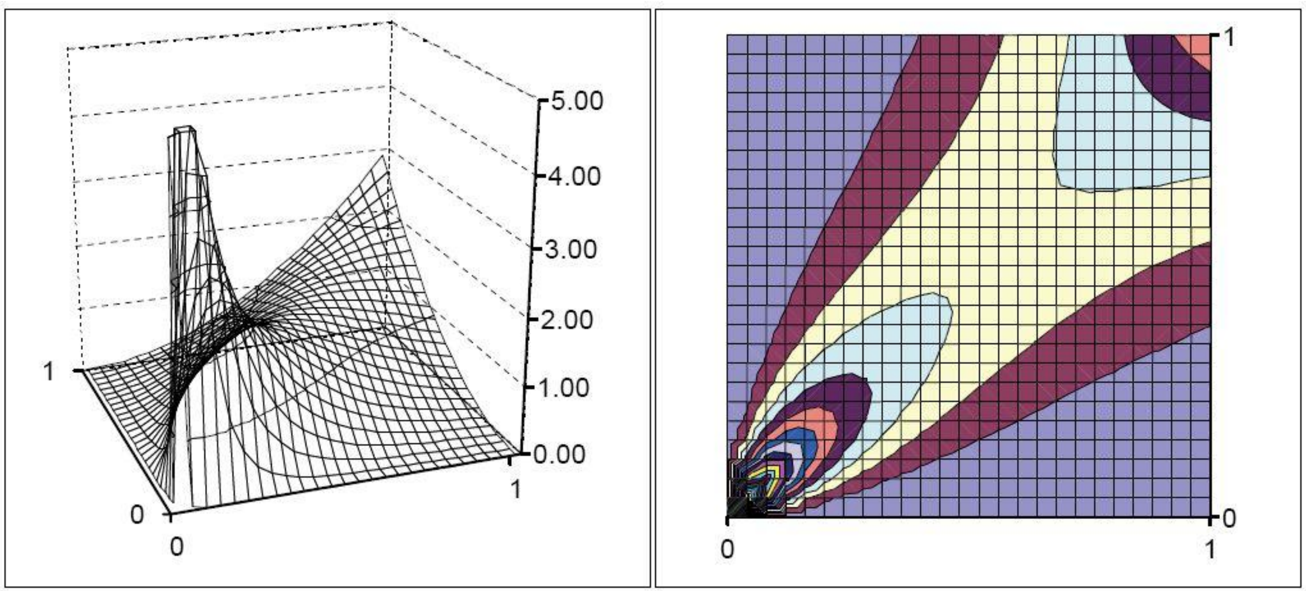
$$\varphi(t) = t^{-\alpha} - 1$$

- CDF

$$C(x, y \mid \alpha) = \left[x^{-\alpha} + y^{-\alpha} - 1 \right]^{-1/\alpha}, \alpha \geq 0$$

- PDF

$\alpha = 1.915$



Images: J. René van Dorp

Gumbel Copula

- Archimedian GF

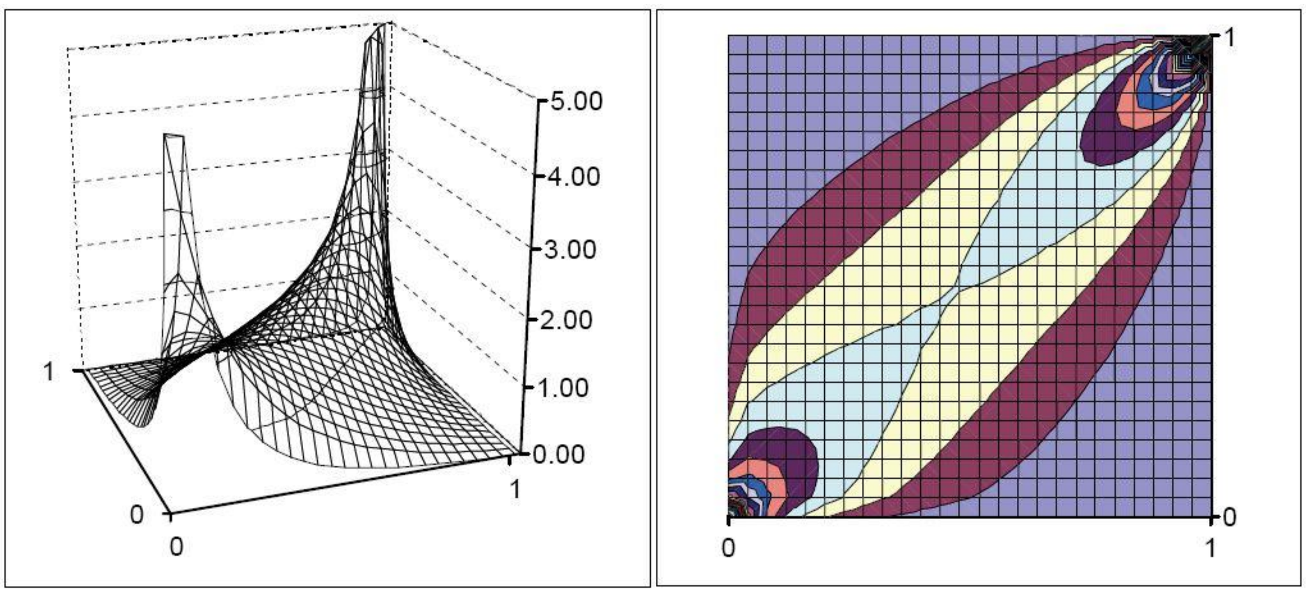
$$\varphi(t) = (-\ln t)^{\alpha}$$

- CDF

$$C(x, y \mid \alpha) = \exp\left\{-\left[(-\ln x)^{\alpha} - (-\ln y)^{\alpha}\right]^{1/\alpha}\right\}, \alpha \geq 1$$

- PDF

$\alpha = 1.997$



Images: J. René van Dorp

And Many More..

116 4 Archimedean Copulas

Table 4.1. One-parameter

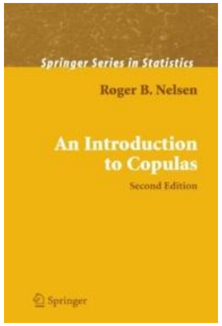
(4.2.#)	$C_\theta(u,v)$	$\varphi_\theta(t)$
1	$\left[\max\left(u^{-\theta}+v^{-\theta}-1,0\right)\right]^{-1/\theta}$	$\frac{1}{\theta}\left(t^{-\theta}-1\right)$
2	$\max\left(1-\left[(1-u)^\theta+(1-v)^\theta\right]^{1/\theta},0\right)$	$(1-t)^\theta$
3	$\frac{uv}{1-\theta(1-u)(1-v)}$	$\ln\frac{1-\theta(1-t)}{t}$
4	$\exp\left(-\left[(-\ln u)^\theta+(-\ln v)^\theta\right]^{1/\theta}\right)$	$(-\ln t)^\theta$
5	$-\frac{1}{\theta}\ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$	$-\ln\frac{e^{-\theta t}-1}{e^{-\theta}-1}$
6	$1-\left[(1-u)^\theta+(1-v)^\theta-(1-u)^\theta(1-v)^\theta\right]^{1/\theta}$	$-\ln\left[1-(1-t)^\theta\right]$
7	$\max(\theta uv+(1-\theta)(u+v-1),0)$	$-\ln[\theta t+(1-\theta)]$
8	$\max\left(\frac{\theta^2uv-(1-u)(1-v)}{\theta^2-(\theta-1)^2(1-u)(1-v)},0\right)$	$\frac{1-t}{1+(\theta-1)t}$
9	$uv\exp(-\theta\ln u\ln v)$	$\ln(1-\theta\ln t)$
10	$uv\big/\left[1+(1-u^\theta)(1-v^\theta)\right]^{1/\theta}$	$\ln\left(2t^{-\theta}-1\right)$
11	$\left[\max\left(u^\theta v^\theta-2(1-u^\theta)(1-v^\theta),0\right)\right]^{1/\theta}$	$\ln\left(2-t^\theta\right)$
12	$\left(1+\left[(u^{-1}-1)^\theta+(v^{-1}-1)^\theta\right]^{1/\theta}\right)^{-1}$	$\left(\frac{1}{t}-1\right)^\theta$
13	$\exp\left(1-\left[(1-\ln u)^\theta+(1-\ln v)^\theta-1\right]^{1/\theta}\right)$	$(1-\ln t)^\theta-1$
14	$\left(1+\left[(u^{-1/\theta}-1)^\theta+(v^{-1/\theta}-1)^\theta\right]^{1/\theta}\right)^{-\theta}$	$(t^{-1/\theta}-1)^\theta$

118 4 Archimedean Copulas

Table 4.1. One-parameter

(4.2.#)	$C_\theta(u,v)$	$\varphi_\theta(t)$
15	$\left\{\max\left(1-\left[(1-u^{1/\theta})^\theta+(1-v^{1/\theta})^\theta\right]^{1/\theta},0\right)\right\}^\theta$	$\left(1-t^{1/\theta}\right)^\theta$
16	$\frac{1}{2}\left(S+\sqrt{S^2+4\theta}\right),\ S=u+v-1-\theta\left(\frac{1}{u}+\frac{1}{v}-1\right)$	$\left(\frac{\theta}{t}+1\right)(1-t)$
17	$\left(1+\frac{[(1+u)^{-\theta}-1][(1+v)^{-\theta}-1]}{2^{-\theta}-1}\right)^{-1/\theta}-1$	$-\ln\frac{(1+t)^{-\theta}-1}{2^{-\theta}-1}$
18	$\max\left(1+\theta/\ln\left[e^{\theta/(u-1)}+e^{\theta/(v-1)}\right],0\right)$	$e^{\theta/(t-1)}$
19	$\theta/\ln\left(e^{\theta/u}+e^{\theta/v}-e^\theta\right)$	$e^{\theta/t}-e^\theta$
20	$\left[\ln\left(\exp(u^{-\theta})+\exp(v^{-\theta})-e\right)\right]^{-1/\theta}$	$\exp\left(t^{-\theta}\right)-e$
21	$1-(1-\{\max([1-(1-u)^\theta]^{1/\theta}+[1-(1-v)^\theta]^{1/\theta}-1,0)\}^\theta)^{1/\theta}$	$1-\left[1-(1-t)^\theta\right]^{1/\theta}$
22	$\max\left(\left[1-(1-u^\theta)\sqrt{1-(1-v^\theta)^2}-(1-v^\theta)\sqrt{1-(1-u^\theta)^2}\right]^{1/\theta},0\right)$	$\arcsin\left(1-t^\theta\right)$

Roger B. Nelsen, “An Introduction to Copulas”, 2nd Edition, Springer (New, York, 2009)



Outline

- Integrated Circuit Design and Test Laboratory at PSU.
- Background
  - Motivation
  - Multinormal vs copula-based multivariate modeling
  - Survey of copulas
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  - Application of the model
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Variable Retention Time

- A bistable atomic defect occurs everywhere in Si.

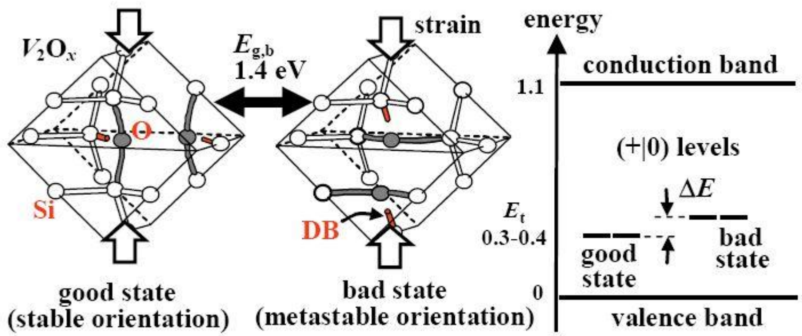
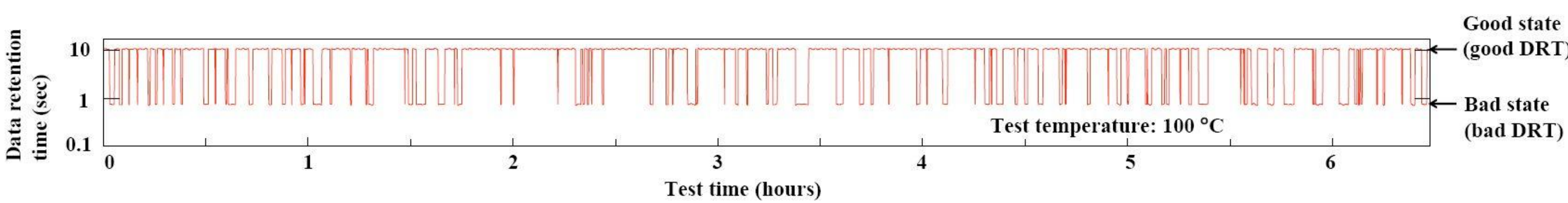


Fig. 5 V_2O_x defect model

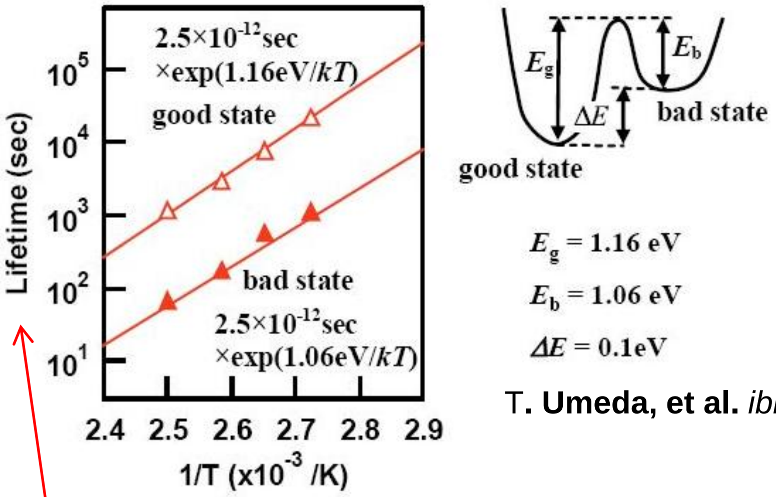
T. Umeda, et al. *"Single silicon vacancy-oxygen complex defect and variable retention time phenomenon in dynamic random access memories."* Applied Physics Letters Vol. 88 253504 (2006)

- When it falls at the near-surface gate/drain boundary of the DRAM capacitor pass xistor a bistable leakage current, and so bistable retention time, occurs.



Variable Retention Time, ct'd

- Retention times range from 100's of microseconds to seconds.
- Difference between min and max retention times varies from bit to bit.
  - In PSU experiment, 82% of measured bits were "stable" (min = max retention time).
- Bits are "stuck" in high or low retention time states for many minutes, or even hours!
- Time-in-state is thermally activated.



T. Umeda, et al. *ibid.*

Fig. 2 Thermal activation of the VRT.

(Time constants for exponential time-in-state distributions.)

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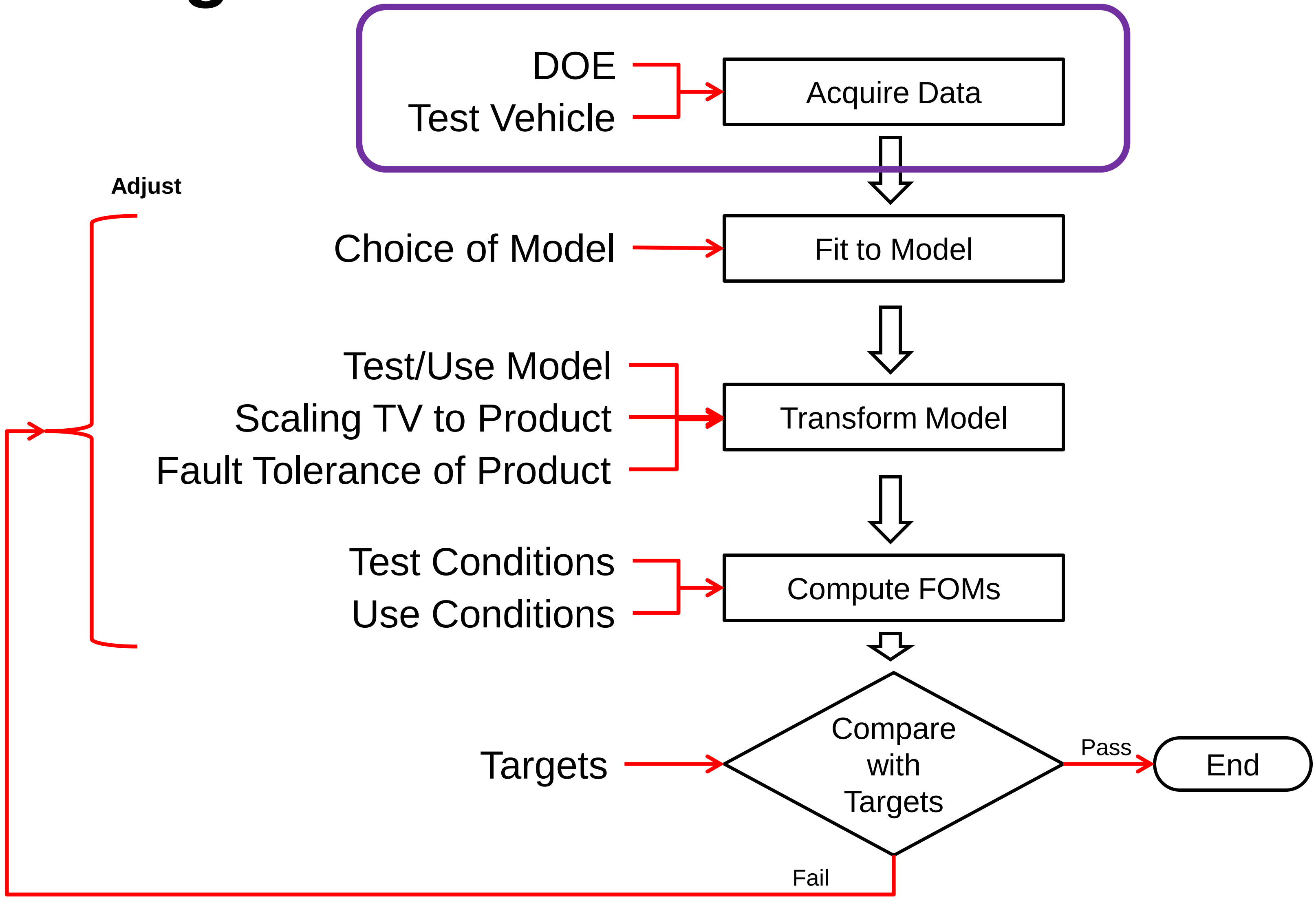
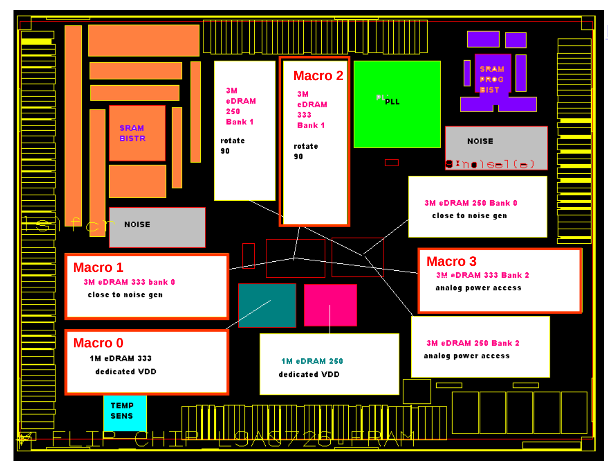
Modeling Miscorrelation



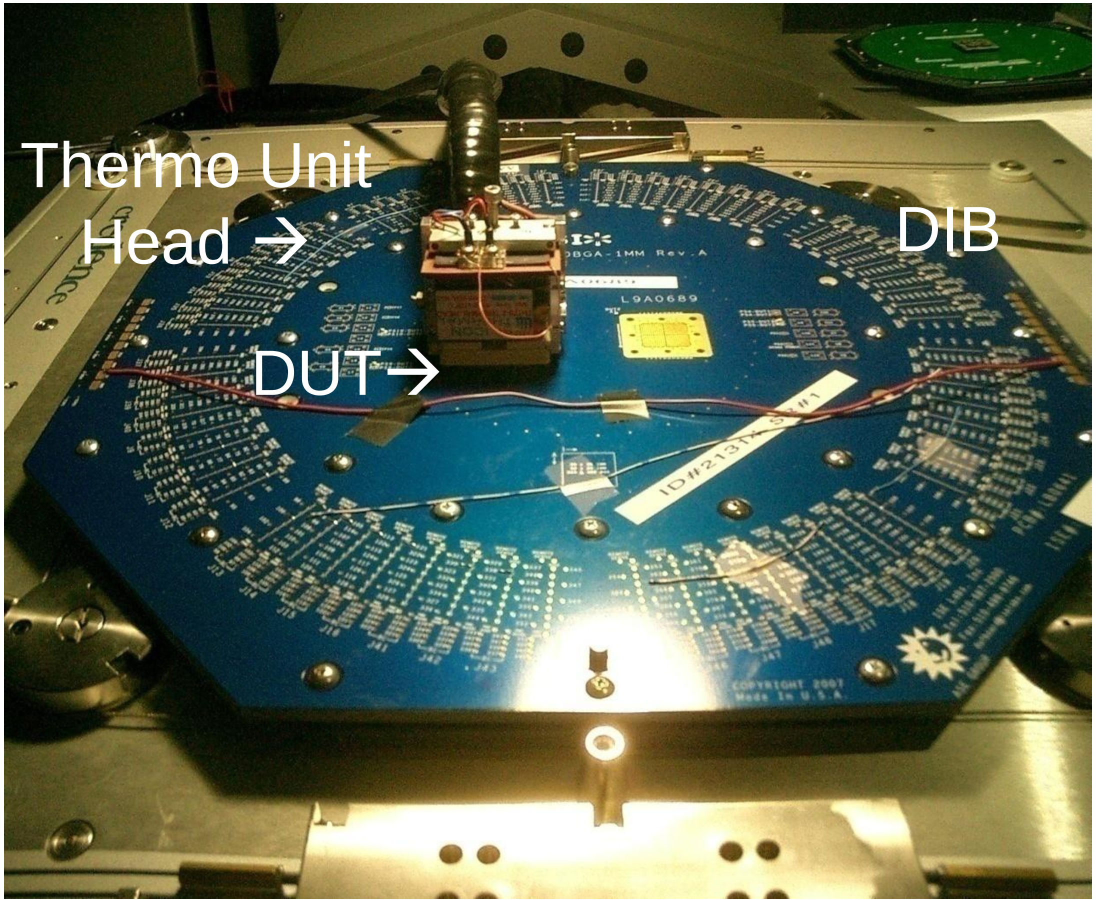
DRAM Experiment

- A 65 nm process DRAM test chip, packaged in BGA, was used.
- 10 random test chips, prescreened for gross faults, were selected.
- Each test chip unit has 4 identical arrays of 1,218,750 bits.
  - Number of bits tested = $10 \times 4 \times 1{,}218{,}750 = 48.75$ Mb
- Tested on Credence Quartet with 145 I/Os, and 7 power supplies using Silicon Thermal Powercool LB300-i for temperature control.
- Retention time for each bit was measured at..
  - 3 temperatures: 105ºC, 115ºC, 125ºC
  - 3 Vdd's: 0.8, 1.0, 1.2 volts
  - 2 Vp's: 0.4, 0.45 volts
- Repeated retention time measurements were made on each bit to characterize retention time variability.



