ECE 510, Lecture 13
Defect Models of Yield and Reliability

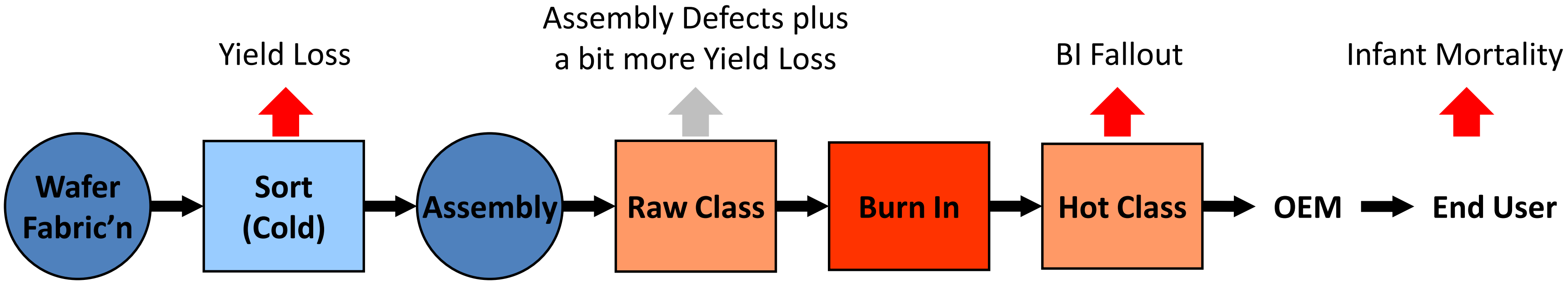
Glenn Shirley
Scott Johnson

Outline

- ● Introduction
- Models of Yield
- Models of Defect Reliability
- Analysis and Synthesis of Lifetest and Burn In

Defect Yield and Reliability

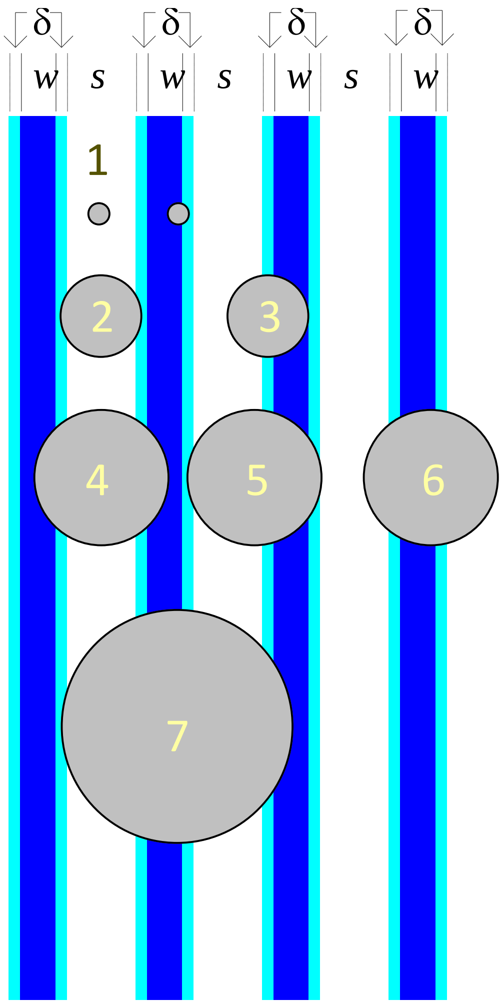
- Defects are inescapable.
  - **The same kinds of defects that degrade yield perceived by the manufacturer, degrade "infant mortality" perceived by end users.**
- Yield is measured at Sort – initial wafer-level testing.
- Infant Mortality is measured by life-test, and controlled by burn in.
  - Life test is an extended burn in designed to acquire detailed reliability data.
- Burn in is a stress preceding final test which activates latent reliability defects (LRDs) so that they may be screened out at final test (Class).
- In these lectures we'll first cover models of Yield, and then cover Infant Mortality.
- Defect models of reliability describe only the left part of the bathtub curve; they don't describe wearout.



Defect Model of Yield and Reliability

- Aspects of defects which affect yield and reliability are
  - Defect density. Number of defects per unit area on a wafer.
  - Spatial variation of defect density
    - Factory-to-factory
    - Lot-to-lot
    - Wafer-to-wafer
    - Across a wafer.
  - Size distribution of defects.
  - Sensitivity of circuits to defects.
- Models are used to
  - Plan for new products by predict yield and reliability figures of merit (FOMs) for hypothetical products and processes.
  - Compute the levels of fault tolerance required to meet yield goals.
  - Calculate burn in times needed to reach required levels of reliability.
  - Design life-test experiments that will provide sufficient data to build reliability models.