Thanks to Satoshi Suzuki for acquiring the data!

Test Environment at PSU



- ICDT Lab at PSU.
- Credence Quartet IC tester.
- One chip per test.
- Temperature controller & sensor (thermocouple).

Temperature Controller

Repeated Bit Retention Measurement

- 12 retention times were measured 5 times for each bit.
- Retention times ranged from 60 au* to 604 au.
  (t_{ret} = 10 + i x 49.5, i =1 to 12)

* *Retention times are given in arbitrary units*

X = 0 (pass), or 1 (fail). X = **X (60 au)**; X = **X (604 au)**

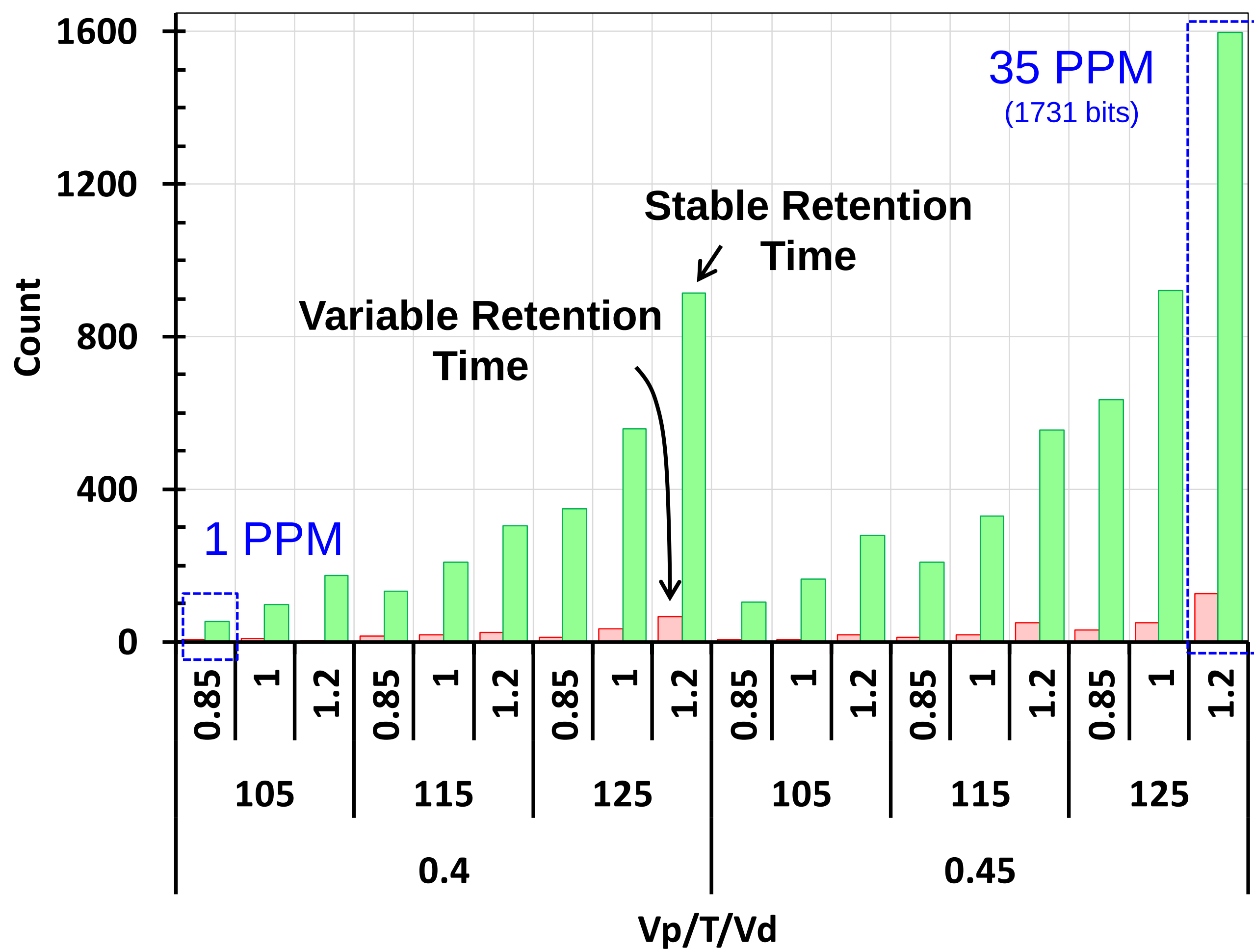
XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX	XXXXXXXXXXXX
Repetition 1	Repetition 2	Repetition 3	Repetition 4	Repetition 5

- Repetitions separated by variable durations, many hours.
- Repetition ensures that any variability will be captured.
- Examples
  - 000000000111 000011111111 000000001111 000000001111 000000000111 } Variable
  - 001101111111 000001111111 001111101111 000001111111 001111111111 } Variable
  - 000000011111 000000011111 000000011111 000000011111 000000011111 } Stable
  - 000000001111 000000001111 000000011111 000000001111 000000011111 } Stable

Variation ≤ 1 is regarded as "stable".

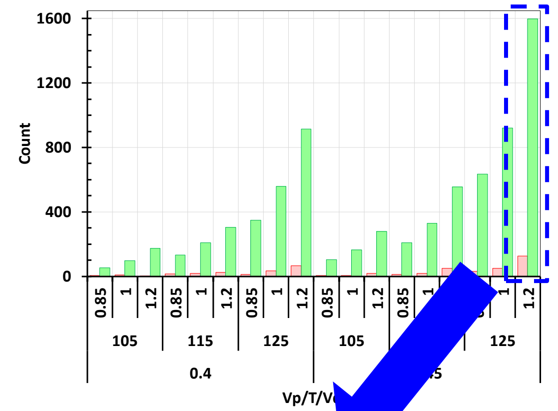
Environmental Dependence

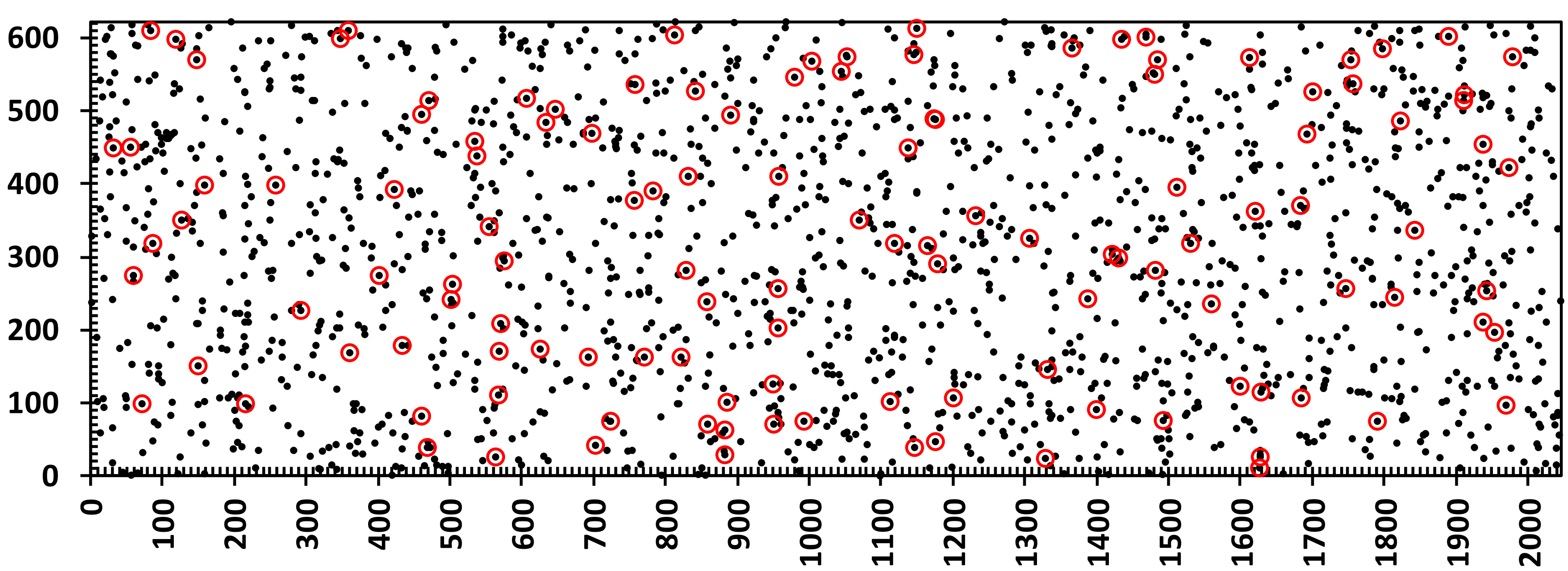
- The sample was from 48,750,000 bits (49Mb).
- The sample was bits with retention times ≤ 604 au.
- Failing bit count was 1 PPM to 35 PPM depending on environmental condition.
- 18% of bits with retention time ≤ 604 au were VRT (pink).



Random Spatial Distribution Seen

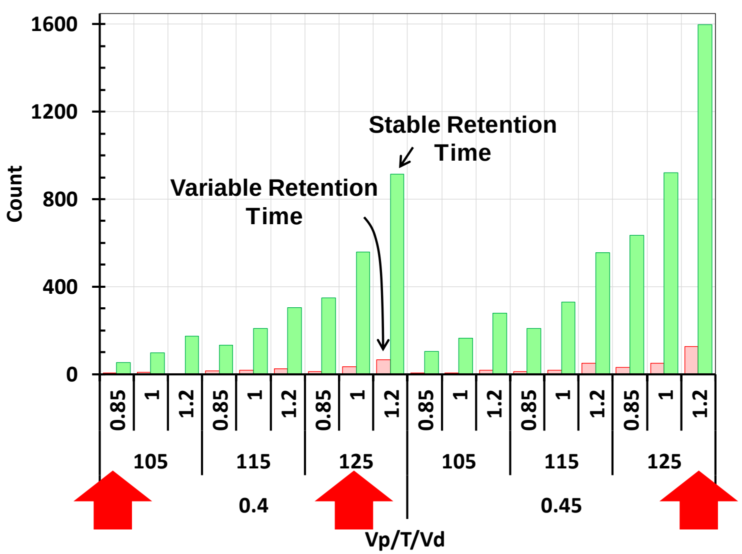
- Bits with retention times ≤ 604 au, sampled from 48,750,000 bits.
- At maximum environmental condition V_{dd} = 1.2, V_p = 0.45, T = 125 °C
- 1731 bit failures, 126 of these (red circles) had variable retention times.





Retention Time Distributions

- Tabulate data assuming $r_{Test}/r_{Use} = t_{min}/t_{max}$ or t_{max}/t_{min} with equal probability.
- Gives symmetrical model, equal margins, easy to fit.
- Will be transformed to realistic Test/Use model later.



Nominal Skew T/Vp/Vd: 105/.4/.85

r_Test (au)													
604	0	0	0	0	0	0	0	0	0	0	0	3	N/A
555	0	0	0	0	0	0	0	0	0	1	3	1	1
505	0	0	0	0	0	0	0	0	1	1	3	5	1
456	0	0	0	0	0	0	0	0	3	4	7	0	0
406	0	0	0	0	0	0	0	3	2	2	0	0	0
357	0	0	0	0	0	0	4	3	1	1	0	0	0
307	0	0	0	0	0	0	2	1	0	0	0	0	0
258	0	0	0	0	0	2	0	0	0	0	0	0	0
208	0	0	0	0	2	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	60	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)

Nominal Skew T/Vp/Vd: 125/.4/1.

r_Test (au)													
604	0	0	0	0	0	0	1	0	1	0	7	23	N/A
555	0	0	0	0	1	0	0	0	0	2	31	38	17
505	0	0	0	0	0	0	0	1	3	38	37	29	7
456	0	0	0	0	0	0	0	3	22	42	17	0	0
406	0	0	0	0	0	0	0	19	24	19	2	0	0
357	0	0	0	0	0	2	12	25	14	2	0	0	0
307	0	0	0	0	0	8	26	18	0	0	0	0	0
258	0	0	0	0	6	24	12	0	0	0	1	0	0
208	0	0	0	3	11	4	0	0	0	0	0	0	0
159	0	0	0	23	7	0	0	0	0	0	0	0	0
109	0	0	2	2	0	0	0	0	0	0	0	0	0
60	0	5	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)

Nominal Skew T/Vp/Vd: 125/.45/1.2

r_Test (au)													
604	0	0	0	0	1	0	1	1	1	1	18	69	N/A
555	0	0	0	0	0	0	0	0	1	14	104	95	59
505	0	0	1	0	0	0	1	2	10	97	92	120	20
456	0	0	0	0	0	0	1	5	46	83	86	14	2
406	0	0	0	0	0	1	4	59	80	56	6	2	3
357	0	0	0	0	0	1	29	68	53	2	1	2	0
307	0	0	0	0	0	36	56	38	2	1	1	0	0
258	0	0	0	0	15	71	20	2	1	0	0	0	0
208	0	0	0	12	38	18	0	0	0	1	0	0	0
159	0	0	8	43	9	0	0	0	0	0	0	0	1
109	0	1	19	6	0	0	0	0	1	0	0	0	0
60	0	11	2	0	0	0	0	0	0	0	0	0	0
0	5	1	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604 r_Use (au)

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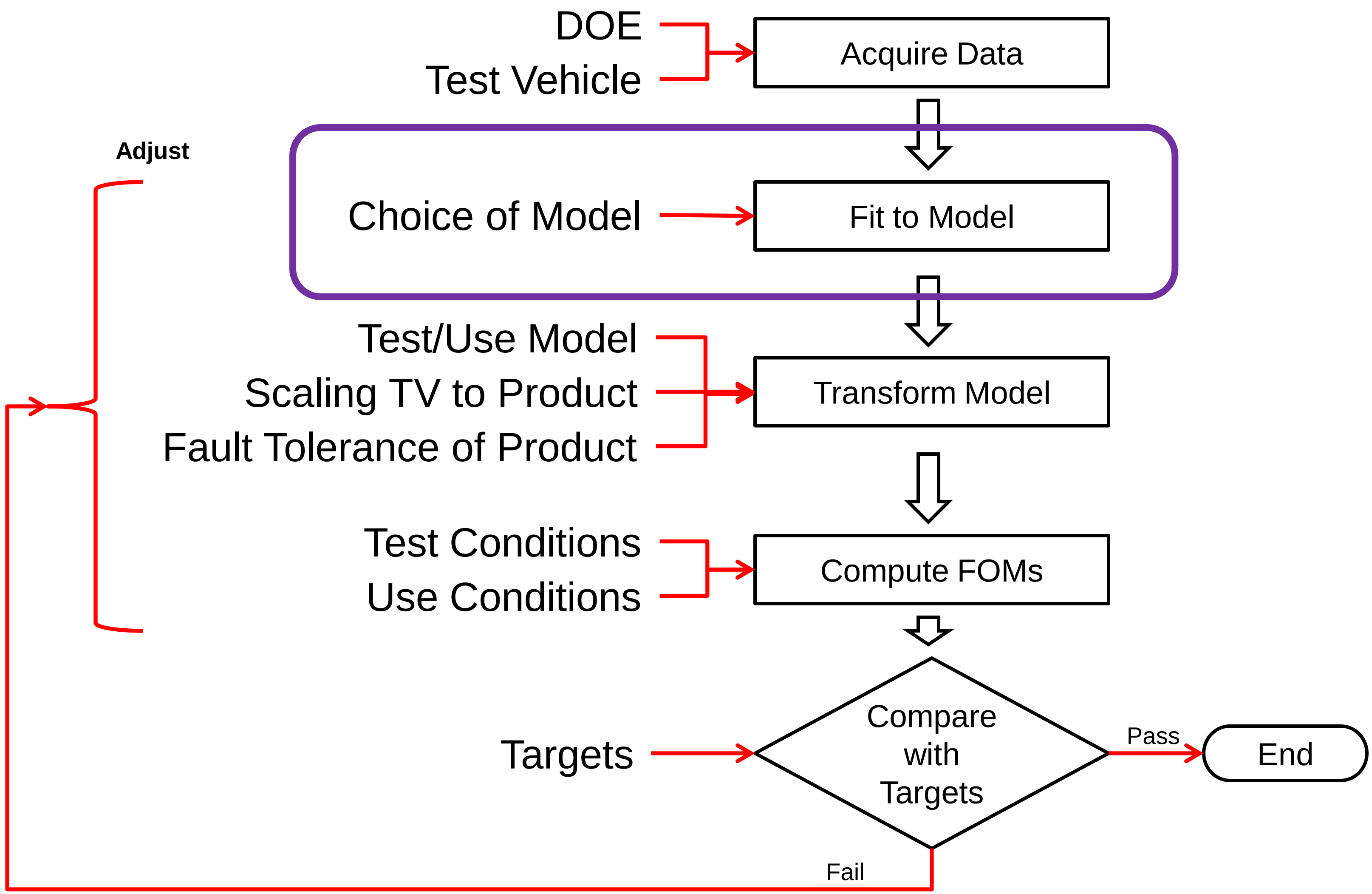


Fit of Marginal Distributions

T = 125 C
Vp = 0.45 V
Vd = 1.2 V
SS = 48750000

$$\text{Probability mass in cell} = \frac{92}{48750000} = 1.89\times10^{-6} = 1.89\ \text{DPPM}$$

Test: F = Cum N/SS (PPM)	Cum N	N	r (au)															
			604	0	0	0	0	1	0	1	1	1	1	18	69	N/A		
33.6	1639	273	555	0	0	0	0	0	0	0	0	1	14	104	95	59		
28.0	1366	343	505	0	0	1	0	0	0	1	2	10	97	92	120	20		
21.0	1023	237	456	0	0	0	0	0	0	1	5	46	83	86	14	2		
16.1	786	211	406	0	0	0	0	0	1	4	59	80	56	6	2	3		
11.8	575	156	357	0	0	0	0	0	1	29	68	53	2	1	2	0		
8.6	419	134	307	0	0	0	0	0	36	56	38	2	1	1	0	0		
5.8	285	109	258	0	0	0	0	15	71	20	2	1	0	0	0	0		
3.6	176	69	208	0	0	0	12	38	18	0	0	0	1	0	0	0		
2.2	107	61	159	0	0	8	43	9	0	0	0	0	0	0	0	1		
0.9	46	27	109	0	1	19	6	0	0	0	0	1	0	0	0	0		
0.4	19	13	60	0	11	2	0	0	0	0	0	0	0	0	0	0		
0.1	6	6	0	5	1	0	0	0	0	0	0	0	0	0	0	0		
				0	60	109	159	208	258	307	357	406	456	505	555	604	r (au)	Use
				5	13	30	61	63	127	112	175	195	255	308	302		N	
				5	18	48	109	172	299	411	586	781	1036	1344	1646		Cum N	
				0.1	0.4	1.0	2.2	3.5	6.1	8.4	12.0	16.0	21.3	27.6	33.8		F=Cum N/SS (PPM)	

$$\text{Marginal cumulative probability} = \frac{299}{48750000} = 6.13\times10^{-6} = 6.13\ \text{DPPM}$$

Weibull Fit of Marginal Dist'ns

$$W = \ln\left[-\ln\left(1-F\right)\right]$$

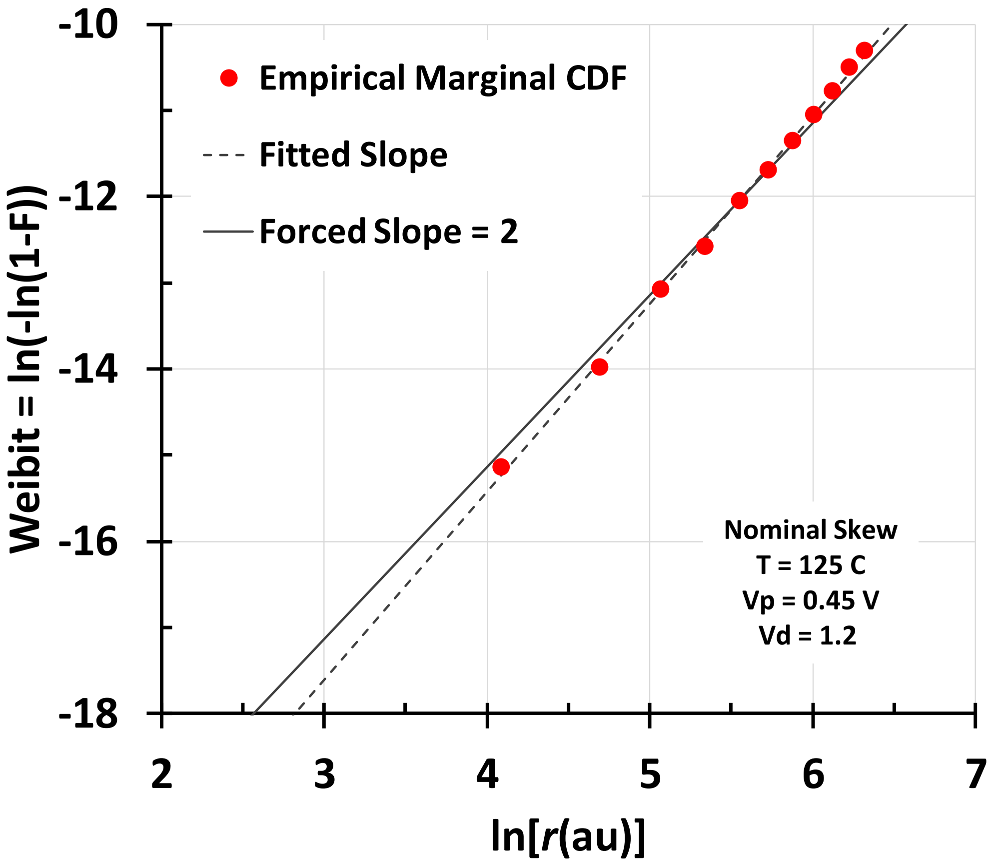
$$F = 1-\exp\left[-\left(\frac{r}{\alpha}\right)^{\beta}\right]$$

$$W = \beta \ln r - \beta \ln \alpha$$

Slope (Forced)

Intercept (Determines α)

- Slope of Weibull plots is close to 2 for all environmental conditions and skews.
- Determine lnα for each of 18 environmental conditions.