Killer vs Latent Reliability Defects

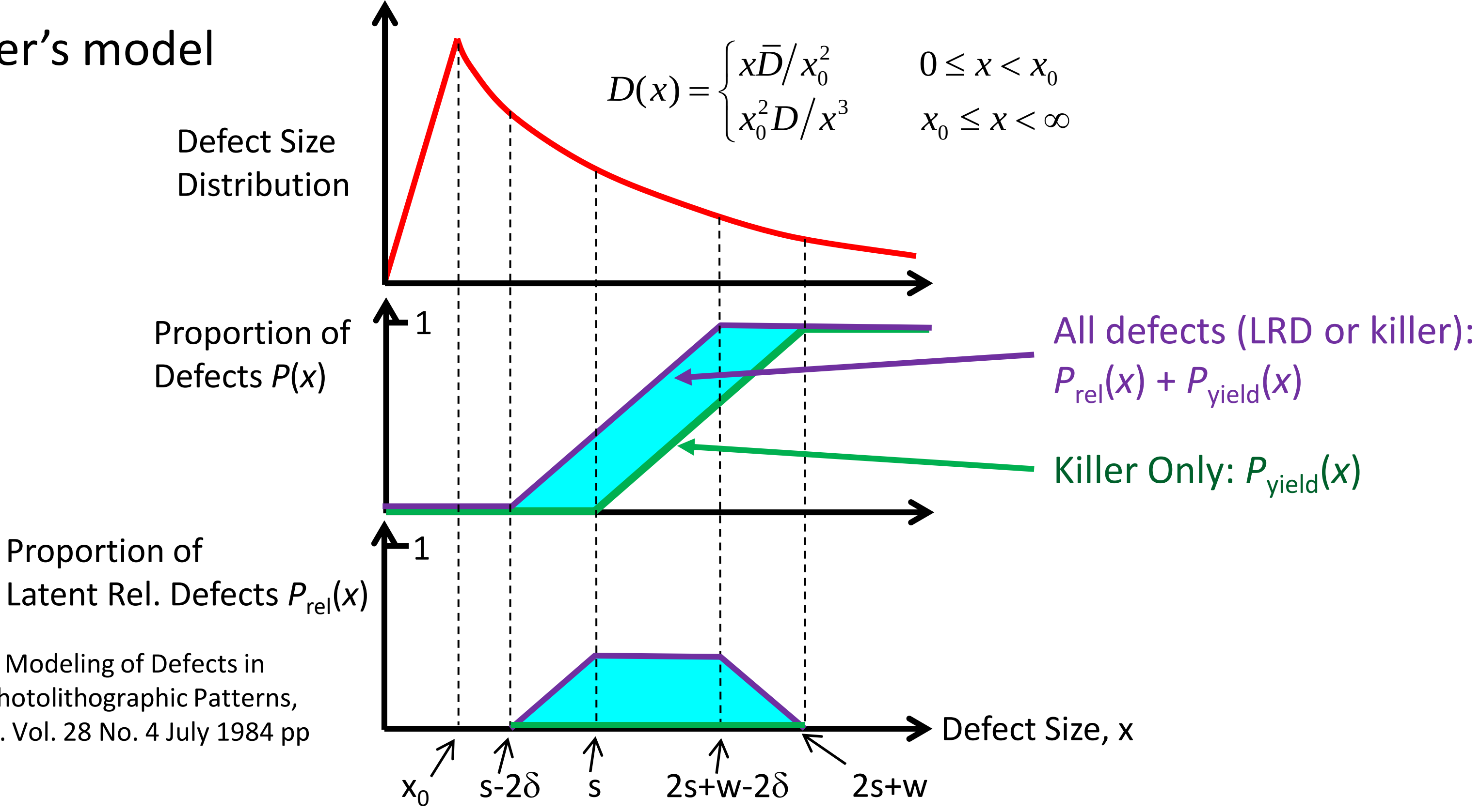


When defect is within δ of line, failure is not immediate but will occur within the specified life of the device.

- Circuit design determines
  - Pattern pitch and space.
  - Different functional blocks have different characteristic pitch/spaces.
- Fab process determines
  - Spatial density of defects, *D* (defects/cm^2)
  - Variation of spatial defect density.
  - Size distribution of defects.
- Ckt design plus size dist’n segregates defects into “killer” and latent reliability defects (LRD).
  - OK, never a yield or reliability defect (1).
  - Sometimes a latent reliability defect (2), sometimes OK (3).
  - Sometimes a killer defect (4), sometimes a latent reliability defect (5), sometimes OK(6).
  - Always a killer defect (7).

Killer vs Latent Reliability Defects

- Defects much smaller or larger than circuit geometry are not latent reliability defects (LRD).
- *Some* defects with size commensurate with circuit geometry are latent reliability defects.
- Typically ~ 1% of defects are latent reliability defects.
- Stapper's model

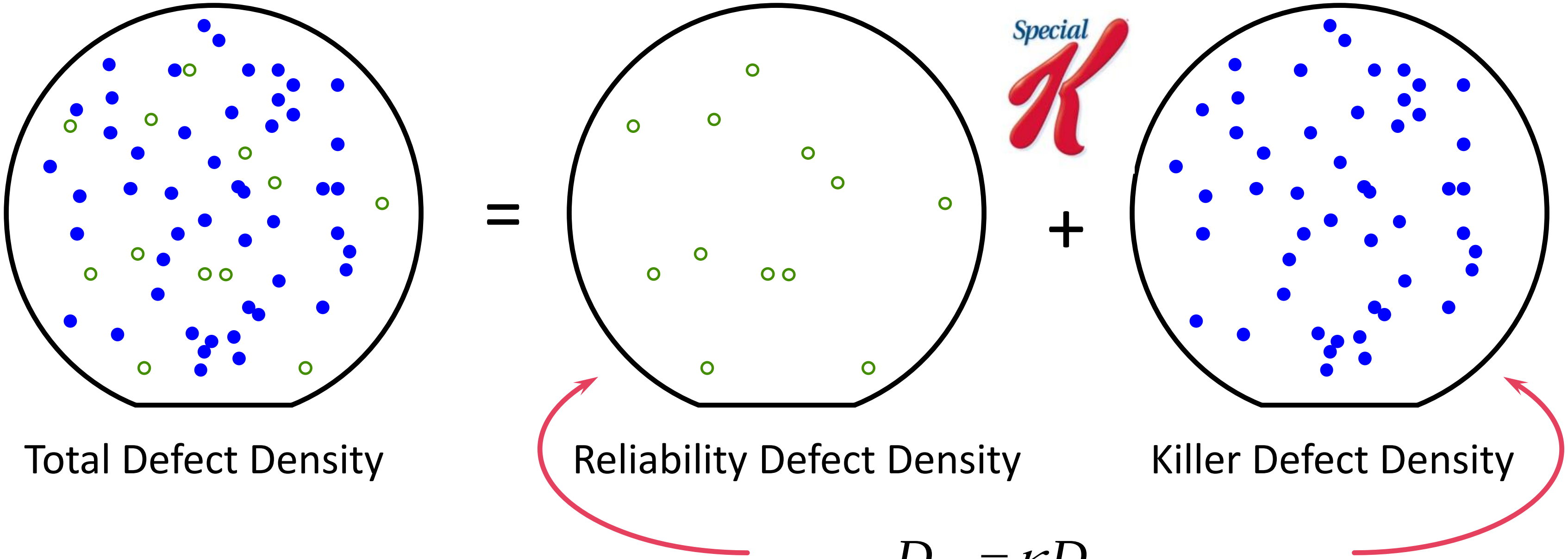


$$D(x) = \begin{cases} x\bar{D}/x_0^2 & 0 \le x < x_0 \\ x_0^2 D/x^3 & x_0 \le x < \infty \end{cases}$$

Charles H. Stapper, Modeling of Defects in Integrated Circuit Photolithographic Patterns, IBM J. Res. Develop. Vol. 28 No. 4 July 1984 pp 461-475

Killer vs Latent Reliability Defects (LRD)

- Defects may be classified as “killer” defects which affect yield or LRD defects which affect reliability.
- Defects of either kind may be clustered. Described by defect density and defect density variance.
- Killer and LRD defects are from the same source, so Yield and Reliability defect densities are proportional: $\kappa = D_{rel}/D_{yield} \approx$ constant (typically ~ 1%).
- D_{yield} is MUCH easier to measure and monitor in manufacturing than D_{rel}.