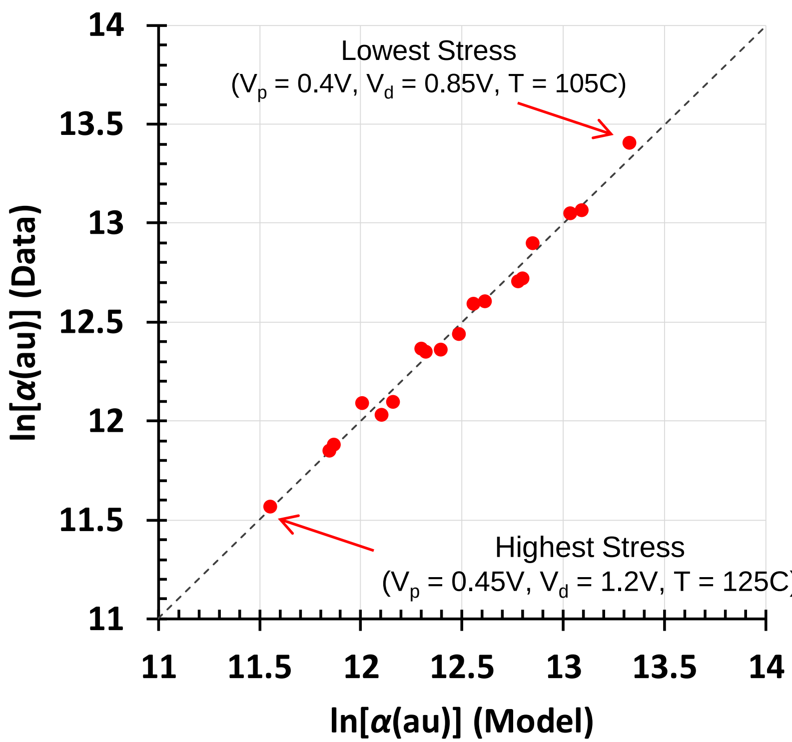
Fit lnα to an Environmental Model

- Determine $\ln\alpha_0$, *a*, *b*, *Q* by least-squares regression of...

... Weibull α at each environmental condition using a reference condition

$$\ln\alpha = \ln\alpha_0 + a(V_p - V_{p0}) + b(V_d - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

- Very good fit.
- lnα is a convenient measure of environmental condition.
- A given α defines a locus of statistically equivalent set points.
  - Gives useful flexibility in test programs.



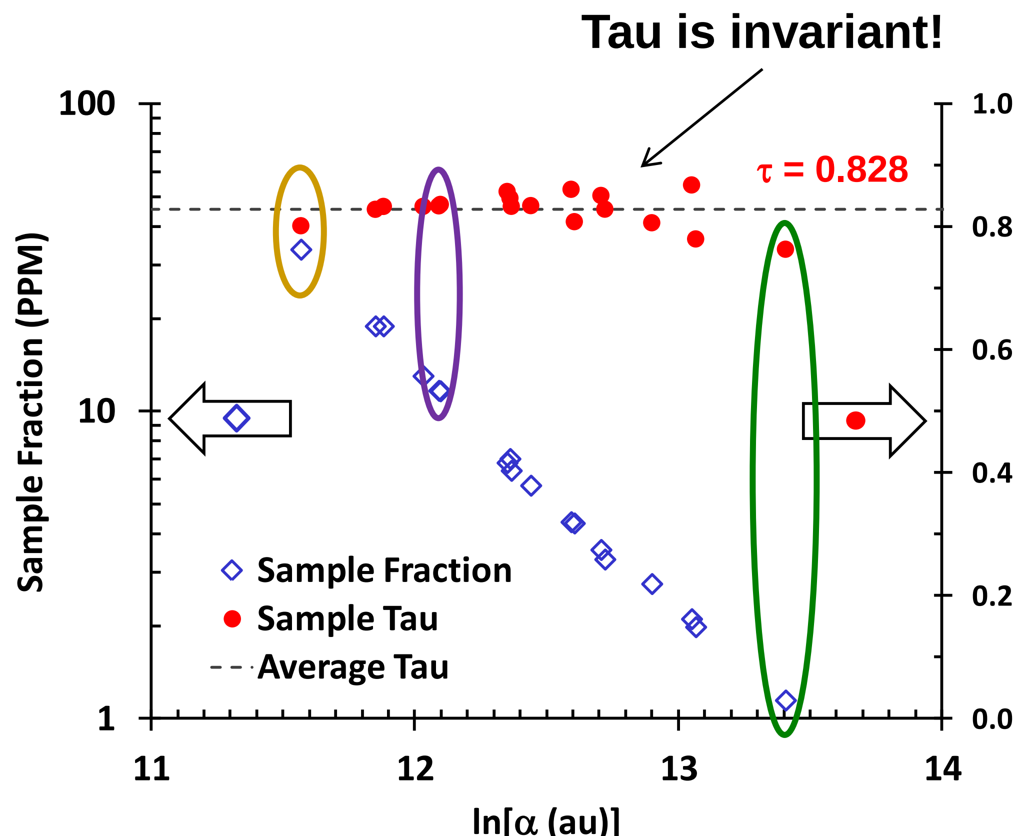
Choice and Fit of Dependency Model

- The environmental condition controls the degree of truncation of data.

r_Test (au)	Nominal Skew T/Vp/Vd: 125/.45/1.2											
604	0	0	0	0	1	0	1	1	1	1	18	69	N/A
555	0	0	0	0	0	0	0	0	1	14	104	95	59
505	0	0	1	0	0	0	1	2	10	97	92	120	20
456	0	0	0	0	0	0	1	5	46	83	86	14	2
406	0	0	0	0	0	1	4	59	80	56	6	2	3
357	0	0	0	0	0	1	29	68	53	2	1	2	0
307	0	0	0	0	0	36	56	38	2	1	1	0	0
258	0	0	0	0	15	71	20	2	1	0	0	0	0
208	0	0	0	12	38	18	0	0	0	1	0	0	0
159	0	0	8	43	9	0	0	0	0	0	0	0	1
109	0	1	19	6	0	0	0	0	1	0	0	0	0
60	0	11	2	0	0	0	0	0	0	0	0	0	0
0	5	1	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604	r_Use (au)

r_Test (au)	Nominal Skew T/Vp/Vd: 125/.4/1.											
604	0	0	0	0	0	0	1	0	1	0	7	23	N/A
555	0	0	0	0	1	0	0	0	0	2	31	38	17
505	0	0	0	0	0	0	0	1	3	38	37	29	7
456	0	0	0	0	0	0	0	3	22	42	17	0	0
406	0	0	0	0	0	0	0	19	24	19	2	0	0
357	0	0	0	0	0	2	12	25	14	2	0	0	0
307	0	0	0	0	0	8	26	18	0	0	0	0	0
258	0	0	0	0	6	24	12	0	0	0	1	0	0
208	0	0	0	3	11	4	0	0	0	0	0	0	0
159	0	0	0	23	7	0	0	0	0	0	0	0	0
109	0	0	2	2	0	0	0	0	0	0	0	0	0
60	0	5	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	59.5	109	159	208	258	307	357	406	456	505	555	604	r_Use (au)

r_Test (au)	Nominal Skew T/Vp/Vd: 105/.4/.85											
604	0	0	0	0	0	0	0	0	0	0	0	3	N/A
555	0	0	0	0	0	0	0	0	0	1	3	1	1
505	0	0	0	0	0	0	0	0	1	1	3	5	1
456	0	0	0	0	0	0	0	0	3	4	7	0	0
406	0	0	0	0	0	0	0	3	2	2	0	0	0
357	0	0	0	0	0	0	4	3	1	1	0	0	0
307	0	0	0	0	0	0	2	1	0	0	0	0	0
258	0	0	0	0	0	2	0	0	0	0	0	0	0
208	0	0	0	0	2	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	60	109	159	208	258	307	357	406	456	505	555	604	r_Use (au)



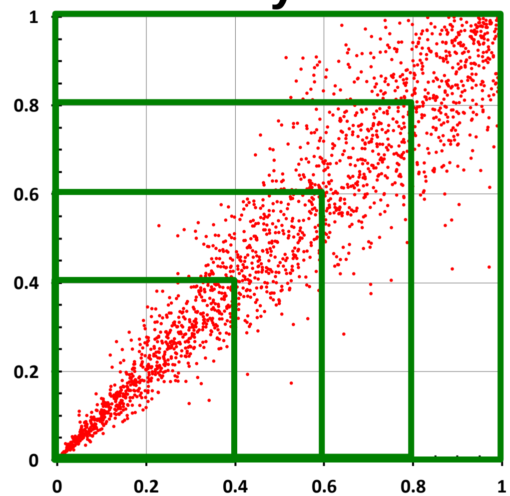
Environmental Condition:

$$\ln\alpha = \ln\alpha_0 + a(V_p - V_{p0}) + b(V_d - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

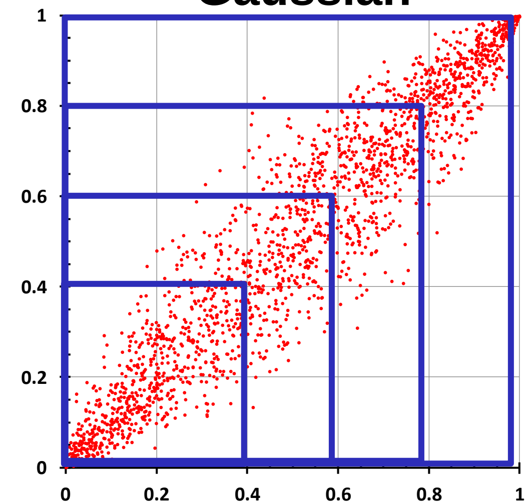
Tau vs Copula Truncation

- Dependence fades away in the tail of the Gaussian copula.
- Dependence is invariant as the Clayton copula is truncated.

Clayton



Gaussian



The Canadian Journal of Statistics
Vol. 33, No. 3, 2005, Pages 465–468
La revue canadienne de statistique

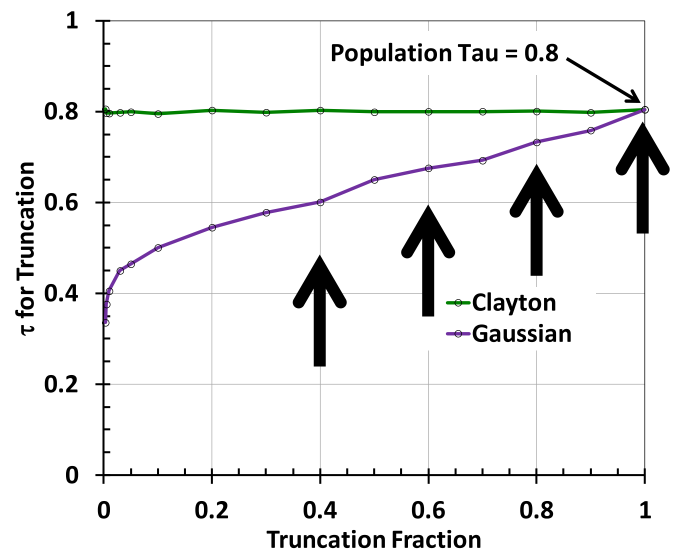
On the preservation of copula structure under truncation

David OAKES

Key words and phrases: Archimedean copula; bivariate distribution; Clayton model; Cook–Johnson model; gamma frailty model; Kendall's tau.

MSC 2000: Primary 62H20; secondary 62P10.

Abstract: The author characterizes the copula associated with the bivariate survival model of Clayton (1978) as the only absolutely continuous copula that is preserved under bivariate truncation.

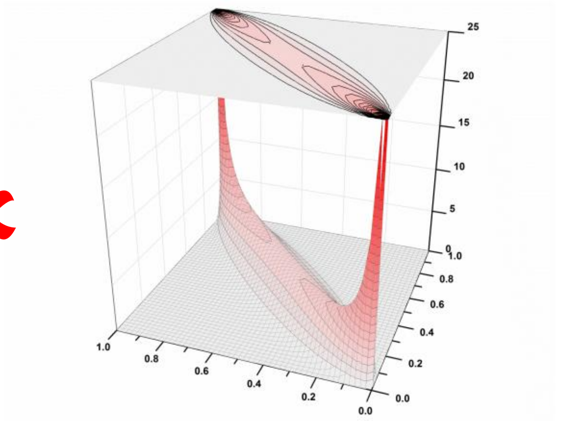


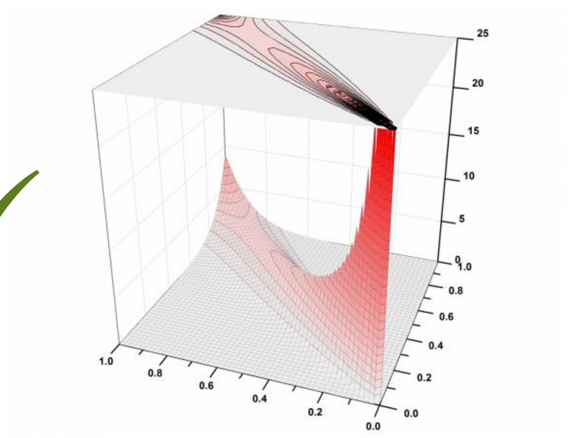
Thanks to Roger Nelsen for pointing this out!

Model, Fitted and Chosen

- Gaussian model rejected because it does not have the correct tail dependence.

Margin	β	2.0
	$\ln[a_0\,(\text{au})]$	11.57
	a (V^{-1})	-5.79
	b (V^{-1})	-1.55
	Q (eV)	0.605
	V_{p0} (V)	0.45
	V_{d0} (V)	1.2
	T_0 (°C)	125.0
Dependence (ρ = 0.999305)	Sample Tau, τ'	0.828
	Clayton Copula, θ	9.74
	Gaussian Copula $(1-\rho)\text{x}10^3$	0.695





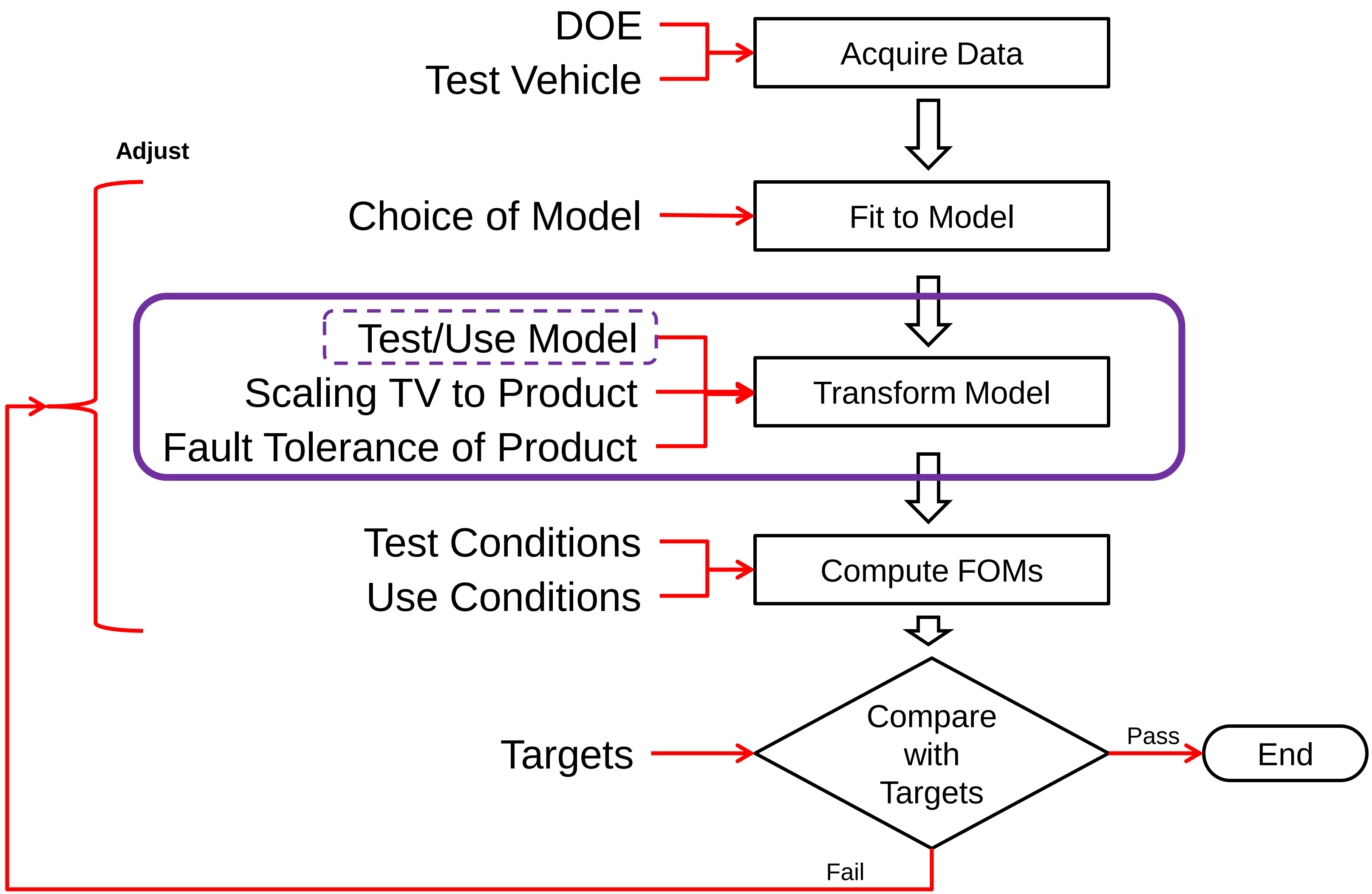
A simple single-parameter copula describes the dependency structure of the DRAM VRT phenomenon across *all* environmental conditions.

$$C(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta} \quad \theta = 9.74$$

Outline

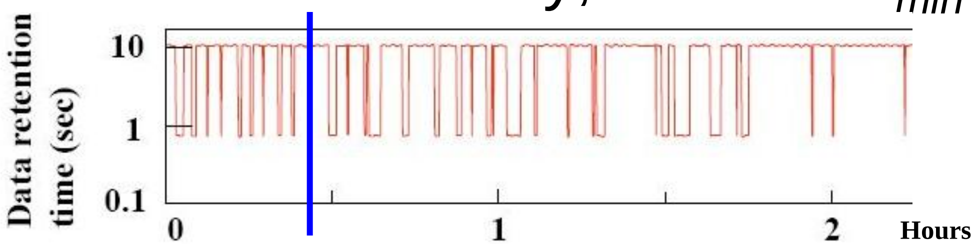
- Integrated Circuit Design and Test Laboratory at PSU.
- Background
  - Motivation
  - Multinormal vs copula-based multivariate modeling
  - Survey of copulas
- DRAM Case Study
  - VRT mechanism
  - Data acquisition
  - Fitting a model
  - Application of the model
- Final Thoughts

Modeling Miscorrelation

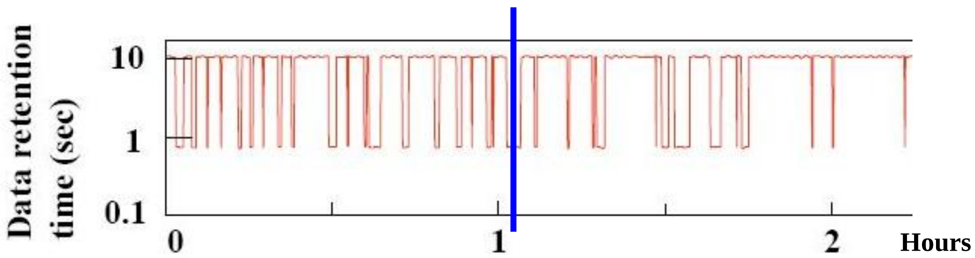


Test/Use Model for a Bit

- Test is momentary, so either r_{min} or r_{max} may be observed.



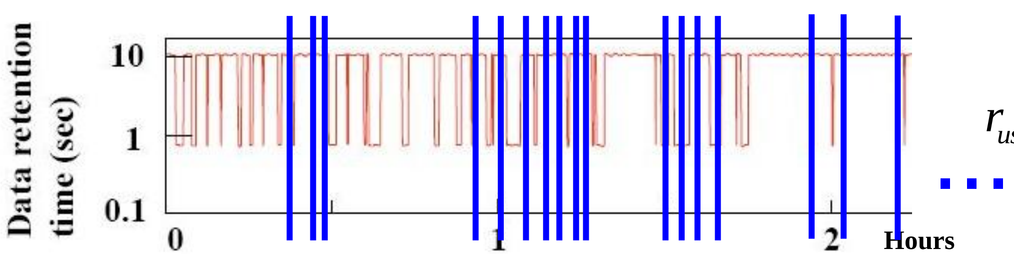
$r_{test} = r_{\max}$ Probability of occurrence: s



$r_{test} = r_{\min}$ Probability of occurrence: $(1 - s)$

s = "duty cycle" of bit variability

- Use is repeated, r_{min} (worst case) will certainly occur.



$r_{use} = r_{\min}$ All the time.

Map Fitted Copula to Test/Use Model

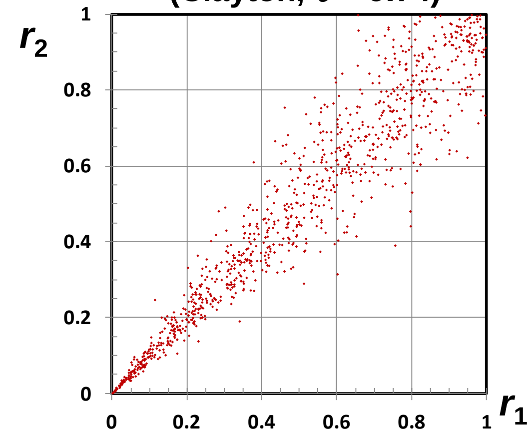
- Each bit has r_{min} and r_{max}.
- r_{min} and r_{max} were mapped to r_1 and r_2 and fitted to an exchangeable copula.