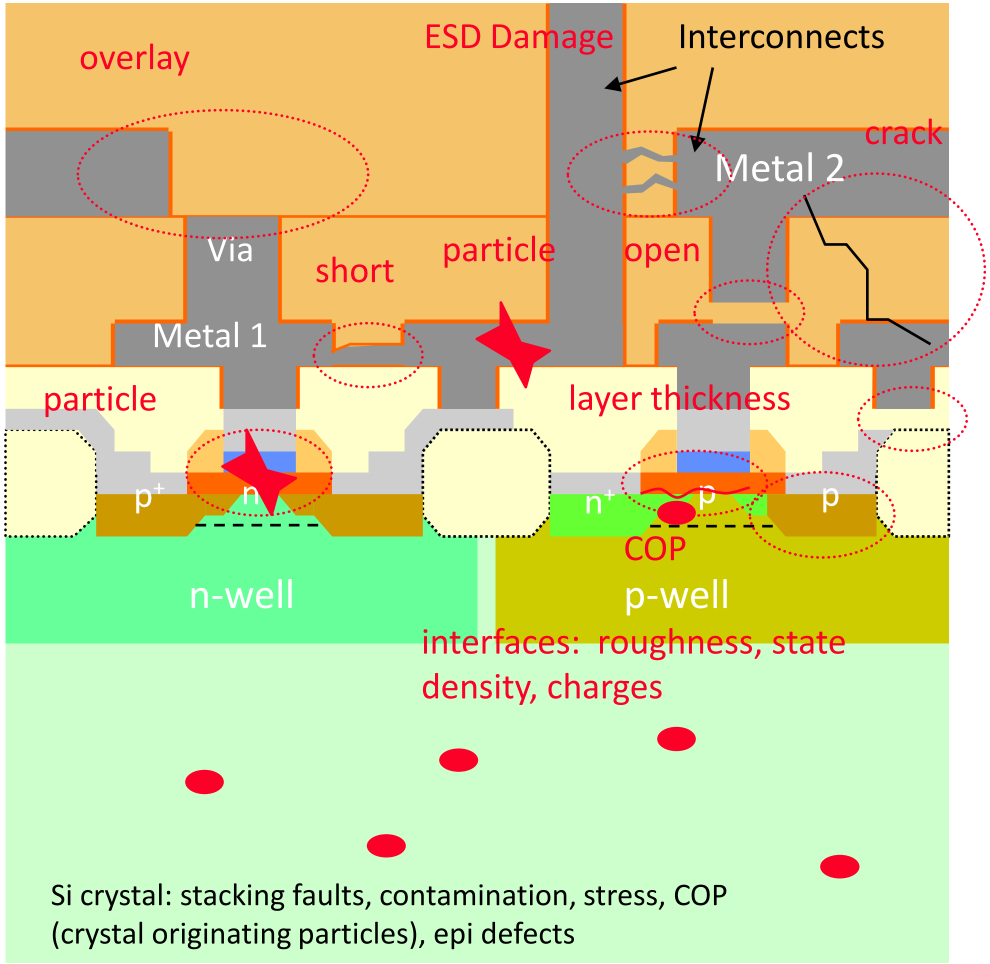
Total Defect Density

Reliability Defect Density

Killer Defect Density

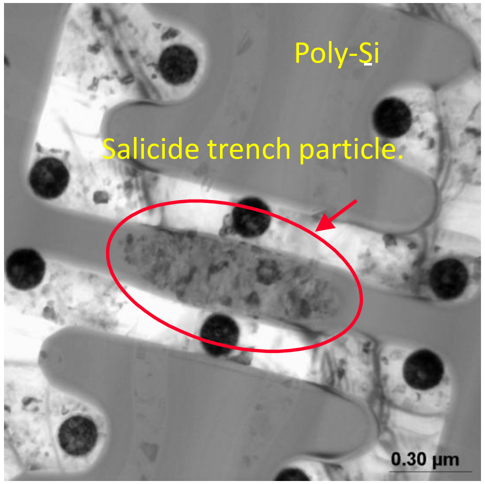
$$D_{rel} = \kappa D_{yield}$$

Ugly Reality



From an ITRS report.

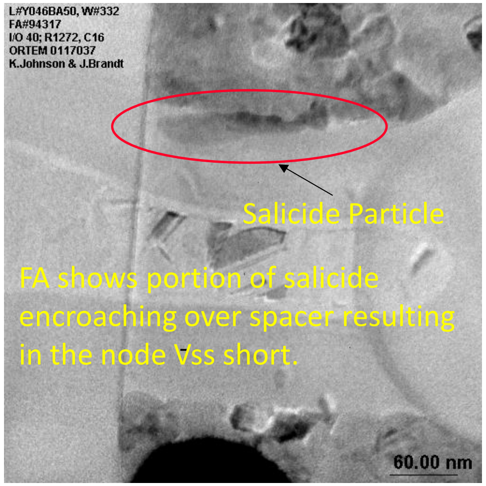
Activated LRDs, Mainly Shorts



Poly-Si

Salicide trench particle.

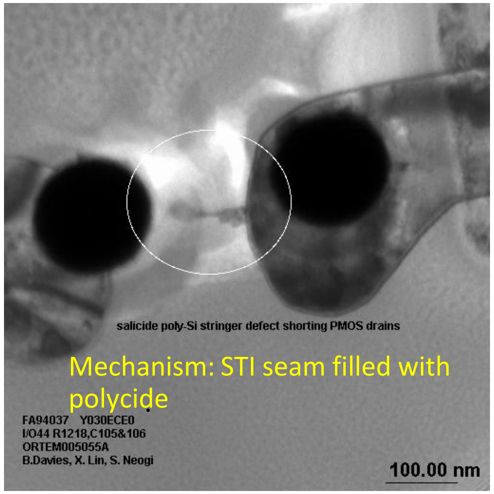
STI Particle



Salicide Particle

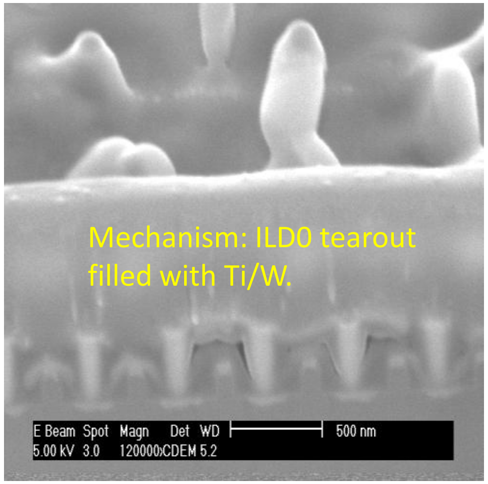
FA shows portion of salicide encroaching over spacer resulting in the node Vss short.

Salicide Encroachment



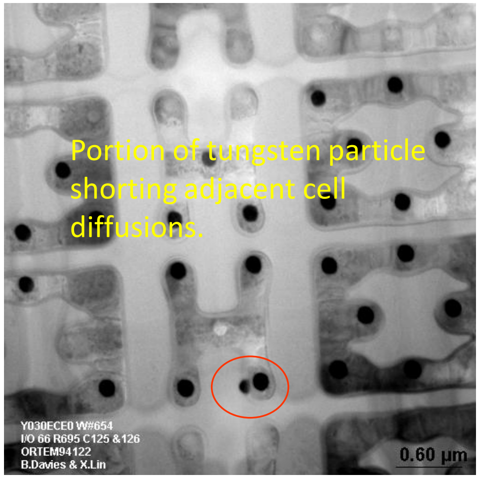
Mechanism: STI seam filled with polycide

Salicide Stringer



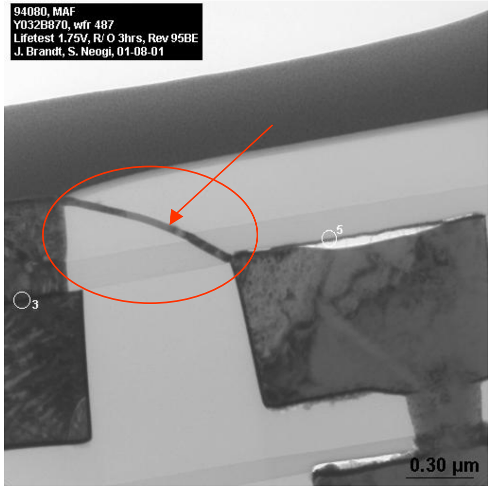
Mechanism: ILD0 tearout filled with Ti/W.

Residual Ti



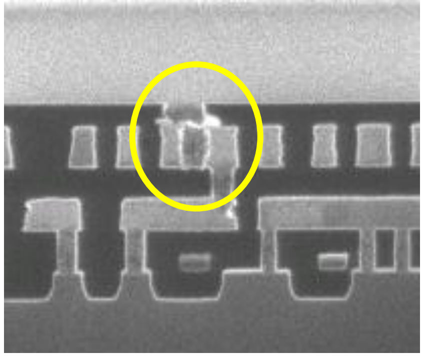
Portion of tungsten particle shorting adjacent cell diffusions.