$C(x,y) = C(y,x)$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} r_{\max} \\ r_{\min} \end{bmatrix} \qquad 50\% \text{ of the time}$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} r_{\min} \\ r_{\max} \end{bmatrix} \qquad 50\% \text{ of the time}$$

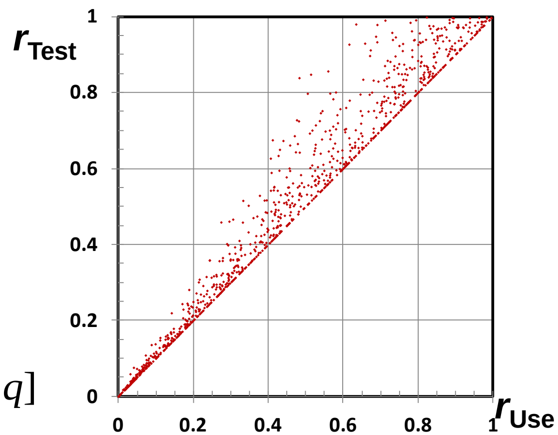
$$C(x,y) = \left[x^{-\theta} + y^{-\theta} - 1\right]^{-1/\theta}$$



- Fitted copula maps to Test/Use model:

$$r_{test} = \begin{cases} r_{\min} = \min[r_1, r_2] & (1-s) \text{ of the time} \\ r_{\max} = \max[r_1, r_2] & s \text{ of the time} \end{cases}$$

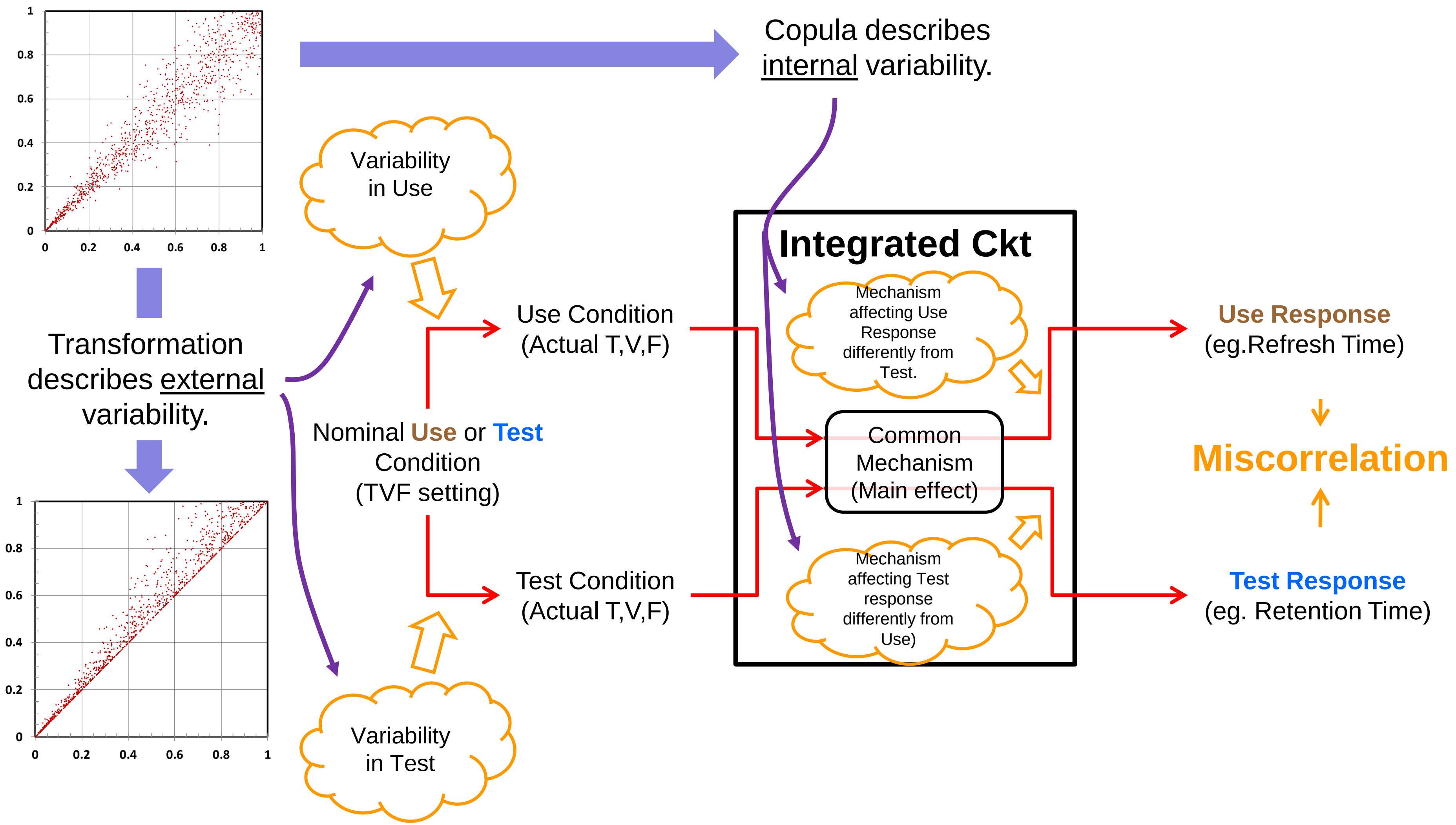
$$r_{use} = r_{\min} = \min[r_1, r_2] \qquad \text{All the time.}$$



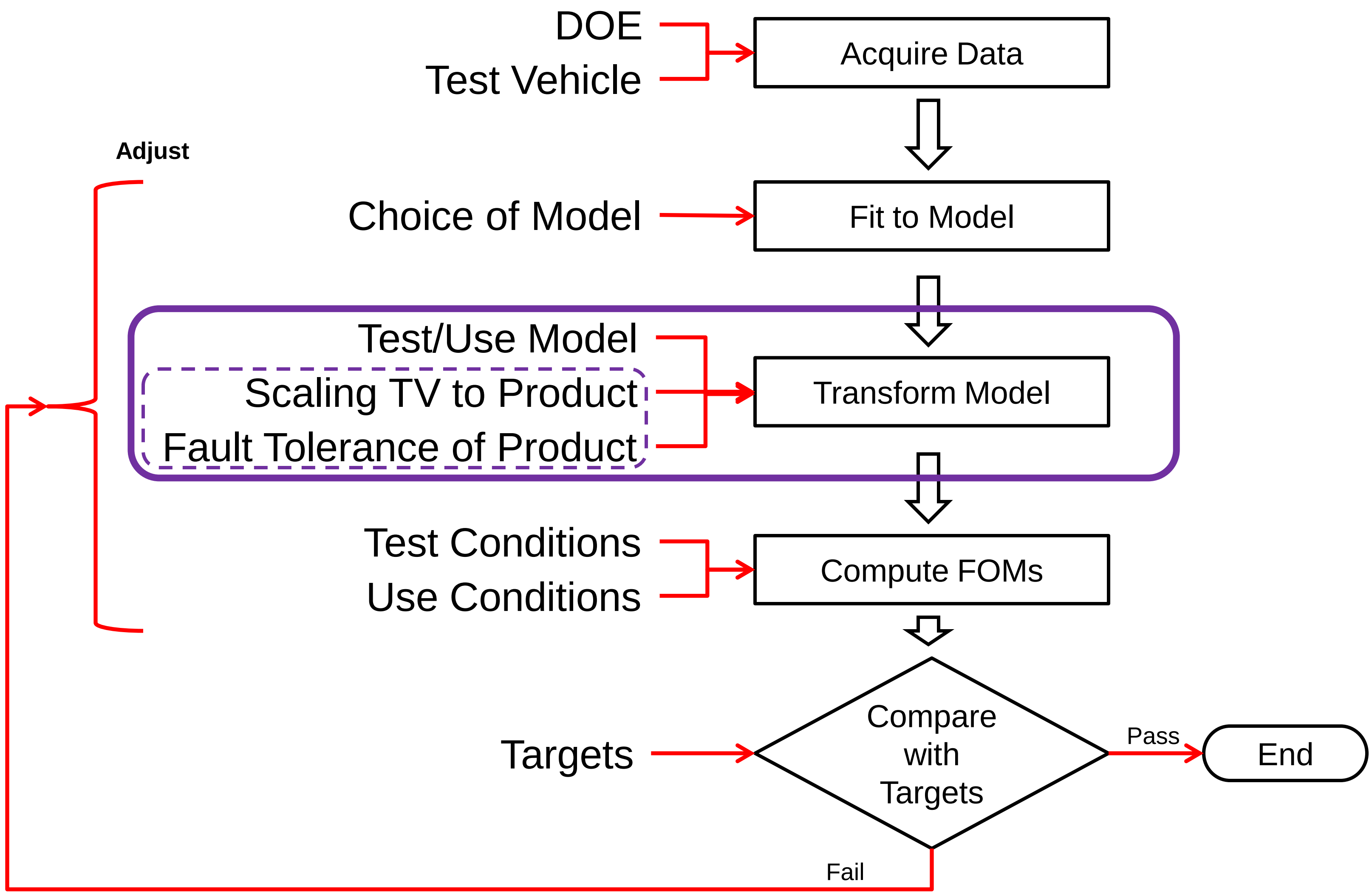
- Mathematically.. (*D* is a pseudo-copula.)

$$D(p,q) = s\left[C(p,q) + C(q,z) - C(p,z)\right] + (1-s)\left[2z - C(z,z)\right], \quad z = \min[p,q]$$

Causes of Miscorrelation



Modeling Miscorrelation



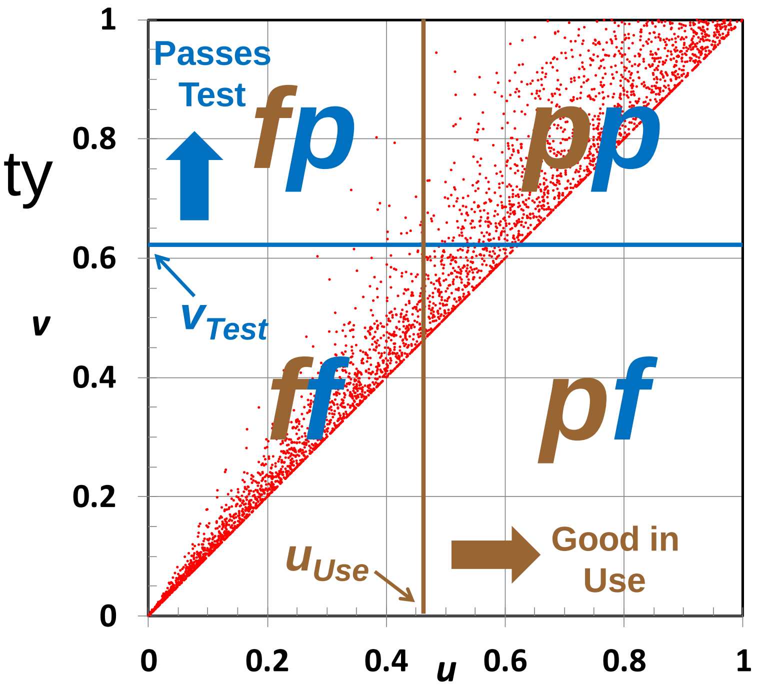
Statistics of Arrays

- Probabilities that a bit falls into each of the 4 categories are, in terms of *D*,

(**First subscript letter** is for Use, **second subscript letter** is for Test.)

$$p_{fp} = D(u,1) - D(u,v) \qquad p_{pf} = D(1,v) - D(u,v)$$

$$p_{ff} = D(u,v) \qquad p_{pp} = 1 - p_{fp} - p_{pf} - p_{ff}$$



- If an array has *N* bits, the probability that it has *exactly* n_{fp}, n_{pf}, and n_{ff} bits in *fp*, *pf*, *ff* categories is

$$\frac{\lambda_{fp}^{n_{fp}} \exp(-\lambda_{fp})}{n_{fp}!} \frac{\lambda_{pf}^{n_{pf}} \exp(-\lambda_{pf})}{n_{pf}!} \frac{\lambda_{ff}^{n_{ff}} \exp(-\lambda_{ff})}{n_{ff}!}$$

where

$$\lambda_{fp} = Np_{fp}, \ \lambda_{pf} = Np_{pf}, \ \lambda_{ff} = Np_{ff}.$$

- Use of Poisson statistics is well justified.

Statistics of Arrays: Example

- Fraction of arrays perfect in Use, irrespective of Test.

$n_{fp} = n_{ff} = 0$ $\qquad$ $0 \le n_{pf} \le \infty$

$$\Pr = \sum_{\substack{n_{fp}=n_{ff}=0 \\ 0 \le n_{pf} \le \infty}} \frac{\lambda_{fp}^{n_{fp}} \exp(-\lambda_{fp})}{n_{fp}!} \frac{\lambda_{pf}^{n_{pf}} \exp(-\lambda_{pf})}{n_{pf}!} \frac{\lambda_{ff}^{n_{ff}} \exp(-\lambda_{ff})}{n_{ff}!}$$

$$= \exp\left[-(\lambda_{fp} + \lambda_{ff})\right] = \exp\left[-ND(u,1)\right] = \exp\left[-N\left(2u - C(u,u)\right)\right]$$

where

$$C(u,v) = \left[u^{-\theta} + v^{-\theta} - 1\right]^{-1/\theta}$$

$$u = 1 - \exp\left[-\left(\frac{r_{use}}{\alpha_{use}}\right)^{\beta}\right]$$

$$\ln \alpha_{use} = \ln \alpha_0 + a(V_{p(use)} - V_{p0}) + b(V_{d(use)} - V_{d0}) + \frac{Q}{k_B}\left(\frac{1}{T_{use}} - \frac{1}{T_0}\right)$$

Fault Tolerance of Arrays

- If an array tolerates up to n_u bad bits in Use and up to n_t bad bits in Test...
- Passes Test, irrespective of Use requires..

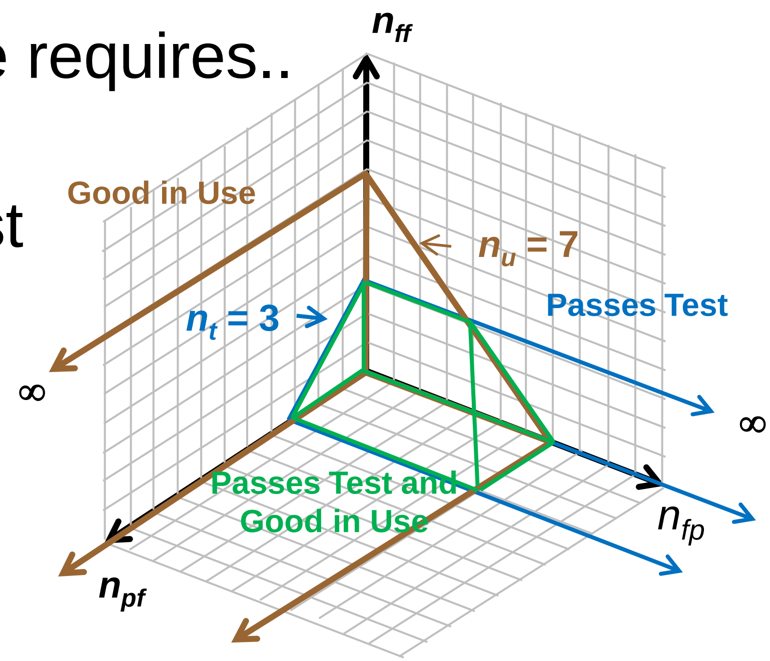
$$n_{pf} + n_{ff} \le n_t \qquad 0 \le n_{fp} \le \infty$$

- Good in Use, irrespective of Test requires..

$$n_{fp} + n_{ff} \le n_u \qquad 0 \le n_{pf} \le \infty$$

- Passes Test and Good in Use requires..

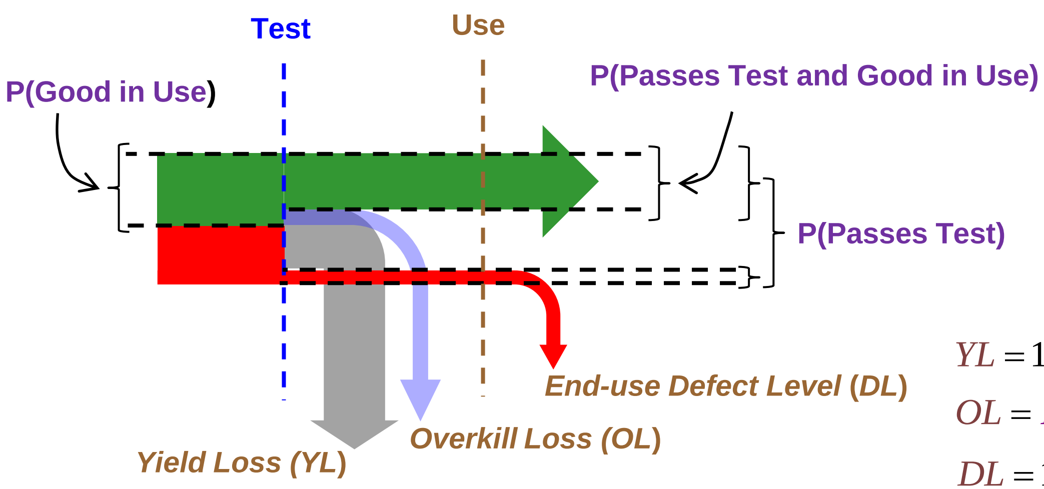
$$n_{pf} + n_{ff} \le n_t \qquad n_{fp} + n_{ff} \le n_u$$



- Sum terms over index space to get *array* probabilities
  - *P*(Good in Use), *P*(Passes Test), *P*(Passes Test and Good in Use)

Calculate FOMs from Probabilities

- Figures of Merit are *designed*
  - With identified stakeholder in mind.
  - So that values *all* lie in the range [0,1].
  - So that more is worse for *all* FOMs.
  - To compare with do-not-exceed targets.

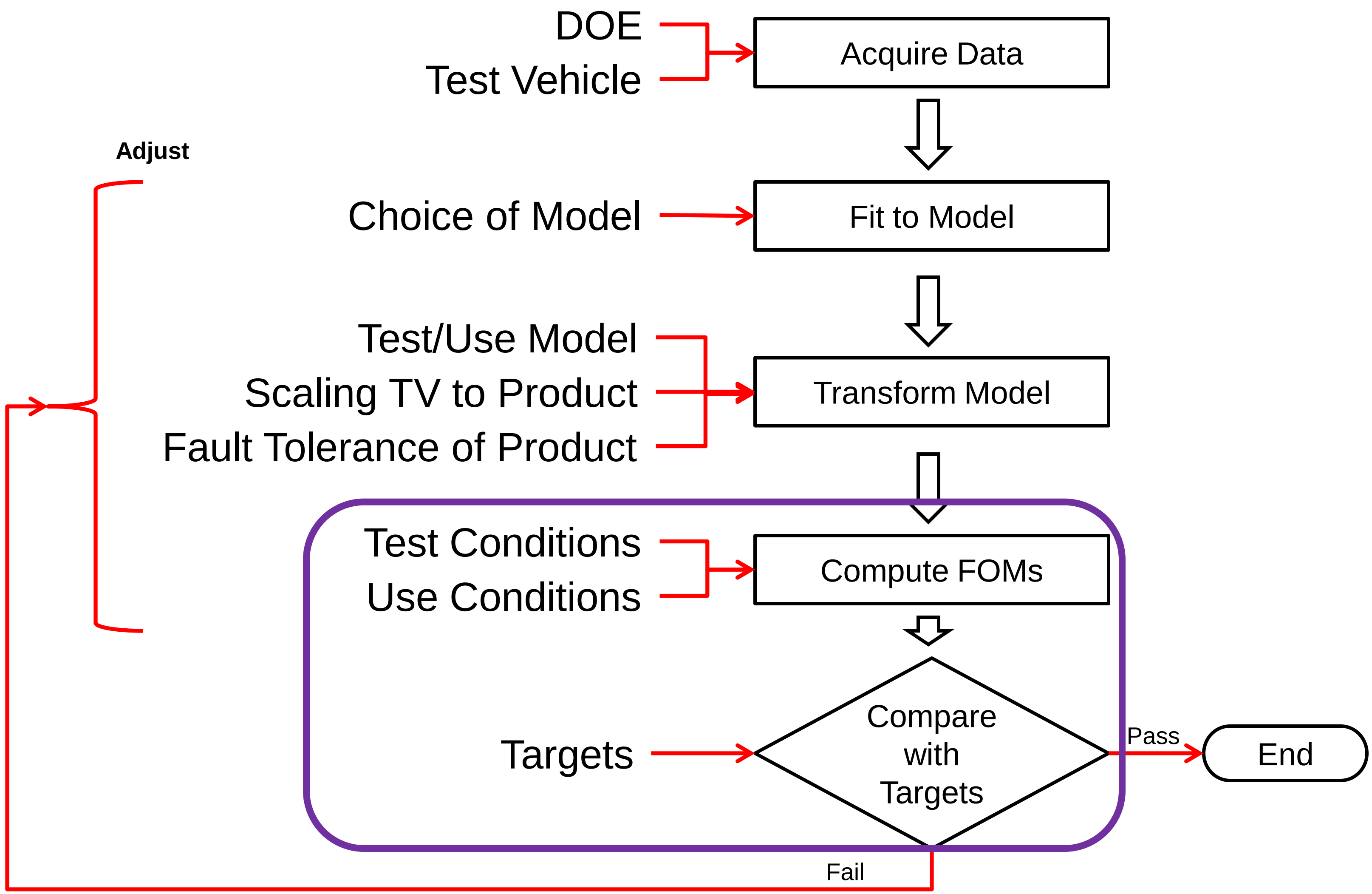


$$YL = 1 - P(\text{Passes Test})$$

$$OL = P(\text{Good in Use}) - P(\text{Passes Test and Good in Use})$$

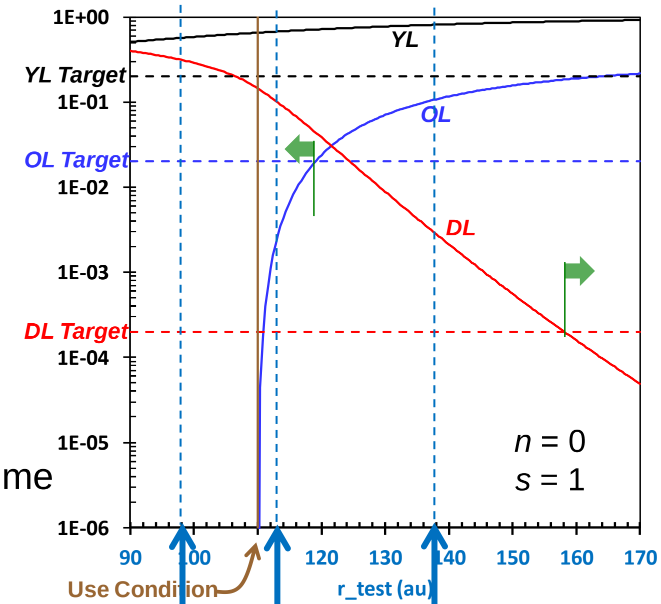
$$DL = 1 - P(\text{Passes Test and Good in Use}) / P(\text{Passes Test})$$

Modeling Miscorrelation



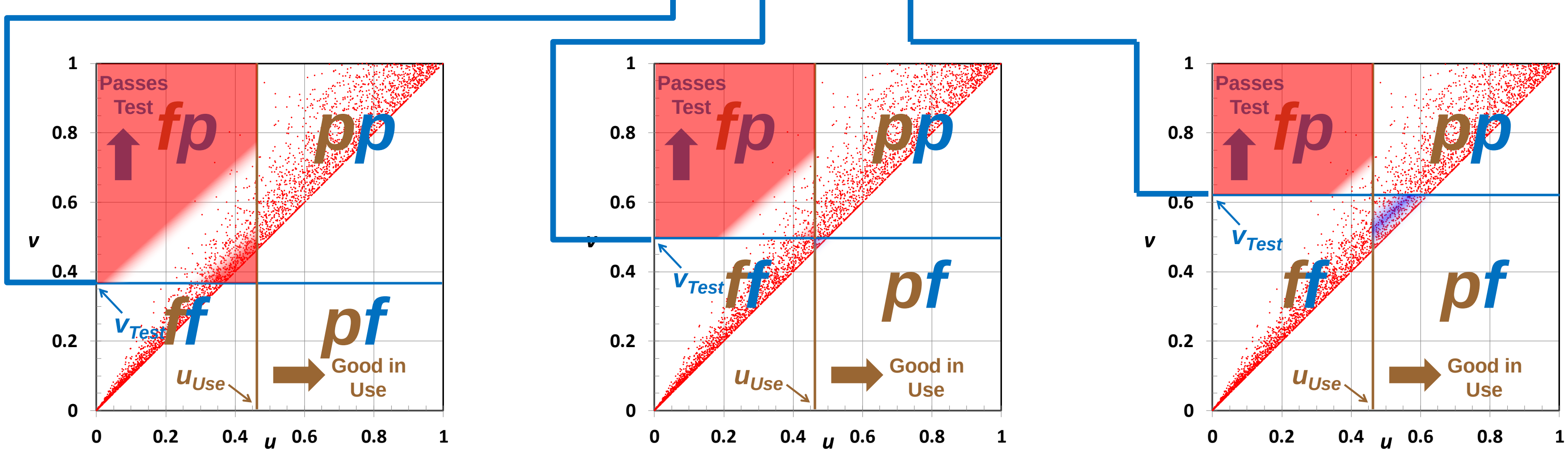
Plot FOMs as Test Sweeps Past Use

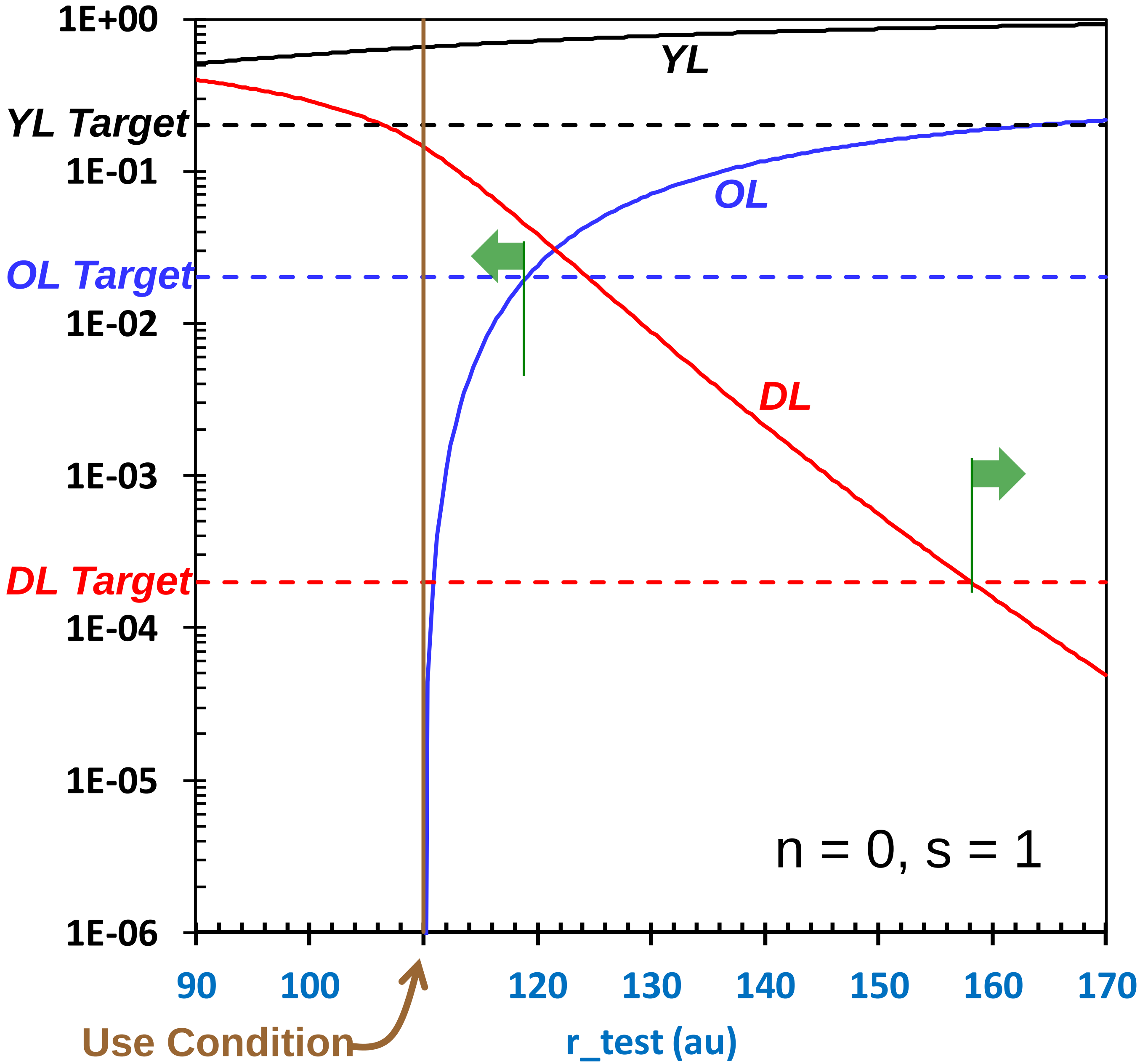
<u>Design</u>
1 Mb Array (2^{20} bits)
Tolerates *n* bad bits (varies) in Test and Use.
<u>Use Condition (Datasheet)</u>
Refresh = 110 au
T = 125 °C
V_p = 0.45 V
V_d = 1.2 V
<u>Test Condition</u>
Same as Use except retention time is swept past Use refresh time spec.

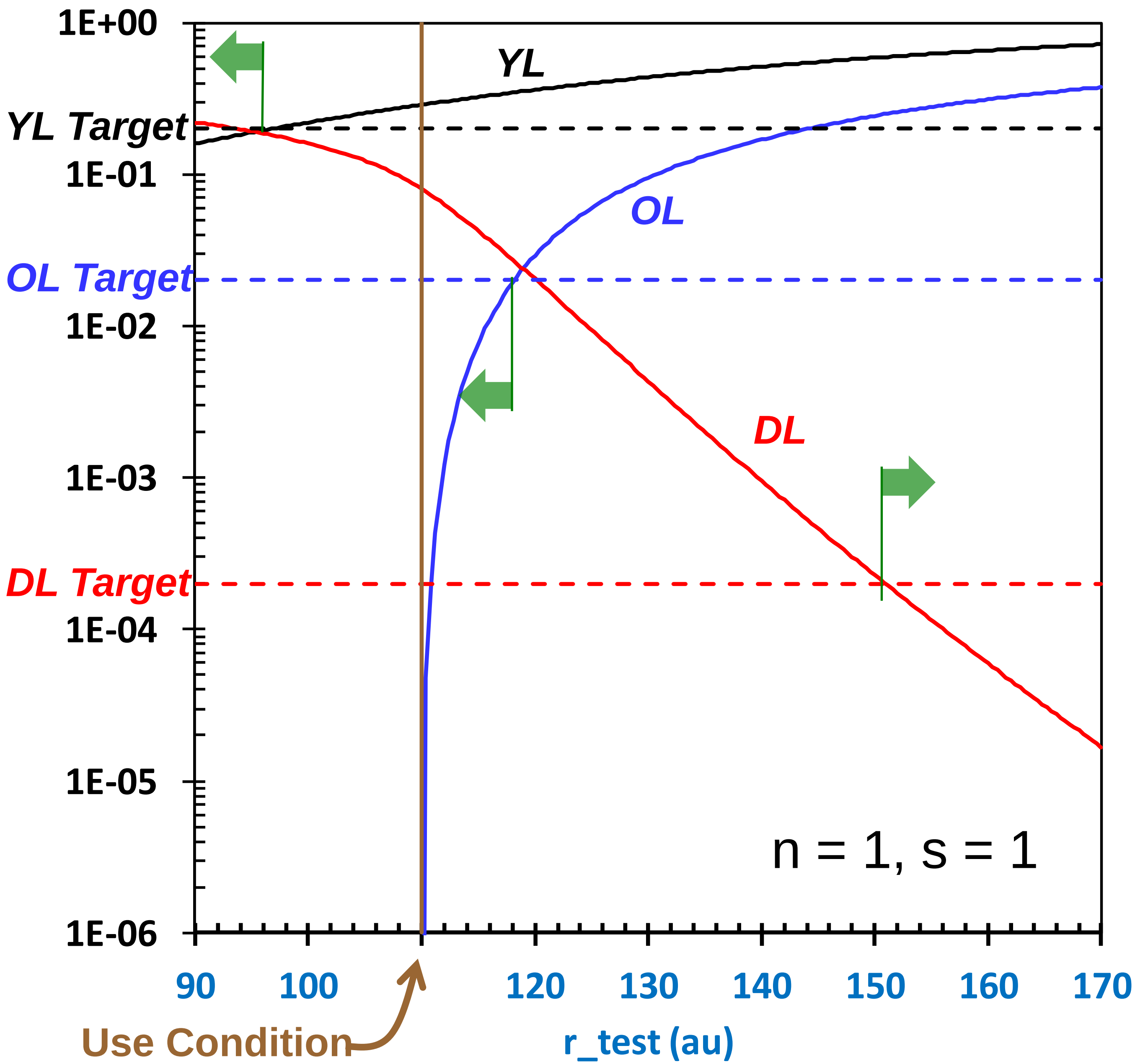


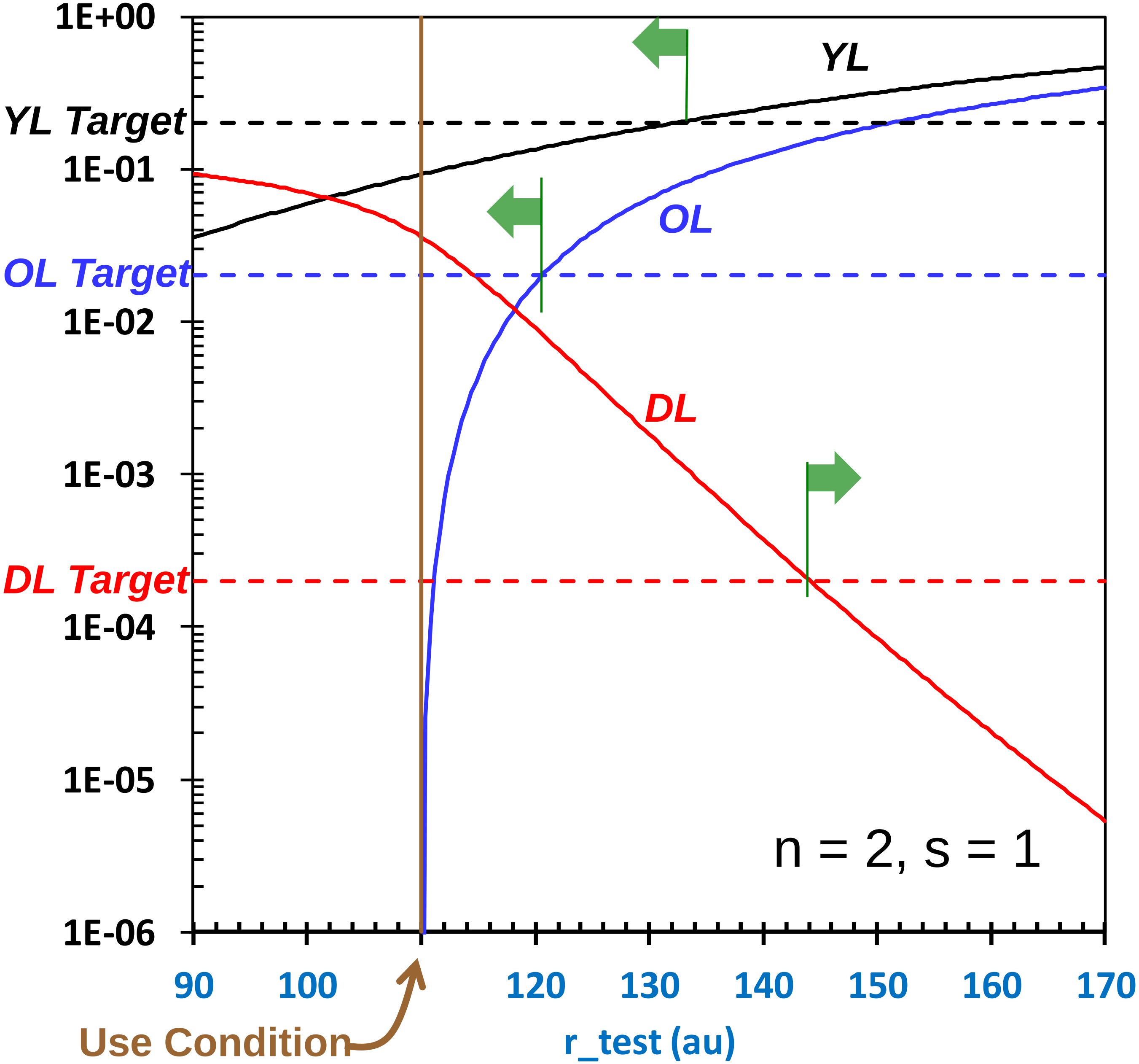
<u>Targets</u>
$YL \leq 20\%$
$OL \leq 2\%$
$DL \leq 200$ PPM
<u>Test Model</u>
Conservative (s = 1)
Less Conservative (s < 1)

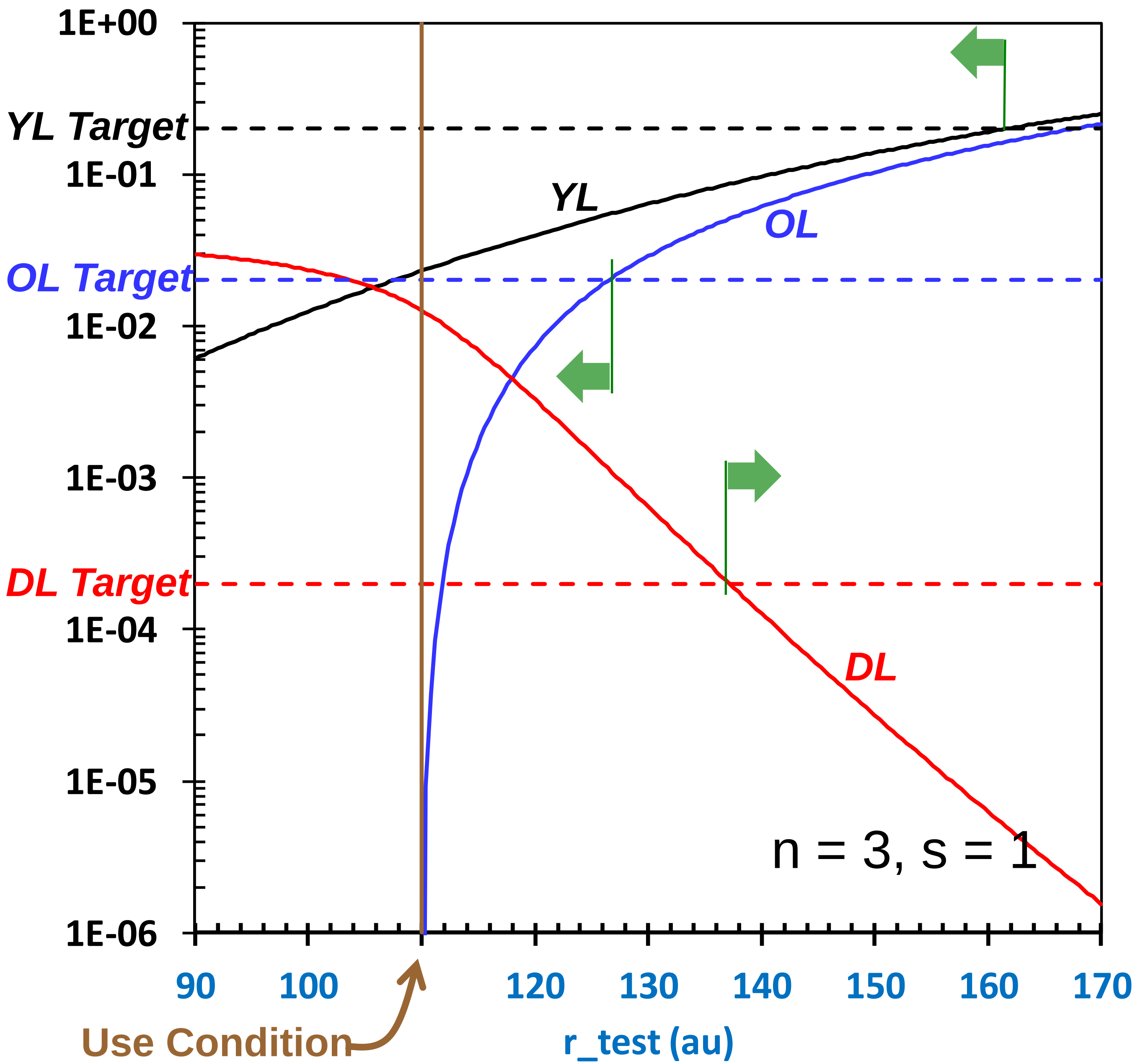
(s is the fraction of time Test finds a bit in the maximum retention time state.)

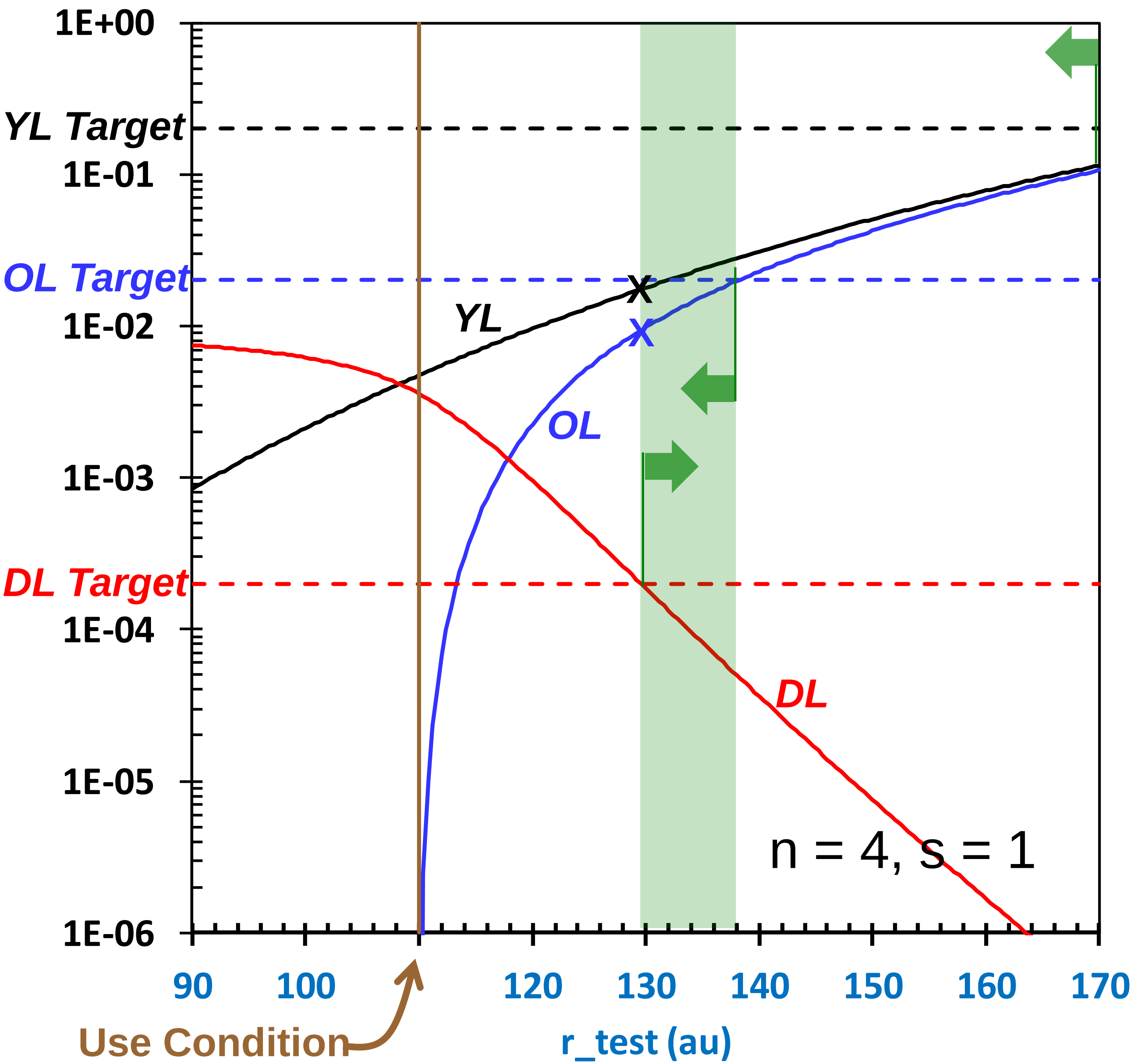


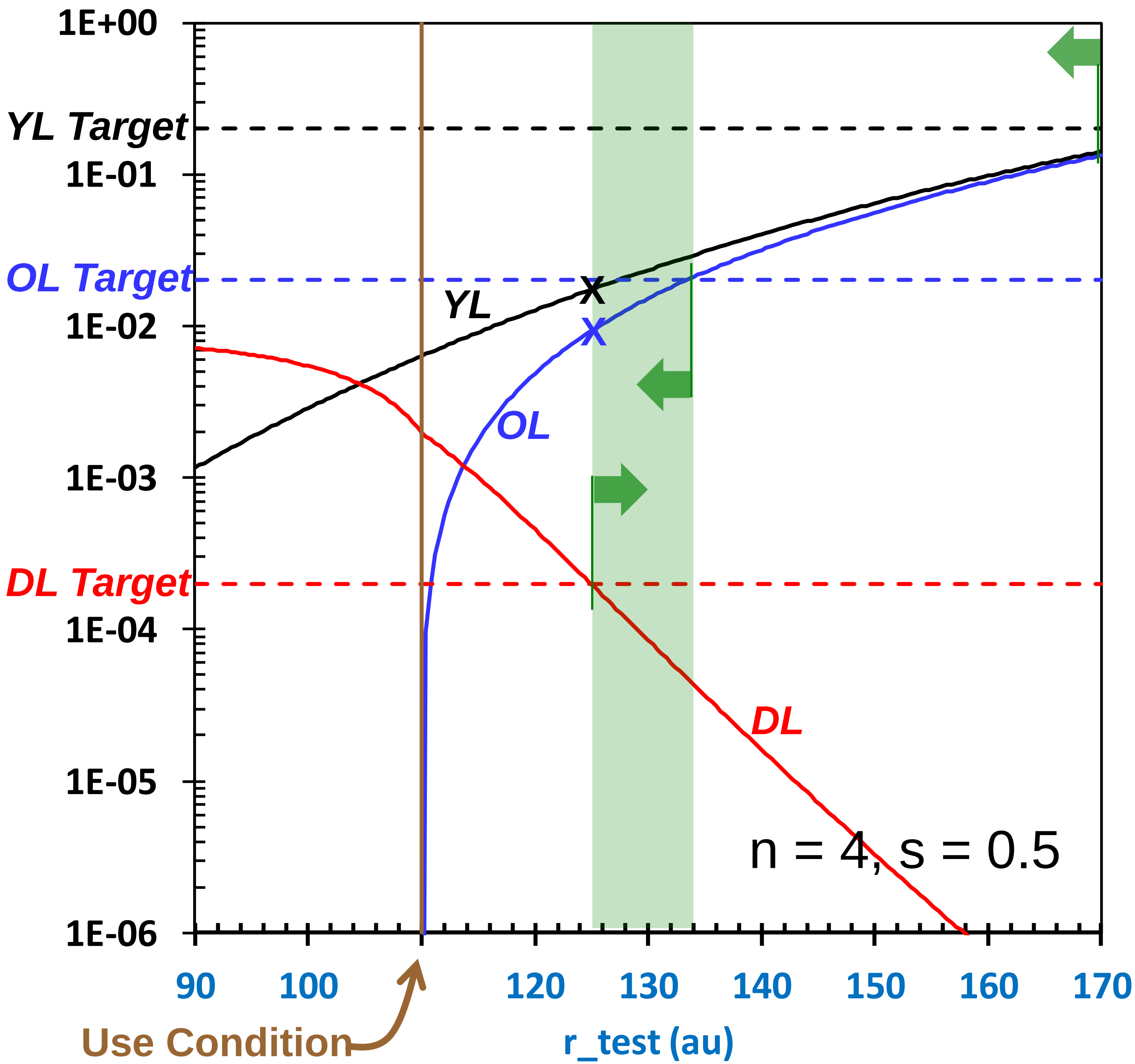

1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL
YL Target
OL
OL Target
DL
DL Target
n = 0, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)