Tungsten Particle

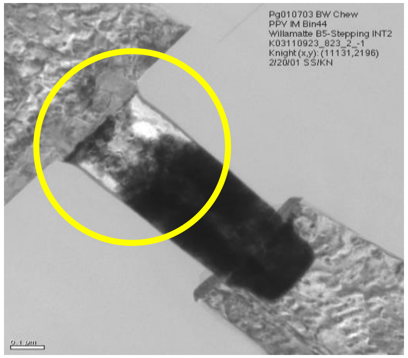


Copper Extrusion

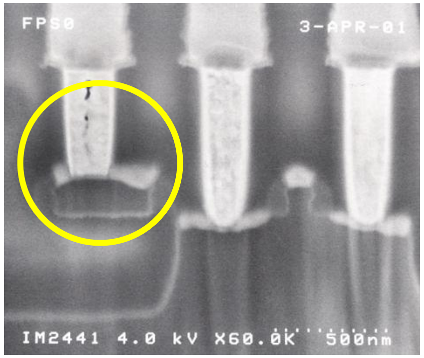
Activated LRDs, Mainly Opens



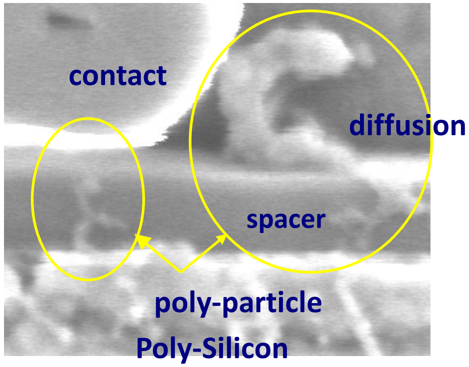
Metal 2 Tungsten Short



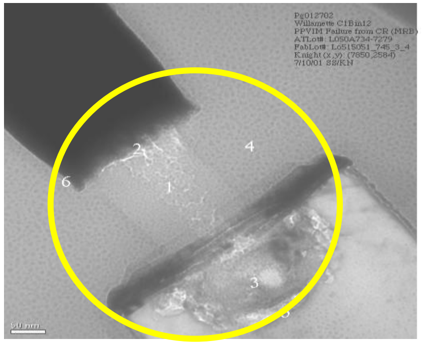
Spongy Via2



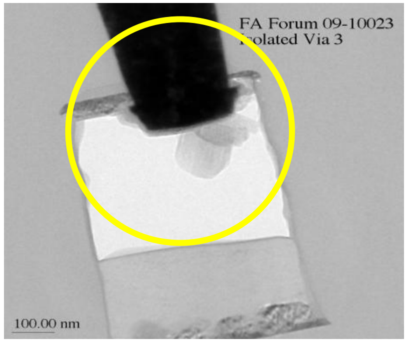
Salicide Punch through



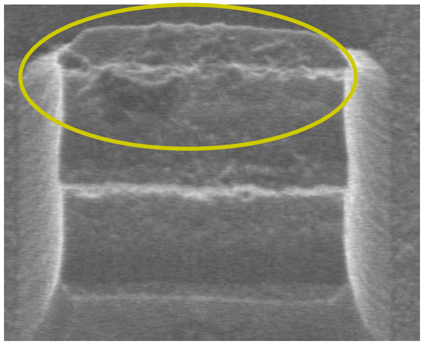
Poly Particle Short



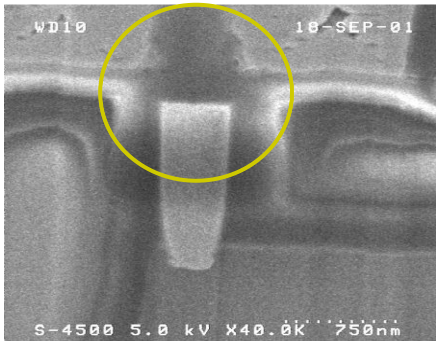
Incomplete filled Via2



Isolated Via3 by Metal Voiding



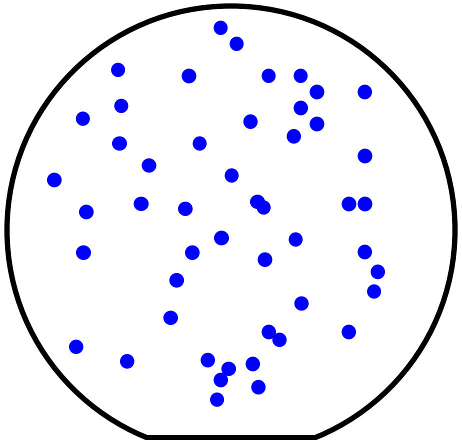
Silicon Abnormality



Missing MT6 at Via5

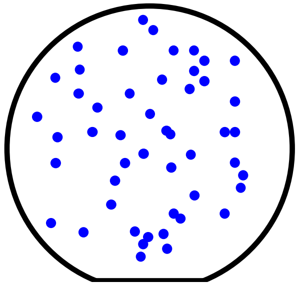
Outline

- Introduction
- ● Models of Yield
- Models of Defect Reliability
- Analysis and Synthesis of Lifetest and Burn In



Killer Defect Density

Defect Models of Yield



Killer Defect Density

- Assumptions for random Yield Model
  - *N* [1000] defects distributed spatially at random across wafers.
  - The silicon area is *W* [1600 cm^2].
  - The die area is *A* [1 cm^2]
  - The defect density is $D = N/W$ [1000/1600 = 0.625 /cm^2]
  - The probability that a defect falls on a die is $p = A/W$ [1/1600 = 0.000626]
  - The average defects per die is $\lambda = Np = NA/W = AD$ [0.625 defects per die]
- The probability that a die has exactly *n* defects is given by the binomial theorem

$$\frac{N!}{(N-n)!n!}p^n(1-p)^{N-n}$$

- Example: *N* = 1000 defects over 1600 dies

$$1600\times\left\{\left(1-\frac{1}{1600}\right)^{1000}\right\}=856$$

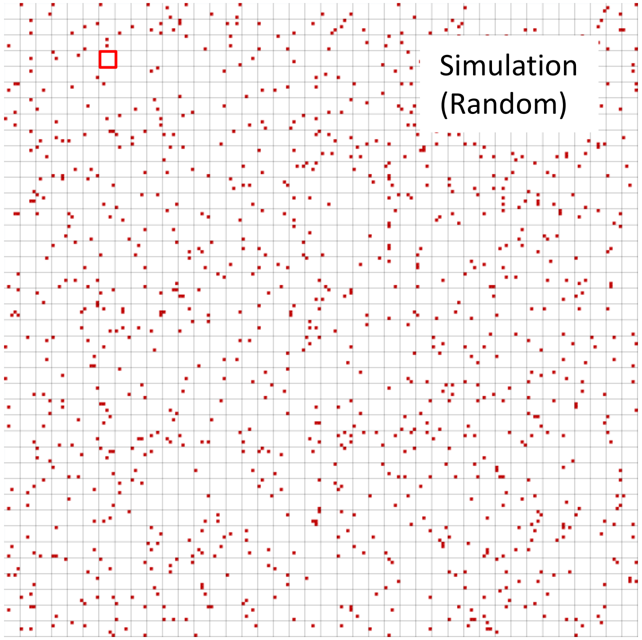
dies with 0 defects

$$1600\times\left\{1000\left(1-\frac{1}{1600}\right)^{999}\left(\frac{1}{1600}\right)\right\}=535$$

dies with 1 defect

$$1600\times\left\{\frac{1000\times999}{2}\left(1-\frac{1}{1600}\right)^{998}\left(\frac{1}{1600}\right)^2\right\}=167$$

dies with 2 defects



Simulation (Random)

Poisson Limit of the Binomial Dist’n

- When $p \to 0$ and $N \to \infty$ in such a way that Np remains finite, it is much more convenient to use the Poisson limit.

$$\frac{N!}{(N-n)!n!} p^n (1-p)^{N-n} \xrightarrow[N\to\infty,\, p\to 0]{} \frac{\lambda^n}{n!} \exp(-\lambda), \quad \lambda = Np$$

$$\lambda = Np = 1000 \times \frac{1}{1600} = 0.625$$

$$1600 \times \left\{ \frac{\lambda^0}{0!} e^{-\lambda} = e^{-0.625} \right\} = 856 \qquad \text{dies with 0 defects}$$

$$1600 \times \left\{ \frac{\lambda^1}{1!} e^{-\lambda} = 0.625 \times e^{-0.625} \right\} = 535 \qquad \text{dies with 1 defect}$$

$$1600 \times \left\{ \frac{\lambda^2}{2!} e^{-\lambda} = \frac{0.625^2 \times e^{-0.625}}{2} \right\} = 167 \qquad \text{dies with 2 defects}$$

- The Poisson limit is nearly always sufficient for yield models.
  - Works well for $N \geq 20$ and $p \leq 0.05$, or if $N \geq 100$ and $p \leq 0.10$
- Details

$$B(n\,|\,N,p) = \frac{N!}{(N-n)!n!} p(1-p)^{N-n}$$

$$= \frac{N(N-1)(N-2)..(N+1-n)}{n!} \left(\frac{\lambda}{N}\right)^n \left(1-\frac{\lambda}{N}\right)^{N-n} = \cancel{N^n} \frac{1\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)..\left(1-\frac{n-1}{N}\right)}{n!} \times \frac{\lambda^n}{\cancel{N^n}} \left(1-\frac{\lambda}{N}\right)^N \left(1-\frac{\lambda}{N}\right)^{-n} = \frac{1\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)..\left(1-\frac{n-1}{N}\right)}{\left(1-\frac{\lambda}{N}\right)^n} \times \frac{\lambda^n}{n!} \times \underbrace{\left(1-\frac{\lambda}{N}\right)^N}$$