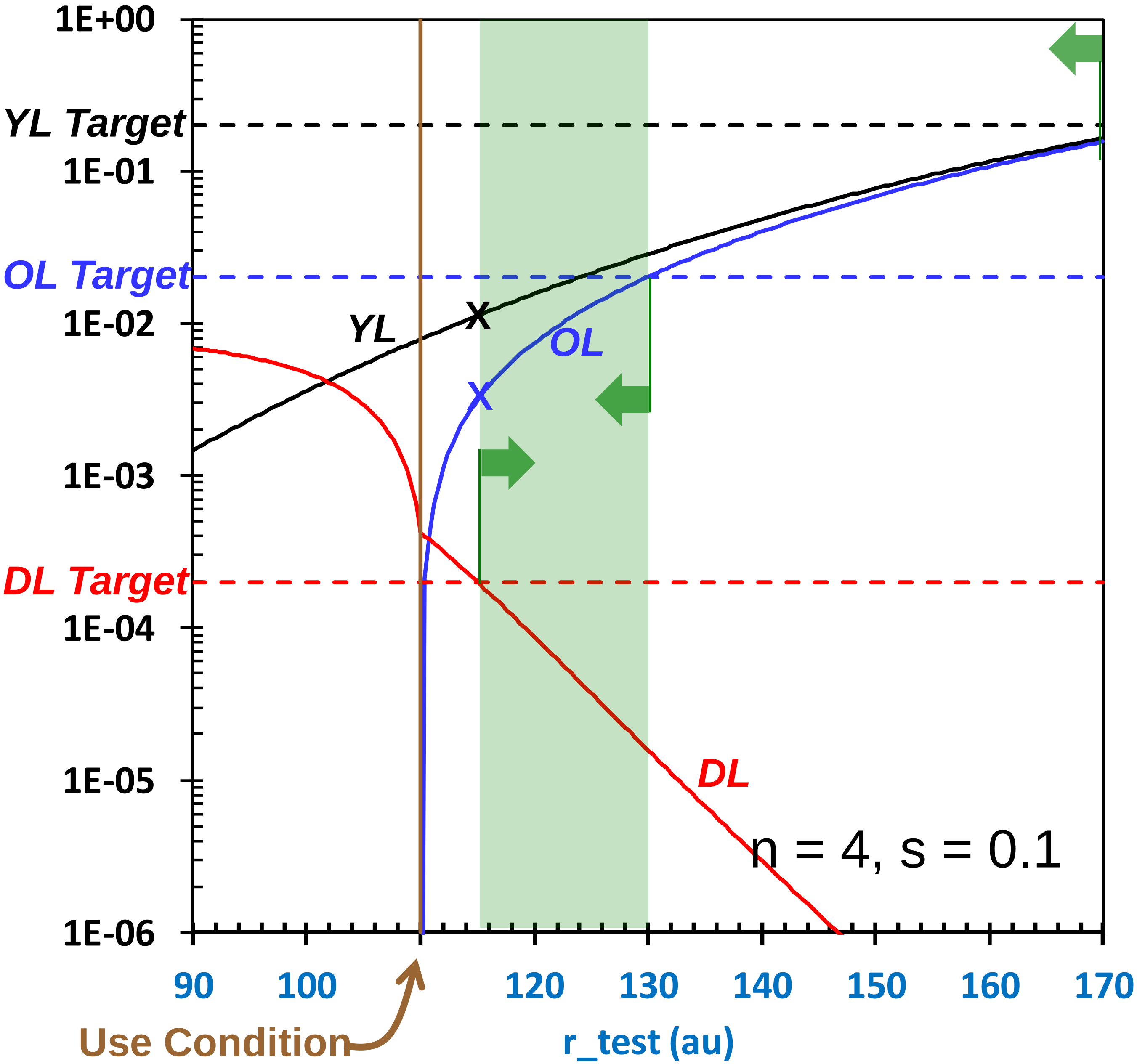

1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL
YL Target
OL
OL Target
DL
DL Target
n = 1, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
OL Target
1E-02
1E-03
DL Target
1E-04
1E-05
1E-06
YL
OL
DL
n = 2, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
YL
OL
OL Target
1E-02
1E-03
DL Target
1E-04
DL
1E-05
n = 3, s = 1
1E-06
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
OL Target
1E-02
1E-03
DL Target
1E-04
1E-05
1E-06
YL
X
X
OL
DL
n = 4, s = 1
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-06
YL Target
OL Target
DL Target
YL
OL
DL
X
X
n = 4, s = 0.5
90
100
120
130
140
150
160
170
Use Condition
r_test (au)


1E+00
YL Target
1E-01
OL Target
1E-02
YL
X
OL
X
1E-03
DL Target
1E-04
1E-05
DL
n = 4, s = 0.1
1E-06
90
100
120
130
140
150
160
170
Use Condition
r_test (au)

Outline

- Integrated Circuit Design and Test Laboratory at PSU.
- Background
  - Motivation
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  - VRT mechanism
  - Data acquisition
  - Fitting a model
  - Application of the model
- Final Thoughts

Final Thoughts

- Copula methods are *necessary*..
  - To capture the phenomenon of dependent extreme values.
  - Eg. The DRAM dependency *cannot* be described by a multinormal distribution.
- Copula methods have great *convenience and flexibility*.
  - *Any* marginal models may be coupled using *any* copula.
  - Marginal and copula models may be fitted independently.
  - Efficiencies in Monte-Carlo synthesis are often available.
- Flexibility leads to the question of *copula choice*.
  - Tail dependences help choose a mathematical form, but...
  - What are the underlying stochastic mechanisms that would enable construction of a copula from first principles?

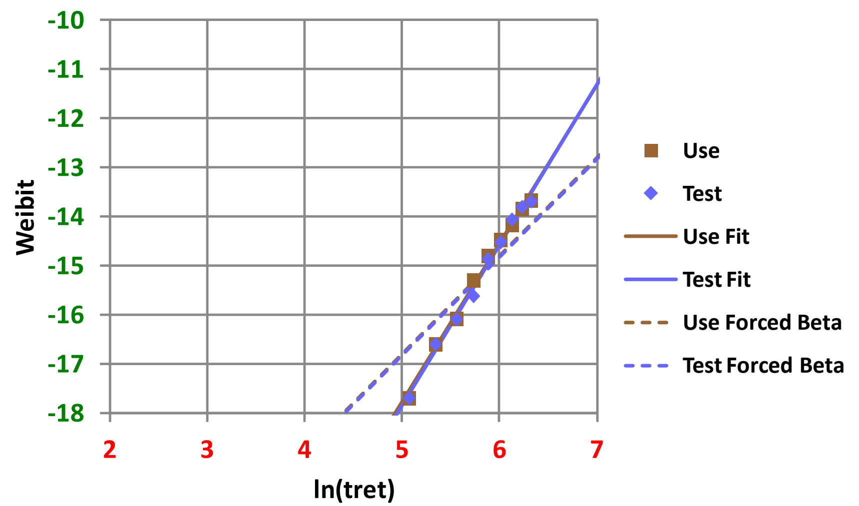
Backup

Data Record

Identity					Environmental Condition			Results of Test			
skew	chip	macro	PX	PY	VP	VDD	temp	IRetMin	IRetMax	IRetDelta	LoopGroups
4	1	0	1	238	0.4	0.85	125	10	11	1	000000000001 000000000011 000000000001 000000000011 000000000001
4	1	0	1	238	0.4	1	125	10	11	1	000000000011 000000000011 000000000011 000000000011 000000000001
4	1	0	1	238	0.4	1.2	125	10	11	1	000000000001 000000000011 000000000011 000000000001 000000000001
4	1	0	1	238	0.45	0.85	125	8	9	1	000000000111 000000001111 000000000111 000000000111 000000001111
4	1	0	1	238	0.45	1	125	9	9	0	000000000111 000000000111 000000000111 000000000111 000000000111
4	1	0	1	238	0.45	1.2	125	10	11	1	000000000011 000000000001 000000000001 000000000011 000000000001
4	1	0	16	520	0.4	0.85	105	4	4	0	000011111111 000011111111 000011111111 000011111111 000011111111
4	1	0	16	520	0.4	0.85	115	3	3	0	000111111111 000111111111 000111111111 000111111111 000111111111
4	1	0	16	520	0.4	0.85	125	2	2	0	001111111111 001111111111 001111111111 001111111111 001111111111
4	1	0	16	520	0.4	1	105	3	3	0	000111111111 000111111111 000111111111 000111111111 000111111111
4	1	0	16	520	0.4	1	115	2	2	0	001111111111 001111111111 001111111111 001111111111 001111111111
4	1	0	16	520	0.4	1	125	1	1	0	011111111111 011111111111 011111111111 011111111111 011111111111
4	1	0	16	520	0.4	1.2	105	2	2	0	001111111111 001111111111 001111111111 001111111111 001111111111
4	1	0	16	520	0.4	1.2	115	1	1	0	011111111111 011111111111 011111111111 011111111111 011111111111
4	1	0	16	520	0.4	1.2	125	1	1	0	011111111111 011111111111 011111111111 011111111111 011111111111
4	1	0	16	520	0.45	0.85	105	3	3	0	000111111111 000111111111 000111111111 000111111111 000111111111
4	1	0	16	520	0.45	0.85	115	2	2	0	001111111111 001111111111 001111111111 001111111111 001111111111
4	1	0	16	520	0.45	0.85	125	1	1	0	011111111111 011111111111 011111111111 011111111111 011111111111
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

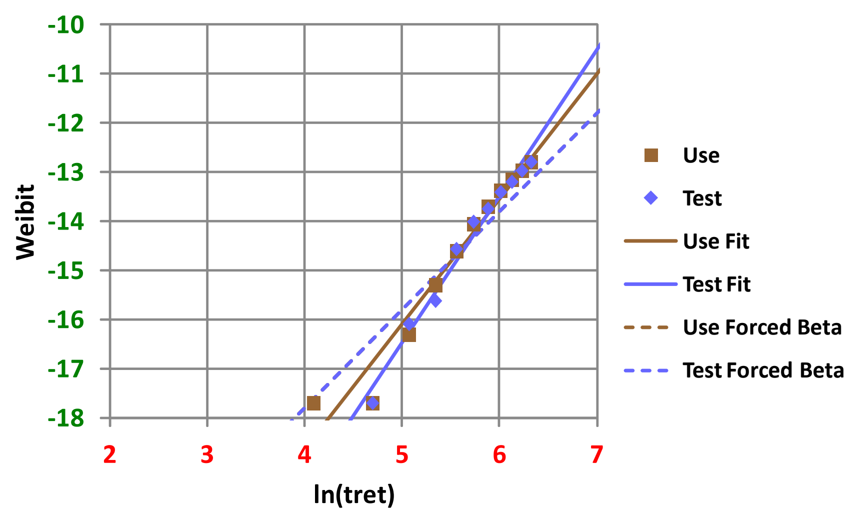
T/Vp/Vd = 105/.4/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	0	0	0	3	0	
6.32	555	-13.69	1.13	55	6	11	0	0	0	0	0	0	0	0	0	1	3	1	1	
6.23	505	-13.81	1.01	49	11	10	0	0	0	0	0	0	0	0	1	1	3	5	1	
6.12	456	-14.06	0.78	38	14	9	0	0	0	0	0	0	0	0	3	4	7	0	0	
6.01	406	-14.52	0.49	24	7	8	0	0	0	0	0	0	0	3	2	2	0	0	0	
5.88	357	-14.87	0.35	17	9	7	0	0	0	0	0	0	4	3	1	1	0	0	0	
5.73	307	-15.62	0.16	8	3	6	0	0	0	0	0	0	2	1	0	0	0	0	0	
5.55	258	-16.09	0.10	5	2	5	0	0	0	0	0	2	0	0	0	0	0	0	0	
5.34	208	-16.60	0.06	3	2	4	0	0	0	0	2	0	0	0	0	0	0	0	0	
5.07	159	-17.70	0.02	1	1	3	0	0	0	1	0	0	0	0	0	0	0	0	0	
4.69	109	#N/A	0.00	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								0	0	1	2	2	6	7	7	9	13	9		N
								0	0	1	3	5	11	18	25	34	47	56		CumN
								0.00	0.00	0.02	0.06	0.10	0.23	0.37	0.51	0.70	0.96	1.15		F
								#N/A	#N/A	-17.70	-16.60	-16.09	-15.30	-14.81	-14.48	-14.18	-13.85	-13.68		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



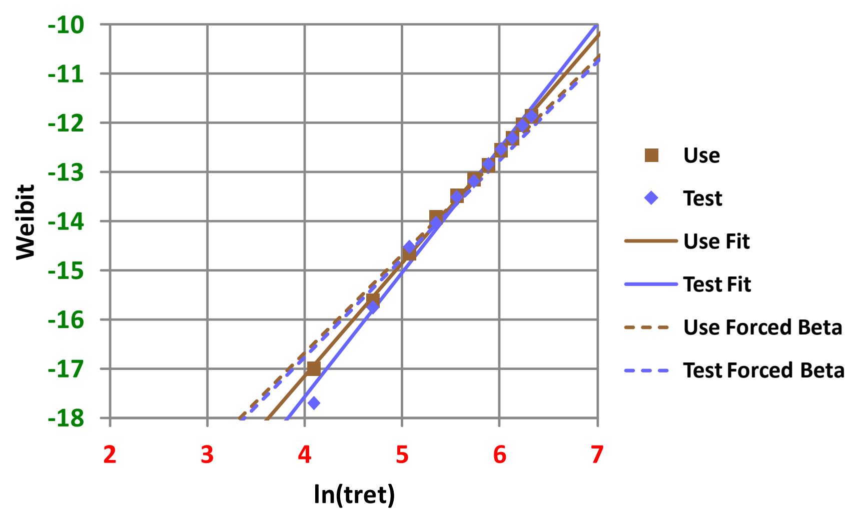
T/Vp/Vd = 115/.4/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	0	1	1	0	2	5	0
6.32	555	-12.80	2.77	135	22	11	0	0	0	0	0	0	0	0	0	1	5	8	8
6.23	505	-12.97	2.32	113	23	10	0	0	0	0	0	0	0	0	1	7	5	8	2
6.12	456	-13.20	1.85	90	17	9	0	0	0	0	0	0	0	1	7	3	6	0	0
6.01	406	-13.41	1.50	73	21	8	0	0	0	0	0	0	0	6	8	7	0	0	0
5.88	357	-13.75	1.07	52	12	7	0	0	0	0	0	0	6	2	2	1	1	0	0
5.73	307	-14.01	0.82	40	17	6	0	0	0	0	0	2	7	6	2	0	0	0	0
5.55	258	-14.57	0.47	23	15	5	0	0	0	0	4	8	3	0	0	0	0	0	0
5.34	208	-15.62	0.16	8	3	4	0	0	0	0	2	1	0	0	0	0	0	0	0
5.07	159	-16.09	0.10	5	4	3	0	0	0	3	1	0	0	0	0	0	0	0	0
4.69	109	-17.70	0.02	1	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								1	0	3	7	11	16	16	21	19	19	21	N
								1	1	4	11	22	38	54	75	94	113	134	CumN
								0.02	0.02	0.08	0.23	0.45	0.78	1.11	1.54	1.93	2.32	2.75	F
								-17.70	-17.70	-16.32	-15.30	-14.61	-14.06	-13.71	-13.38	-13.16	-12.97	-12.80	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



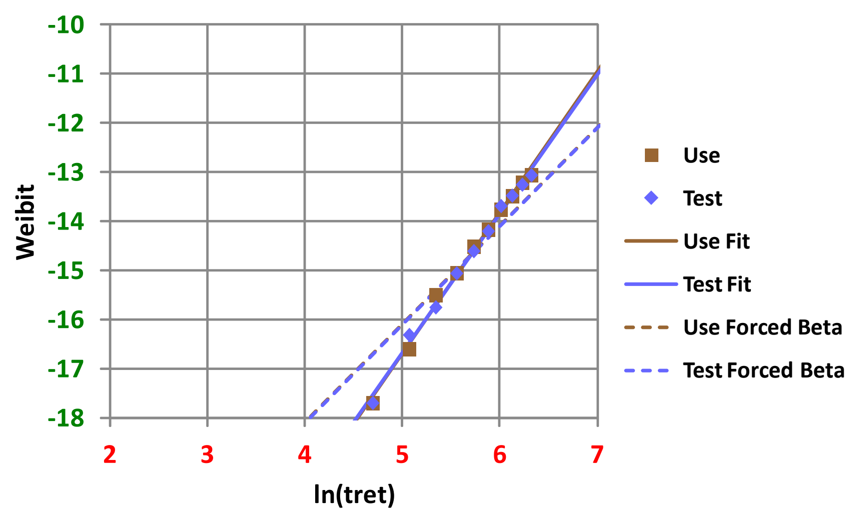
T/Vp/Vd = 125/.4/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	1	0	0	0	0	0	3	12	0
6.32	555	-11.86	7.04	343	59	11	0	0	0	0	0	0	0	0	0	0	28	15	16
6.23	505	-12.05	5.83	284	64	10	0	0	0	0	0	0	0	0	1	12	23	27	1
6.12	456	-12.31	4.51	220	45	9	0	0	0	0	0	0	0	1	10	19	14	1	0
6.01	406	-12.54	3.59	175	46	8	0	0	0	0	0	0	0	8	22	15	1	0	0
5.88	357	-12.84	2.65	129	38	7	0	0	0	0	0	1	7	17	12	1	0	0	0
5.73	307	-13.19	1.87	91	25	6	0	0	0	0	0	5	15	5	0	0	0	0	0
5.55	258	-13.51	1.35	66	27	5	0	0	0	1	5	16	5	0	0	0	0	0	0
5.34	208	-14.04	0.80	39	15	4	0	0	0	0	13	2	0	0	0	0	0	0	0
5.07	159	-14.52	0.49	24	17	3	0	0	2	11	4	0	0	0	0	0	0	0	0
4.69	109	-15.76	0.14	7	6	2	0	1	4	1	0	0	0	0	0	0	0	0	0
4.09	60	-17.70	0.02	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								2	6	13	23	24	27	31	45	47	69	55		N
								2	8	21	44	68	95	126	171	218	287	342		CumN
								0.04	0.16	0.43	0.90	1.39	1.95	2.58	3.51	4.47	5.89	7.02		F
								-17.01	-15.62	-14.66	-13.92	-13.48	-13.15	-12.87	-12.56	-12.32	-12.04	-11.87		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