$$\xrightarrow[N\to\infty]{} 1 \times \frac{\lambda^n}{n!} \exp(-\lambda)$$

Note: $\lim_{N\to\infty} \left(1-\frac{\lambda}{N}\right)^N = \exp(-\lambda)$

Fault Tolerance

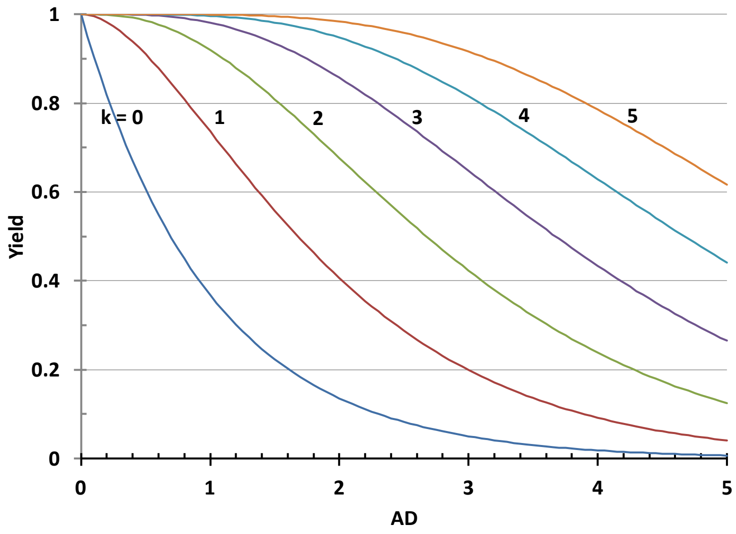
- If a die must be perfect to be "good" the yield is the probability that a die has 0 defects

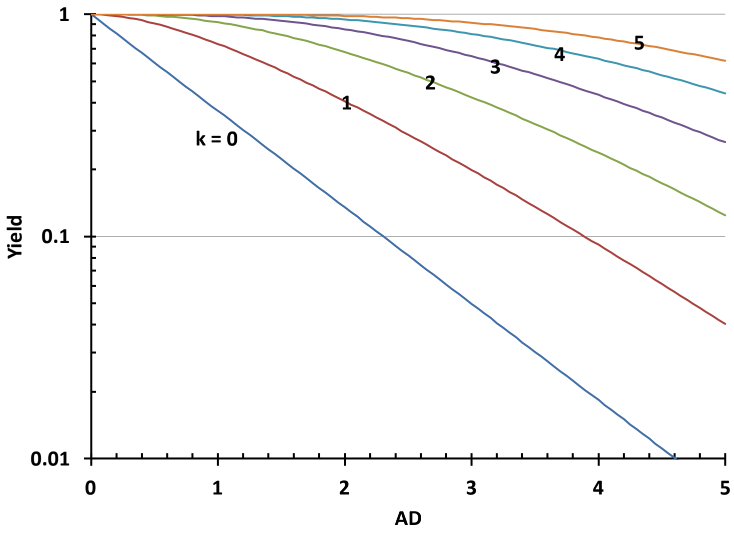
$$Y = \exp(-\lambda), \qquad \lambda = AD \equiv \text{average number of defects per die}$$

- If a die can be good with up to (and including) *k* defects, then the yield is the sum of probabilities of 0, 1, 2, .. *k* defects

Excel:

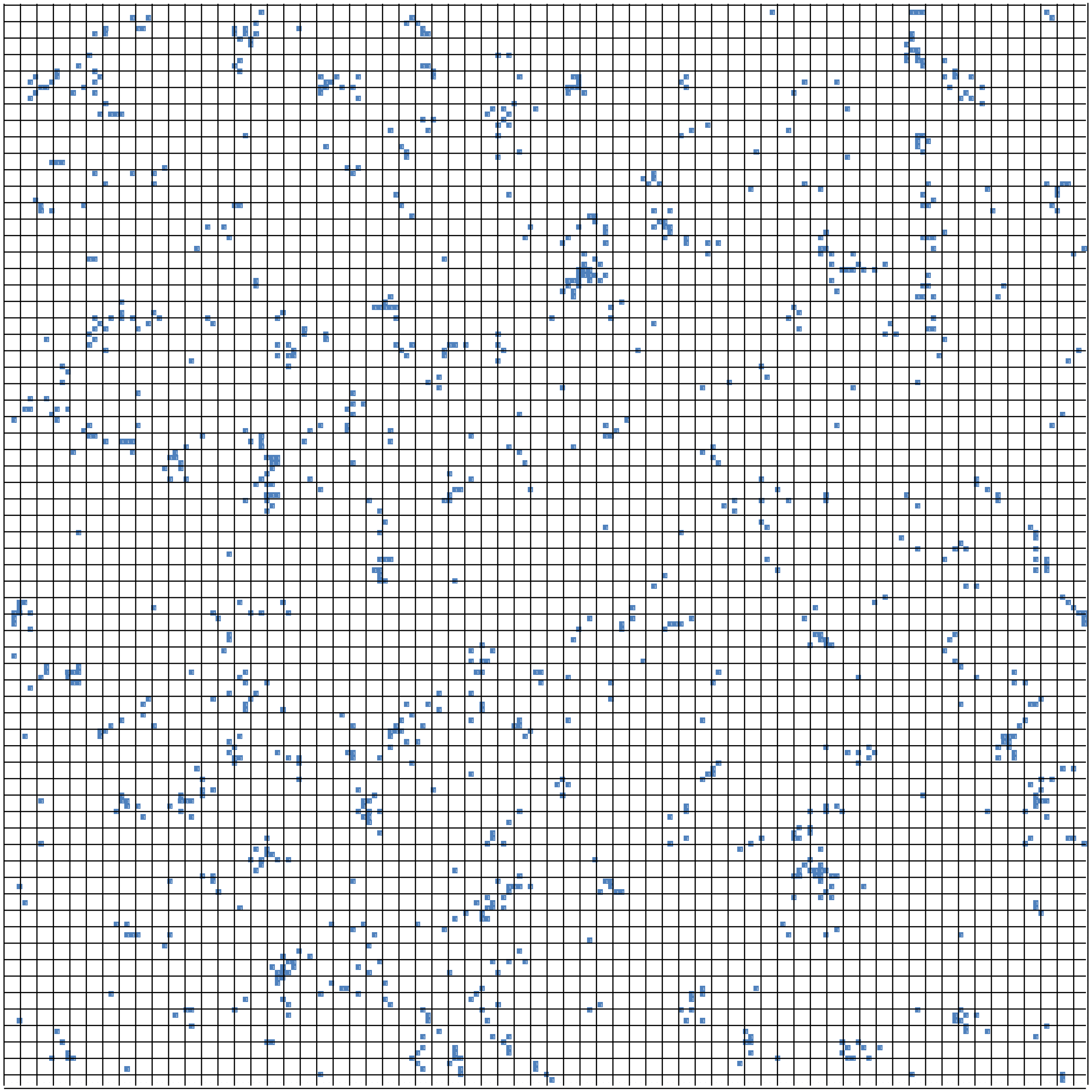
$$Y = \sum_{i=0}^{k} \frac{\lambda^i}{i!} \exp(-\lambda) = \mathrm{POISSON}\left(k, \lambda, \mathrm{TRUE}\right)$$