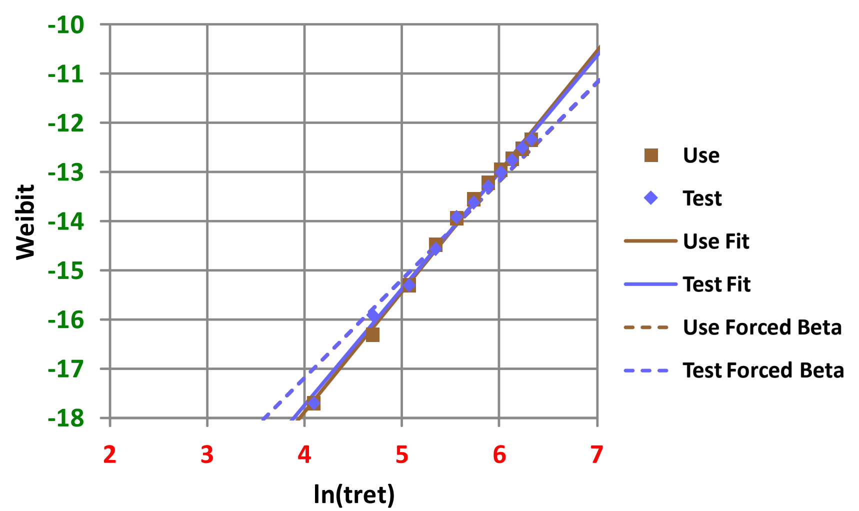
T/Vp/Vd = 105/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	0	0	0	5	0	
6.32	555	-13.07	2.11	103	18	11	0	0	0	0	0	0	0	0	0	0	9	6	3	
6.23	505	-13.26	1.74	85	17	10	0	0	0	0	0	0	0	0	1	3	8	4	1	
6.12	456	-13.48	1.39	68	13	9	0	0	0	0	0	0	0	0	1	7	4	0	1	
6.01	406	-13.69	1.13	55	22	8	0	0	0	0	0	0	1	3	12	6	0	0	0	
5.88	357	-14.21	0.68	33	11	7	0	0	0	0	0	0	1	7	3	0	0	0	0	
5.73	307	-14.61	0.45	22	8	6	0	0	0	0	0	0	8	0	0	0	0	0	0	
5.55	258	-15.06	0.29	14	7	5	0	0	0	0	2	5	0	0	0	0	0	0	0	
5.34	208	-15.76	0.14	7	3	4	0	0	0	0	3	0	0	0	0	0	0	0	0	
5.07	159	-16.32	0.08	4	3	3	0	0	0	2	1	0	0	0	0	0	0	0	0	
4.69	109	-17.70	0.02	1	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0	
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								0	1	2	6	5	10	10	17	16	21	15		N
								0	1	3	9	14	24	34	51	67	88	103		CumN
								0.00	0.02	0.06	0.18	0.29	0.49	0.70	1.05	1.37	1.81	2.11		F
								#N/A	-17.70	-16.60	-15.50	-15.06	-14.52	-14.18	-13.77	-13.50	-13.22	-13.07		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



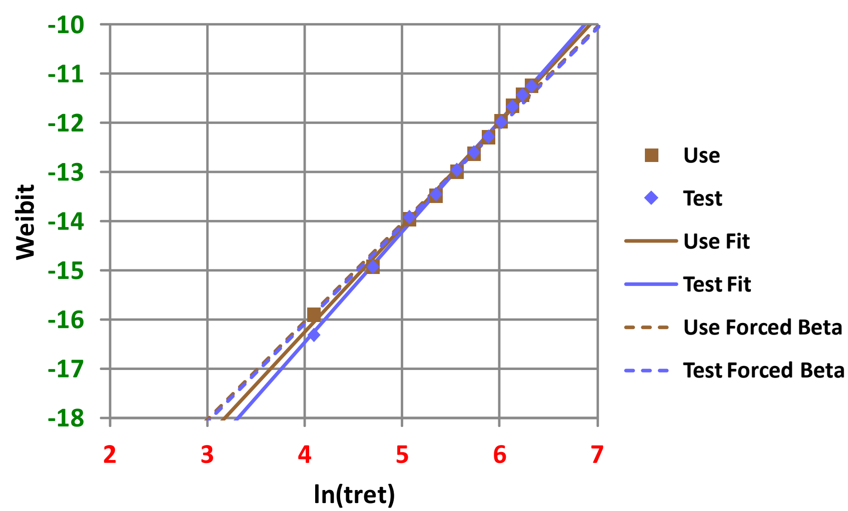
T/Vp/Vd = 115/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	0	0	1	0	1	0	0	0	2	0
6.32	555	-12.34	4.39	214	33	11	0	0	0	0	0	0	0	0	0	2	6	21	4
6.23	505	-12.50	3.71	181	41	10	0	0	0	0	0	0	0	0	2	7	19	12	1
6.12	456	-12.76	2.87	140	31	9	0	0	0	0	0	0	0	1	10	11	7	2	0
6.01	406	-13.01	2.24	109	27	8	0	0	0	0	0	0	0	10	8	9	0	0	0
5.88	357	-13.30	1.68	82	23	7	0	0	0	0	0	0	3	13	7	0	0	0	0
5.73	307	-13.62	1.21	59	15	6	0	0	0	0	0	3	12	0	0	0	0	0	0
5.55	258	-13.92	0.90	44	21	5	0	0	0	0	4	12	5	0	0	0	0	0	0
5.34	208	-14.57	0.47	23	12	4	0	0	0	1	9	2	0	0	0	0	0	0	0
5.07	159	-15.30	0.23	11	5	3	0	0	0	4	1	0	0	0	0	0	0	0	0
4.69	109	-15.91	0.12	6	5	2	0	0	3	2	0	0	0	0	0	0	0	0	0
4.09	60	-17.70	0.02	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12** Use
								1	3	7	14	18	20	25	27	29	32	37	N
								1	4	11	25	43	63	88	115	144	176	213	CumN
								0.02	0.08	0.23	0.51	0.88	1.29	1.81	2.36	2.95	3.61	4.37	F
								-17.70	-16.32	-15.30	-14.48	-13.94	-13.56	-13.22	-12.96	-12.73	-12.53	-12.34	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



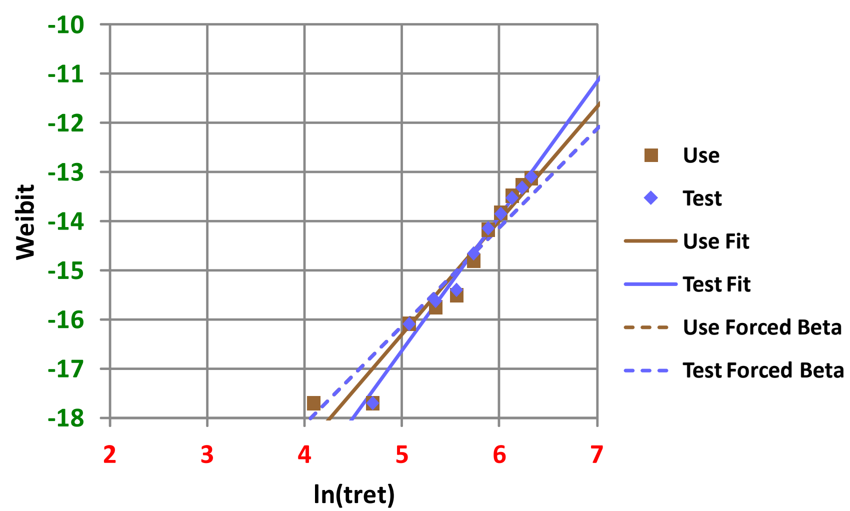
T/Vp/Vd = 125/.45/.85

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	1	0	0	0	0	3	27	0	
6.32	555	-11.26	12.92	630	104	11	0	0	0	0	0	0	0	0	1	5	36	39	23	
6.23	505	-11.44	10.79	526	110	10	0	0	0	0	0	0	0	0	3	33	39	35	0	
6.12	456	-11.67	8.53	416	112	9	0	0	0	0	0	0	0	2	27	48	28	6	1	
6.01	406	-11.99	6.24	304	78	8	0	0	0	0	0	0	0	17	36	24	0	1	0	
5.88	357	-12.28	4.64	226	62	7	0	0	0	0	0	0	8	32	19	2	1	0	0	
5.73	307	-12.60	3.36	164	49	6	0	0	0	0	0	8	28	12	0	0	0	0	1	
5.55	258	-12.96	2.36	115	45	5	0	0	0	0	4	28	13	0	0	0	0	0	0	
5.34	208	-13.45	1.44	70	26	4	0	0	0	1	19	6	0	0	0	0	0	0	0	
5.07	159	-13.92	0.90	44	28	3	0	0	2	23	2	0	0	0	0	0	0	0	1	
4.69	109	-14.93	0.33	16	12	2	0	2	8	2	0	0	0	0	0	0	0	0	0	
4.09	60	-16.32	0.08	4	4	1	0	4	0	0	0	0	0	0	0	0	0	0	0	
						0	1	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								6	10	26	26	43	49	63	86	112	107	108		N
								6	16	42	68	111	160	223	309	421	528	636		CumN
								0.12	0.33	0.86	1.39	2.28	3.28	4.57	6.34	8.64	10.83	13.05		F
								-15.91	-14.93	-13.96	-13.48	-12.99	-12.63	-12.30	-11.97	-11.66	-11.43	-11.25		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



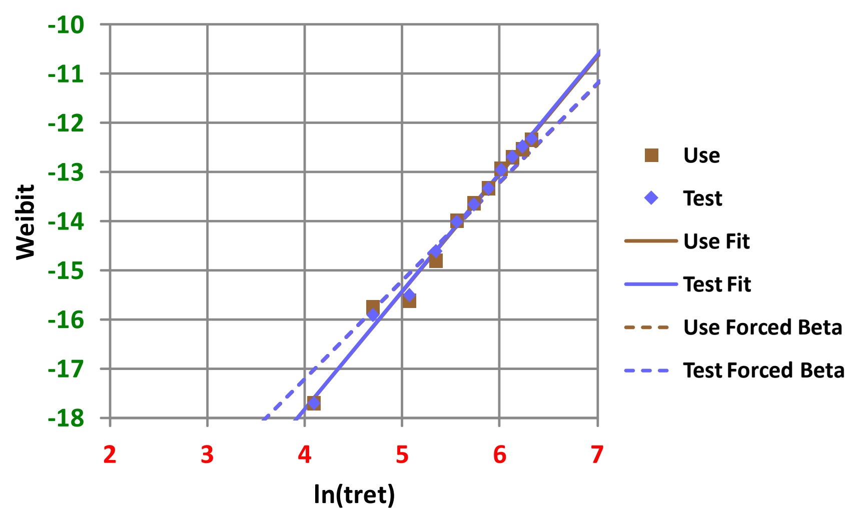
T/Vp/Vd = 105/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	0	0	0	0	0	4	0
6.32	555	-13.10	2.05	100	20	11	0	0	0	0	0	0	0	0	0	1	10	4	5
6.23	505	-13.32	1.64	80	15	10	0	0	0	0	0	0	0	0	2	5	2	4	2
6.12	456	-13.53	1.33	65	18	9	0	0	0	0	0	0	0	1	5	7	4	1	0
6.01	406	-13.85	0.96	47	12	8	0	0	0	0	0	0	0	3	3	6	0	0	0
5.88	357	-14.15	0.72	35	14	7	0	0	0	0	0	0	2	7	4	1	0	0	0
5.73	307	-14.66	0.43	21	11	6	0	0	0	0	0	1	5	5	0	0	0	0	0
5.55	258	-15.40	0.21	10	2	5	0	0	0	0	0	0	2	0	0	0	0	0	0
5.34	208	-15.62	0.16	8	3	4	0	0	0	1	1	1	0	0	0	0	0	0	0
5.07	159	-16.09	0.10	5	4	3	0	0	0	3	1	0	0	0	0	0	0	0	0
4.69	109	-17.70	0.02	1	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0
4.09	60	#N/A	0.00	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use
								1	0	4	2	2	9	16	14	20	16	13	N
								1	1	5	7	9	18	34	48	68	84	97	CumN
								0.02	0.02	0.10	0.14	0.18	0.37	0.70	0.98	1.39	1.72	1.99	F
								-17.70	-17.70	-16.09	-15.76	-15.50	-14.81	-14.18	-13.83	-13.48	-13.27	-13.13	Weibit
								60	109	159	208	258	307	357	406	456	505	555	tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	ln(tRet)



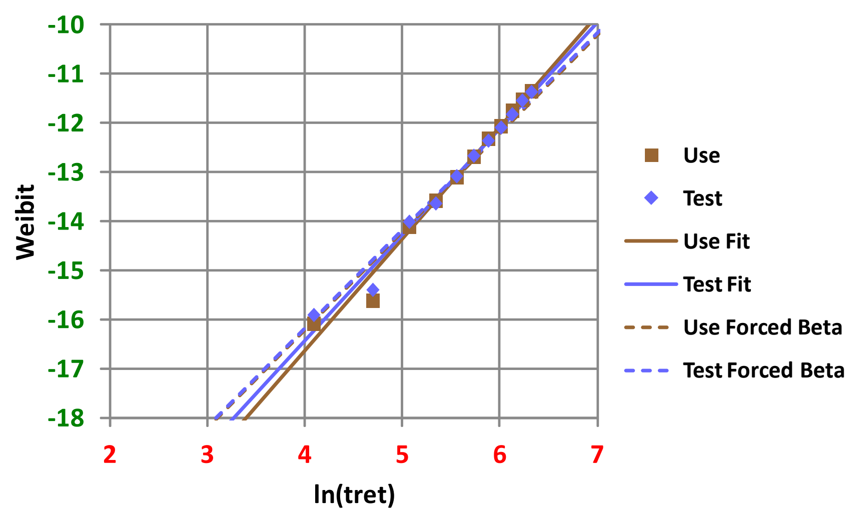
T/Vp/Vd = 115/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	1	0	0	0	2	4	0	
6.32	555	-12.32	4.45	217	32	11	0	0	0	0	0	0	0	1	0	1	7	13	10	
6.23	505	-12.48	3.79	185	35	10	0	0	0	0	0	0	0	0	1	9	7	16	2	
6.12	456	-12.69	3.08	150	35	9	0	0	0	0	0	0	0	2	12	9	9	3	0	
6.01	406	-12.96	2.36	115	36	8	0	0	0	0	0	0	0	6	17	11	1	1	0	
5.88	357	-13.33	1.62	79	22	7	0	0	0	0	0	0	8	5	9	0	0	0	0	
5.73	307	-13.66	1.17	57	17	6	0	0	0	0	0	4	5	7	0	0	0	0	1	
5.55	258	-14.01	0.82	40	18	5	0	0	0	0	2	13	3	0	0	0	0	0	0	
5.34	208	-14.61	0.45	22	13	4	0	0	0	0	7	6	0	0	0	0	0	0	0	
5.07	159	-15.50	0.18	9	3	3	0	0	1	1	1	0	0	0	0	0	0	0	0	
4.69	109	-15.91	0.12	6	5	2	0	0	5	0	0	0	0	0	0	0	0	0	0	
4.09	60	-17.70	0.02	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								1	6	1	10	23	17	21	39	30	26	37		N
								1	7	8	18	41	58	79	118	148	174	211		CumN
								0.02	0.14	0.16	0.37	0.84	1.19	1.62	2.42	3.04	3.57	4.33		F
								-17.70	-15.76	-15.62	-14.81	-13.99	-13.64	-13.33	-12.93	-12.71	-12.54	-12.35		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



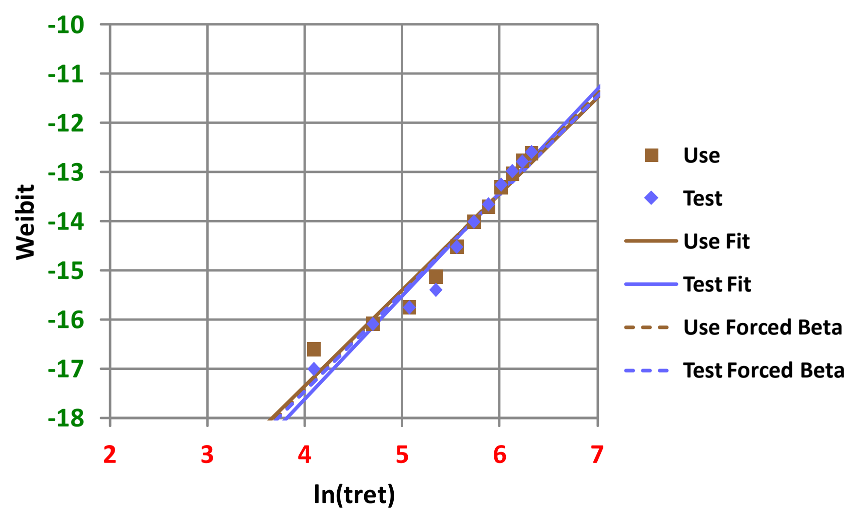
T/Vp/Vd = 125/.4/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	1	0	1	0	7	23	0
6.32	555	-11.37	11.49	560	89	11	0	0	0	0	1	0	0	0	0	2	31	38	17
6.23	505	-11.55	9.66	471	115	10	0	0	0	0	0	0	0	1	3	38	37	29	7
6.12	456	-11.83	7.30	356	84	9	0	0	0	0	0	0	0	3	22	42	17	0	0
6.01	406	-12.10	5.58	272	64	8	0	0	0	0	0	0	0	19	24	19	2	0	0
5.88	357	-12.36	4.27	208	55	7	0	0	0	0	0	2	12	25	14	2	0	0	0
5.73	307	-12.67	3.14	153	52	6	0	0	0	0	0	8	26	18	0	0	0	0	0
5.55	258	-13.09	2.07	101	43	5	0	0	0	0	6	24	12	0	0	0	1	0	0
5.34	208	-13.64	1.19	58	18	4	0	0	0	3	11	4	0	0	0	0	0	0	0
5.07	159	-14.01	0.82	40	30	3	0	0	0	23	7	0	0	0	0	0	0	0	0
4.69	109	-15.40	0.21	10	4	2	0	0	2	2	0	0	0	0	0	0	0	0	0
4.09	60	-15.91	0.12	6	6	1	0	5	1	0	0	0	0	0	0	0	0	0	0
						0	1	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								5	3	28	25	38	51	66	64	103	95	90		N
								5	8	36	61	99	150	216	280	383	478	568		CumN
								0.10	0.16	0.74	1.25	2.03	3.08	4.43	5.74	7.86	9.81	11.65		F
								-16.09	-15.62	-14.12	-13.59	-13.11	-12.69	-12.33	-12.07	-11.75	-11.53	-11.36		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



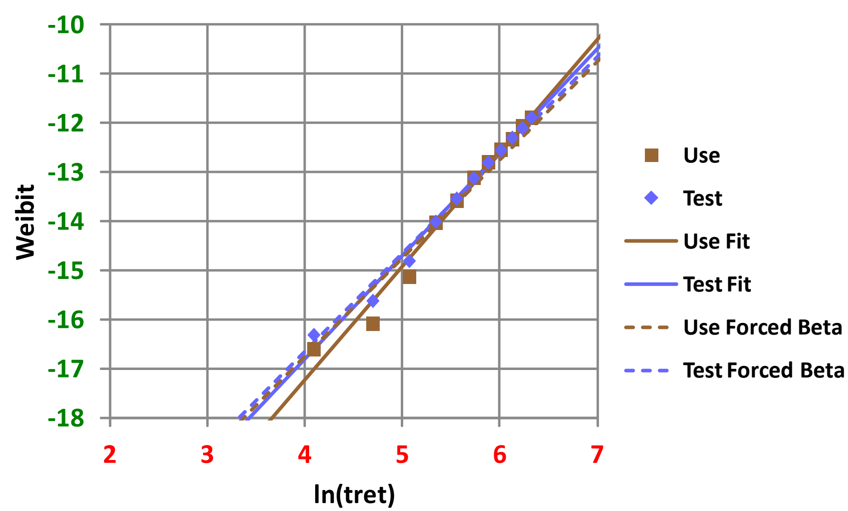
T/Vp/Vd = 105/.45/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	0	0	0	0	0	0	4	0	
6.32	555	-12.60	3.38	165	29	11	0	0	0	0	0	0	0	1	0	0	13	9	6	
6.23	505	-12.79	2.79	136	24	10	0	0	0	0	0	0	0	0	0	4	10	9	1	
6.12	456	-12.98	2.30	112	27	9	0	0	0	0	0	0	0	0	4	13	9	1	0	
6.01	406	-13.26	1.74	85	28	8	0	0	0	0	0	0	0	8	12	7	0	0	1	
5.88	357	-13.66	1.17	57	17	7	0	0	0	0	0	0	1	4	11	1	0	0	0	
5.73	307	-14.01	0.82	40	16	6	0	0	0	0	0	4	11	1	0	0	0	0	0	
5.55	258	-14.52	0.49	24	14	5	0	0	0	0	3	7	4	0	0	0	0	0	0	
5.34	208	-15.40	0.21	10	3	4	0	0	0	0	3	0	0	0	0	0	0	0	0	
5.07	159	-15.76	0.14	7	2	3	0	0	0	2	0	0	0	0	0	0	0	0	0	
4.69	109	-16.09	0.10	5	3	2	0	1	2	0	0	0	0	0	0	0	0	0	0	
4.09	60	-17.01	0.04	2	2	1	0	2	0	0	0	0	0	0	0	0	0	0	0	
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								3	2	2	6	11	16	14	27	25	32	23		N
								3	5	7	13	24	40	54	81	106	138	161		CumN
								0.06	0.10	0.14	0.27	0.49	0.82	1.11	1.66	2.17	2.83	3.30		F
								-16.60	-16.09	-15.76	-15.14	-14.52	-14.01	-13.71	-13.31	-13.04	-12.77	-12.62		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



T/Vp/Vd = 115/.45/1.

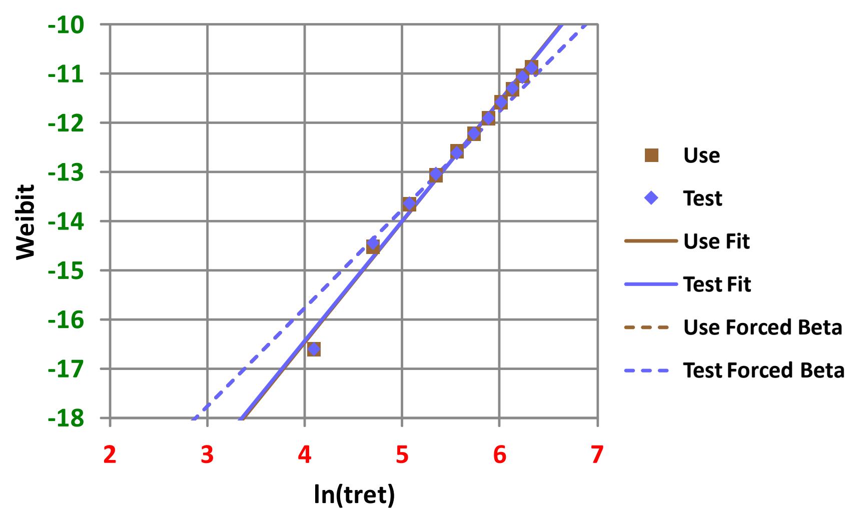
ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	0	1	0	0	0	0	4	9	0	
6.32	555	-11.90	6.81	332	63	**11**	0	0	0	0	0	0	0	0	0	0	26	**27**	10	
6.23	505	-12.11	5.52	269	48	**10**	0	0	0	0	0	0	0	0	0	15	**19**	12	2	
6.12	456	-12.30	4.53	221	50	**9**	0	0	0	0	0	0	0	2	9	**18**	16	4	1	
6.01	406	-12.56	3.51	171	38	**8**	0	0	0	0	0	0	0	11	**18**	8	1	0	0	
5.88	357	-12.81	2.73	133	36	**7**	0	0	0	0	0	0	9	**15**	12	0	0	0	0	
5.73	307	-13.13	1.99	97	33	**6**	0	0	0	0	1	4	**20**	8	0	0	0	0	0	
5.55	258	-13.54	1.31	64	24	**5**	0	0	0	0	2	**14**	8	0	0	0	0	0	0	
5.34	208	-14.01	0.82	40	22	**4**	0	0	0	2	**17**	3	0	0	0	0	0	0	0	
5.07	159	-14.81	0.37	18	10	**3**	0	0	0	**4**	6	0	0	0	0	0	0	0	0	
4.69	109	-15.62	0.16	8	4	**2**	0	0	**2**	2	0	0	0	0	0	0	0	0	0	
4.09	60	-16.32	0.08	4	4	**1**	1	**3**	0	0	0	0	0	0	0	0	0	0	0	
						0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	
							0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Use**
								3	2	8	26	22	37	36	39	41	66	52		**N**
								3	5	13	39	61	98	134	173	214	280	332		**CumN**
								0.06	0.10	0.27	0.80	1.25	2.01	2.75	3.55	4.39	5.74	6.81		**F**
								-16.60	-16.09	-15.14	-14.04	-13.59	-13.12	-12.80	-12.55	-12.34	-12.07	-11.90		**Weibit**
								60	109	159	208	258	307	357	406	456	505	555		**tRet**
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		**ln(tRet)**



T/Vp/Vd = 125/.45/1.

ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	1	0	0	1	0	1	0	10	42	0
6.32	555	-10.88	18.75	914	154	11	0	0	0	0	0	0	0	0	0	3	66	49	36
6.23	505	-11.07	15.59	760	155	10	0	0	0	0	0	0	0	0	3	39	58	48	7
6.12	456	-11.30	12.41	605	149	9	0	0	0	0	0	0	1	1	41	53	47	6	0
6.01	406	-11.58	9.35	456	127	8	0	0	0	0	0	0	0	28	53	39	7	0	0
5.88	357	-11.91	6.75	329	88	7	0	0	0	0	0	0	21	37	27	1	0	0	2
5.73	307	-12.22	4.94	241	79	6	0	0	0	0	0	20	34	21	4	0	0	0	0
5.55	258	-12.61	3.32	162	56	5	0	0	0	0	9	31	15	1	0	0	0	0	0
5.34	208	-13.04	2.17	106	48	4	0	0	0	5	30	13	0	0	0	0	0	0	0
5.07	159	-13.64	1.19	58	32	3	0	0	3	22	7	0	0	0	0	0	0	0	0
4.69	109	-14.44	0.53	26	23	2	0	0	18	5	0	0	0	0	0	0	0	0	0
4.09	60	-16.60	0.06	3	3	1	0	3	0	0	0	0	0	0	0	0	0	0	0
						0	4	0	0	0	0	0	0	0	0	0	0	0	0

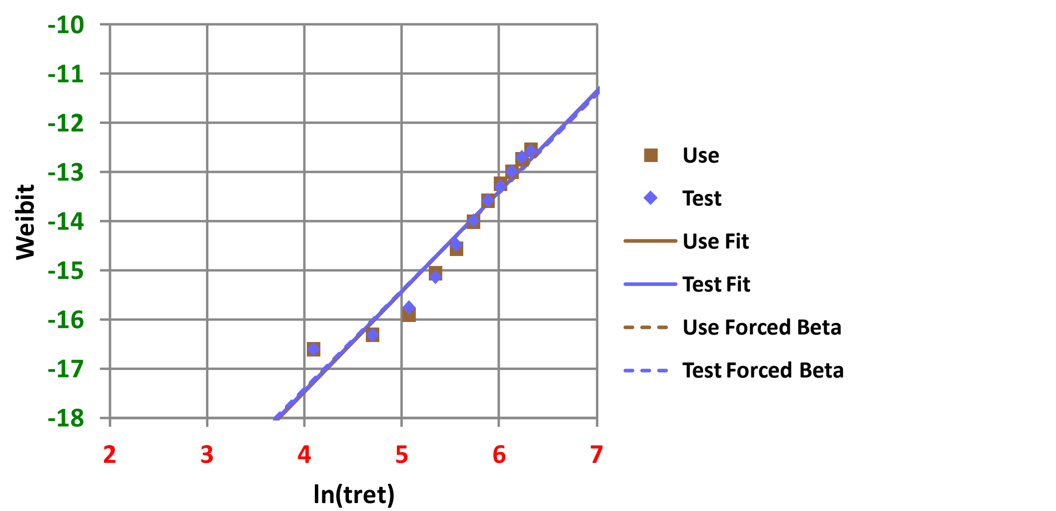
Use	0	1	2	3	4	5	6	7	8	9	10	11	12
N		3	21	33	46	64	72	88	129	135	188	145	
CumN		3	24	57	103	167	239	327	456	591	779	924	
F		0.06	0.49	1.17	2.11	3.43	4.90	6.71	9.35	12.12	15.98	18.95	
Weibit		-16.60	-14.52	-13.66	-13.07	-12.58	-12.23	-11.91	-11.58	-11.32	-11.04	-10.87	
tRet		60	109	159	208	258	307	357	406	456	505	555	
ln(tRet)		4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	



T/Vp/Vd = 105/.4/1.2

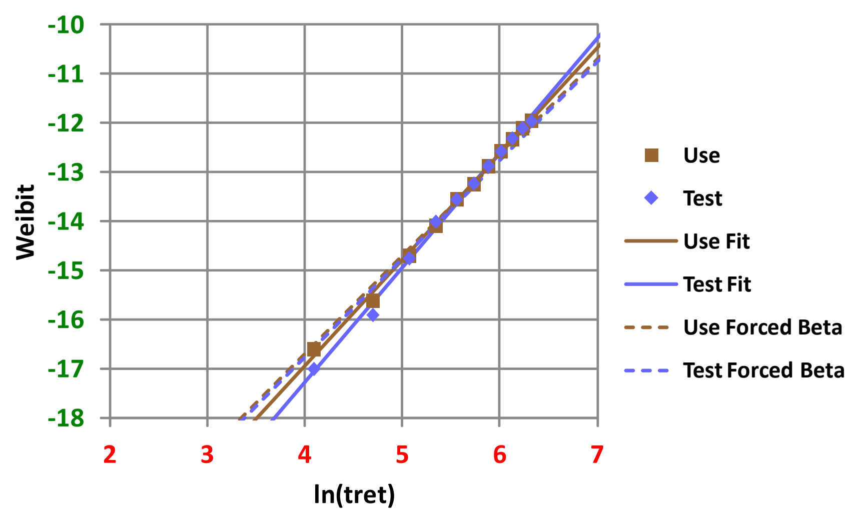
ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	0	0	0	0	0	0	0	0	8	0
6.32	555	-12.58	3.45	168	19	11	0	0	0	0	0	0	0	0	0	0	3	13	3
6.23	505	-12.70	3.06	149	38	10	0	0	0	0	0	0	0	0	0	10	18	10	0
6.12	456	-12.99	2.28	111	29	9	0	0	0	0	0	0	0	1	9	10	9	0	0
6.01	406	-13.30	1.68	82	20	8	0	0	0	0	0	0	0	5	10	4	1	0	0
5.88	357	-13.58	1.27	62	21	7	0	0	0	0	0	0	5	10	6	0	0	0	0
5.73	307	-13.99	0.84	41	16	6	0	0	0	0	0	4	6	5	1	0	0	0	0
5.55	258	-14.48	0.51	25	12	5	0	0	0	0	2	4	6	0	0	0	0	0	0
5.34	208	-15.14	0.27	13	6	4	0	0	0	0	5	1	0	0	0	0	0	0	0
5.07	159	-15.76	0.14	7	3	3	0	0	0	2	1	0	0	0	0	0	0	0	0
4.69	109	-16.32	0.08	4	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0
4.09	60	-16.60	0.06	3	3	1	0	3	0	0	0	0	0	0	0	0	0	0	0
						0	1	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use

	0	1	2	3	4	5	6	7	8	9	10	11	12	Use
		3	1	2	8	9	17	21	26	24	31	31		N
		3	4	6	14	23	40	61	87	111	142	173		CumN
		0.06	0.08	0.12	0.29	0.47	0.82	1.25	1.78	2.28	2.91	3.55		F
		-16.60	-16.32	-15.91	-15.06	-14.57	-14.01	-13.59	-13.24	-12.99	-12.75	-12.55		Weibit
		60	109	159	208	258	307	357	406	456	505	555		tRet
		4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



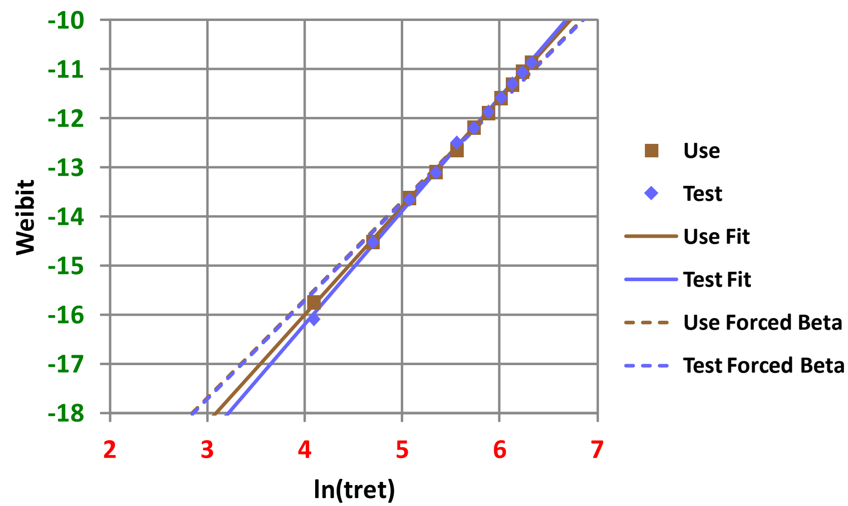
T/Vp/Vd = 115/.4/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	0	0	1	0	0	0	1	2	16	0
6.32	555	-11.97	6.34	309	41	11	0	0	0	0	0	0	0	0	1	2	15	15	8
6.23	505	-12.11	5.50	268	50	10	0	0	0	0	0	0	0	0	0	15	15	13	7
6.12	456	-12.32	4.47	218	51	9	0	0	0	0	0	0	0	0	15	16	17	2	1
6.01	406	-12.58	3.43	167	43	8	0	0	0	0	0	0	0	14	16	10	3	0	0
5.88	357	-12.88	2.54	124	37	7	0	0	0	0	0	0	8	13	12	3	0	0	1
5.73	307	-13.24	1.78	87	24	6	0	0	0	0	0	4	9	11	0	0	0	0	0
5.55	258	-13.56	1.29	63	23	5	0	0	0	0	2	15	6	0	0	0	0	0	0
5.34	208	-14.01	0.82	40	21	4	0	0	0	2	13	6	0	0	0	0	0	0	0
5.07	159	-14.76	0.39	19	13	3	0	0	1	10	2	0	0	0	0	0	0	0	0
4.69	109	-15.91	0.12	6	4	2	0	0	4	0	0	0	0	0	0	0	0	0	0
4.09	60	-17.01	0.04	2	2	1	0	2	0	0	0	0	0	0	0	0	0	0	0
						0	2	1	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 (Use)
						N		3	5	12	17	26	23	38	44	47	52	46	
						CumN		3	8	20	37	63	86	124	168	215	267	313	
						F		0.06	0.16	0.41	0.76	1.29	1.76	2.54	3.45	4.41	5.48	6.42	
						Weibit		-16.60	-15.62	-14.71	-14.09	-13.56	-13.25	-12.88	-12.58	-12.33	-12.11	-11.96	
						tRet		60	109	159	208	258	307	357	406	456	505	555	
						ln(tRet)		4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	



T/Vp/Vd = 125/.4/1.2

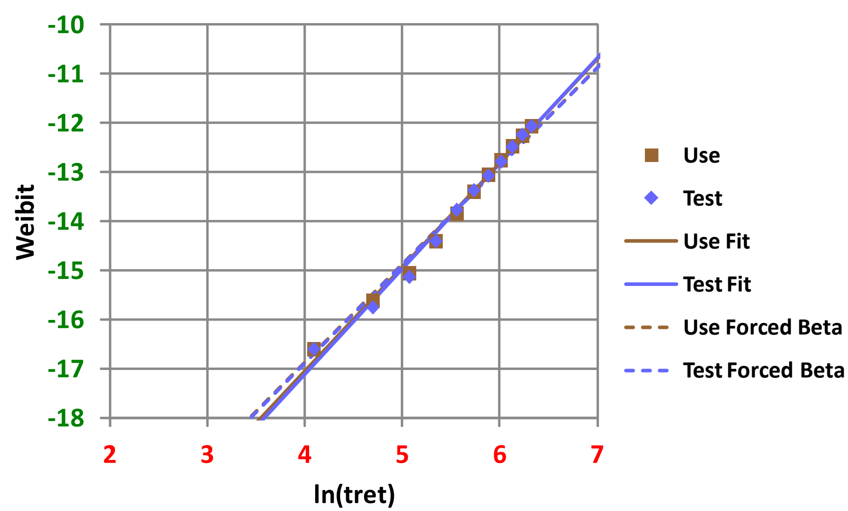
ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	1	0	0	1	0	2	1	6	38	0	
6.32	555	-10.87	19.06	929	167	11	0	0	0	0	0	0	0	0	2	7	61	61	36	
6.23	505	-11.07	15.63	762	156	10	0	0	0	0	0	0	0	0	5	36	59	49	7	
6.12	456	-11.30	12.43	606	152	9	0	0	0	0	0	0	0	3	38	54	48	4	5	
6.01	406	-11.58	9.31	454	112	8	0	0	0	0	0	0	2	24	38	39	8	0	1	
5.88	357	-11.87	7.02	342	97	7	0	0	0	0	0	0	22	39	34	1	0	0	1	
5.73	307	-12.20	5.03	245	63	6	0	0	0	0	0	10	32	19	0	0	0	0	2	
5.55	258	-12.50	3.73	182	83	5	0	0	0	1	6	40	33	1	1	0	0	0	1	
5.34	208	-13.11	2.03	99	42	4	0	0	0	7	28	6	0	0	0	0	0	0	1	
5.07	159	-13.66	1.17	57	33	3	0	0	5	21	7	0	0	0	0	0	0	0	0	
4.69	109	-14.52	0.49	24	19	2	0	4	10	5	0	0	0	0	0	0	0	0	0	
4.09	60	-16.09	0.10	5	5	1	0	3	2	0	0	0	0	0	0	0	0	0	0	
						0	4	0	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								7	17	35	41	56	90	86	120	138	182	152		N
								7	24	59	100	156	246	332	452	590	772	924		CumN
								0.14	0.49	1.21	2.05	3.20	5.05	6.81	9.27	12.10	15.84	18.95		F
								-15.76	-14.52	-13.62	-13.10	-12.65	-12.20	-11.90	-11.59	-11.32	-11.05	-10.87		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



T/Vp/Vd = 105/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test	0	1	2	3	4	5	6	7	8	9	10	11	12
						12	0	0	0	0	0	0	0	0	1	0	0	17	0
6.32	555	-12.07	5.74	280	45	11	0	0	0	0	0	0	0	0	0	5	8	**17**	15
6.23	505	-12.24	4.82	235	51	10	0	0	0	0	0	0	0	1	1	9	**23**	14	3
6.12	456	-12.49	3.77	184	48	9	0	0	0	0	0	0	0	1	12	**19**	13	3	0
6.01	406	-12.79	2.79	136	34	8	0	0	0	0	0	0	0	8	**15**	11	0	0	0
5.88	357	-13.08	2.09	102	26	7	0	0	0	0	0	0	6	**12**	7	1	0	0	0
5.73	307	-13.37	1.56	76	25	6	0	0	0	0	0	3	**15**	6	0	0	0	0	1
5.55	258	-13.77	1.05	51	24	5	0	0	0	0	2	**14**	6	2	0	0	0	0	0
5.34	208	-14.41	0.55	27	14	4	0	0	0	0	**11**	3	0	0	0	0	0	0	0
5.07	159	-15.14	0.27	13	6	3	0	0	1	**5**	0	0	0	0	0	0	0	0	0
4.69	109	-15.76	0.14	7	4	2	0	0	**3**	1	0	0	0	0	0	0	0	0	0
4.09	60	-16.60	0.06	3	3	1	0	**2**	1	0	0	0	0	0	0	0	0	0	0
						0	**1**	1	0	0	0	0	0	0	0	0	0	0	0

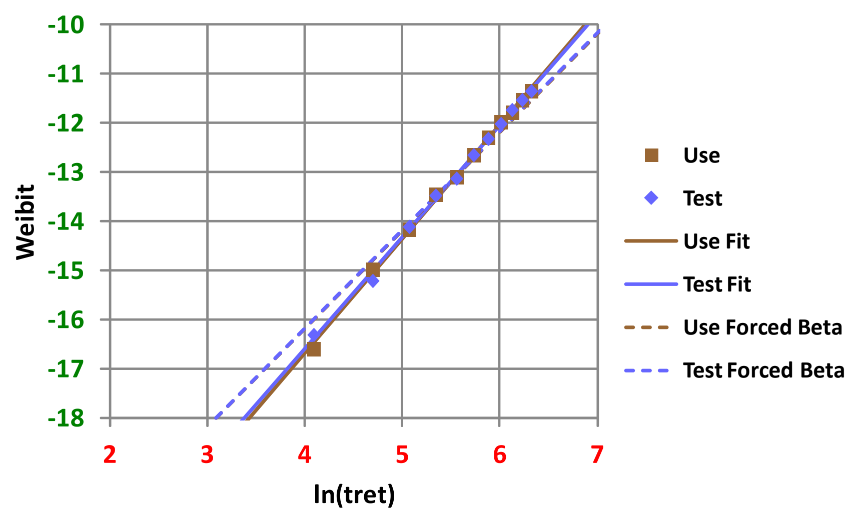
Use	0	1	2	3	4	5	6	7	8	9	10	11	12
N		3	5	6	13	20	27	30	36	45	44	51	
CumN		3	8	14	27	47	74	104	140	185	229	280	
F		0.06	0.16	0.29	0.55	0.96	1.52	2.13	2.87	3.79	4.70	5.74	
Weibit		-16.60	-15.62	-15.06	-14.41	-13.85	-13.40	-13.06	-12.76	-12.48	-12.27	-12.07	
tRet		60	109	159	208	258	307	357	406	456	505	555	
ln(tRet)		4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32	



T/Vp/Vd = 115/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test													
						12	0	0	0	1	0	0	0	0	0	1	10	23	0
6.32	555	-11.36	11.67	569	94	11	0	0	0	0	0	0	0	1	1	1	32	37	22
6.23	505	-11.54	9.74	475	87	10	0	0	0	0	0	0	0	0	2	20	25	30	10
6.12	456	-11.74	7.96	388	96	9	0	0	0	0	0	0	0	1	29	26	32	8	0
6.01	406	-12.03	5.99	292	76	8	0	0	0	0	0	0	2	18	35	15	6	0	0
5.88	357	-12.33	4.43	216	61	7	0	0	0	0	0	0	12	32	16	0	1	0	0
5.73	307	-12.66	3.18	155	59	6	0	0	0	0	1	10	34	13	0	0	0	0	1
5.55	258	-13.14	1.97	96	28	5	0	0	0	0	6	15	7	0	0	0	0	0	0
5.34	208	-13.48	1.39	68	32	4	0	0	0	2	25	5	0	0	0	0	0	0	0
5.07	159	-14.12	0.74	36	24	3	0	0	4	16	3	0	0	1	0	0	0	0	0
4.69	109	-15.22	0.25	12	8	2	0	1	7	0	0	0	0	0	0	0	0	0	0
4.09	60	-16.32	0.08	4	4	1	1	2	1	0	0	0	0	0	0	0	0	0	0
						0	4	0	0	0	0	0	0	0	0	0	0	0	0
							0	1	2	3	4	5	6	7	8	9	10	11	12 Use

	0	1	2	3	4	5	6	7	8	9	10	11	12	Use
		3	12	19	35	30	55	66	83	63	106	98		N
		3	15	34	69	99	154	220	303	366	472	570		CumN
		0.06	0.31	0.70	1.42	2.03	3.16	4.51	6.22	7.51	9.68	11.69		F
		-16.60	-14.99	-14.18	-13.47	-13.11	-12.67	-12.31	-11.99	-11.80	-11.55	-11.36		Weibit
		60	109	159	208	258	307	357	406	456	505	555		tRet
		4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)



T/Vp/Vd = 125/.45/1.2

ln(tRet)	tRet	Weibit	F	CumN	N	Test														
						12	0	0	0	0	1	0	1	1	1	1	18	69	0	
6.32	555	-10.30	33.50	1633	273	11	0	0	0	0	0	0	0	0	1	14	104	95	59	
6.23	505	-10.49	27.90	1360	343	10	0	0	1	0	0	0	1	2	10	97	92	120	20	
6.12	456	-10.78	20.86	1017	237	9	0	0	0	0	0	0	1	5	46	83	86	14	2	
6.01	406	-11.04	16.00	780	211	8	0	0	0	0	0	1	4	59	80	56	6	2	3	
5.88	357	-11.36	11.67	569	156	7	0	0	0	0	0	1	29	68	53	2	1	2	0	
5.73	307	-11.68	8.47	413	134	6	0	0	0	0	0	36	56	38	2	1	1	0	0	
5.55	258	-12.07	5.72	279	109	5	0	0	0	0	15	71	20	2	1	0	0	0	0	
5.34	208	-12.57	3.49	170	69	4	0	0	0	12	38	18	0	0	0	1	0	0	0	
5.07	159	-13.09	2.07	101	61	3	0	0	8	43	9	0	0	0	0	0	0	0	1	
4.69	109	-14.01	0.82	40	27	2	0	1	19	6	0	0	0	0	1	0	0	0	0	
4.09	60	-15.14	0.27	13	13	1	0	11	2	0	0	0	0	0	0	0	0	0	0	
						0	5	1	0	0	0	0	0	0	0	0	0	0	0	
							0	1	2	3	4	5	6	7	8	9	10	11	12	Use
								13	30	61	63	127	112	175	195	255	308	302		N
								13	43	104	167	294	406	581	776	1031	1339	1641		CumN
								0.27	0.88	2.13	3.43	6.03	8.33	11.92	15.92	21.15	27.47	33.66		F
								-15.14	-13.94	-13.06	-12.58	-12.02	-11.70	-11.34	-11.05	-10.76	-10.50	-10.30		Weibit
								60	109	159	208	258	307	357	406	456	505	555		tRet
								4.09	4.69	5.07	5.34	5.55	5.73	5.88	6.01	6.12	6.23	6.32		ln(tRet)