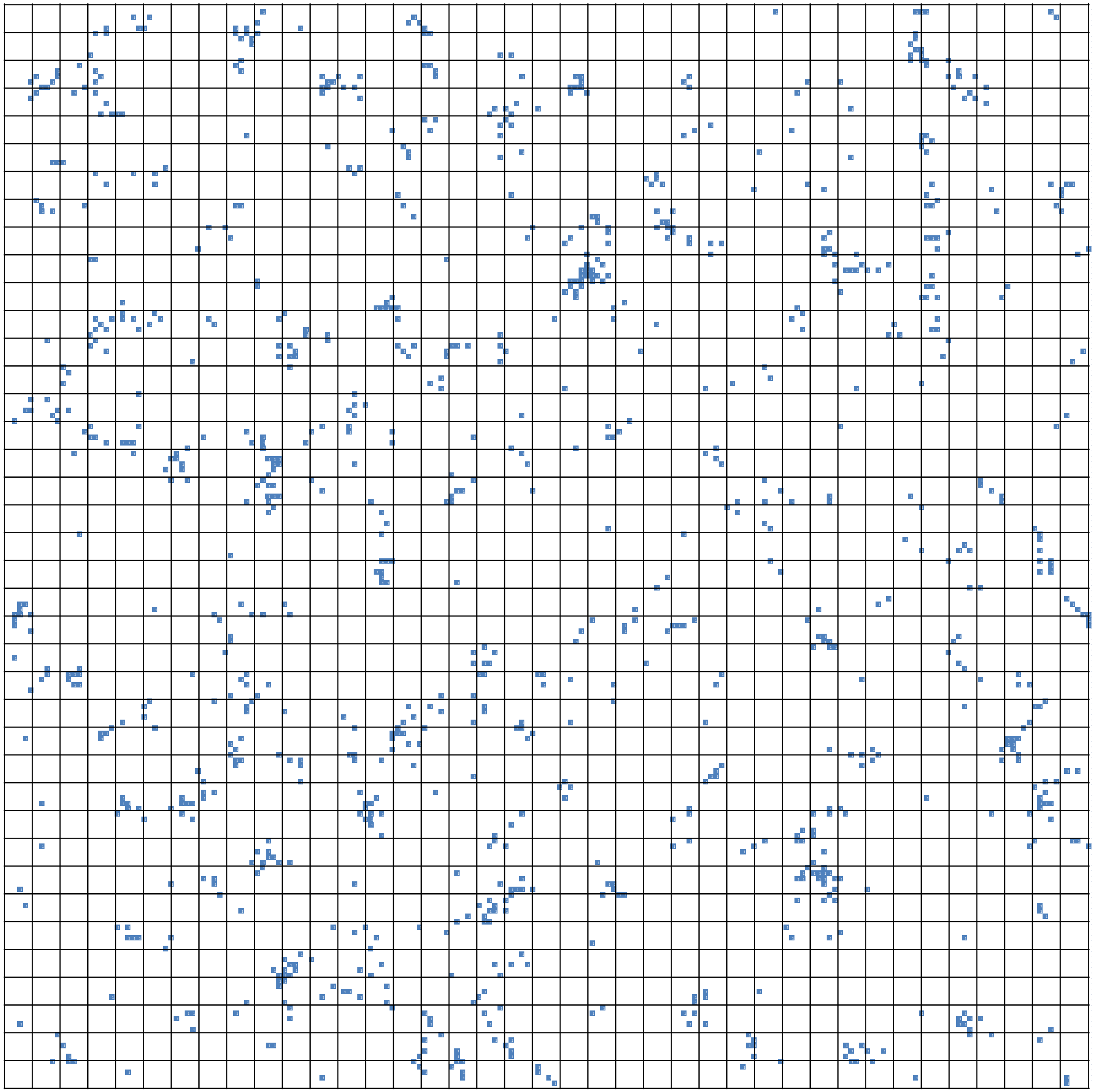


Effect of Defect Clustering

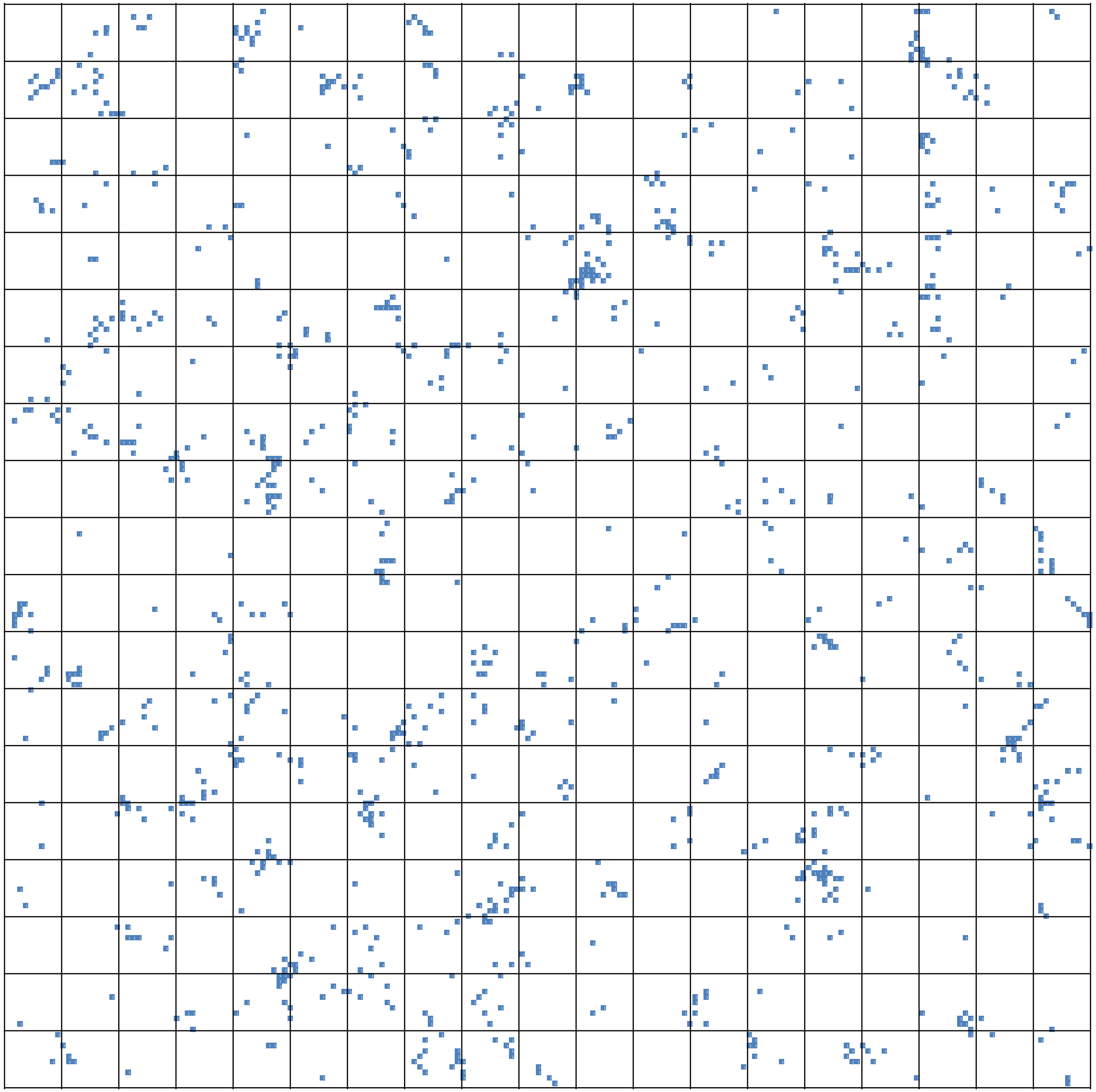
3x3



5x5

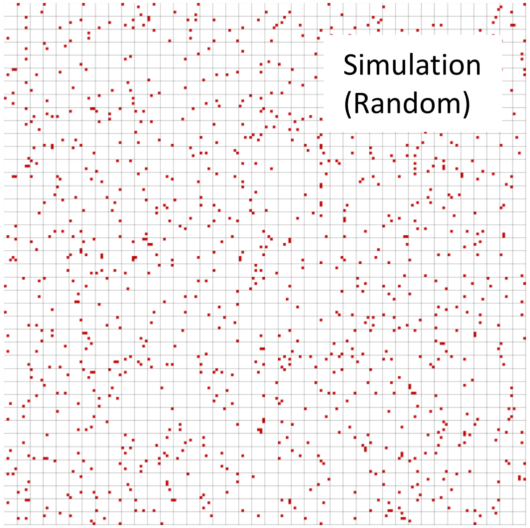


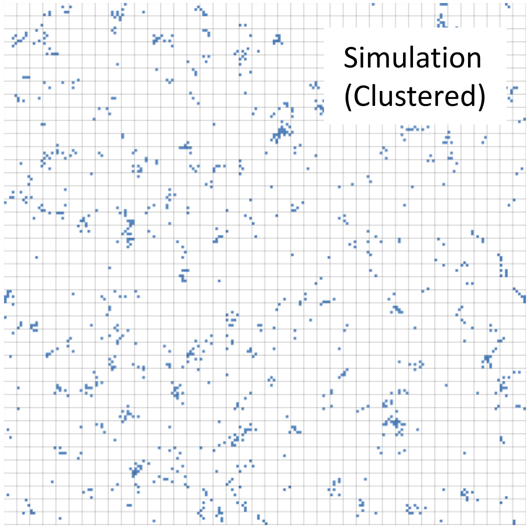
10x10

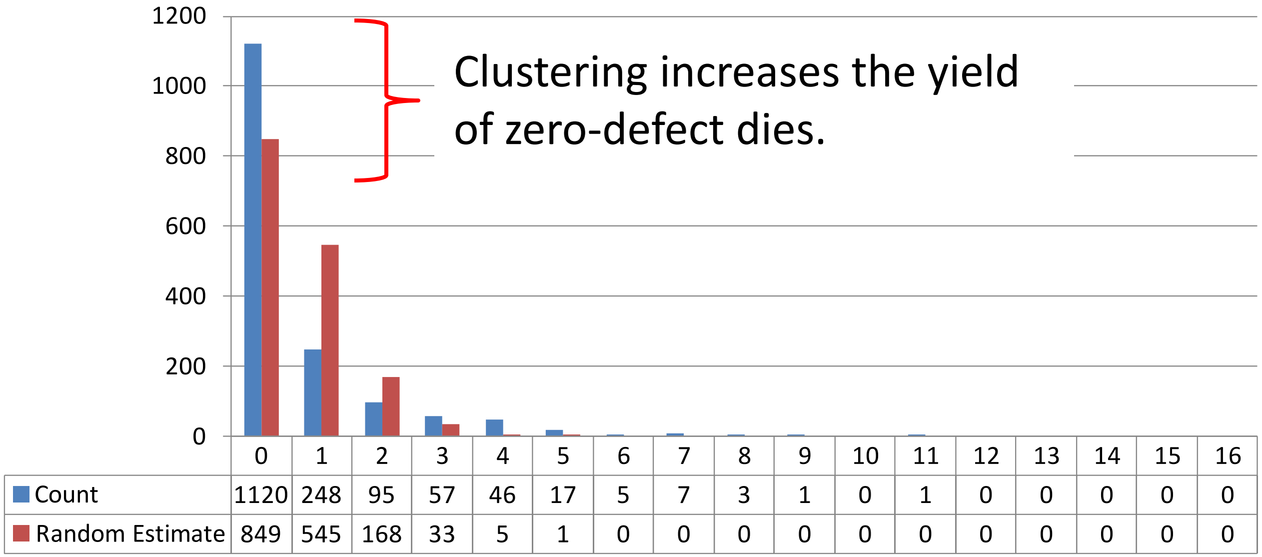


Effect of Defect Clustering

- 1000 defects were synthetically distributed on a 200x200 grid.
- 2 cases: 1) Random, 2) Clustered according to a special algorithm.
- 5x5 dies were superimposed on the grid.
- Counts of defect-free, 1-defect etc. dies were made.


Simulation (Random)


Simulation (Clustered)

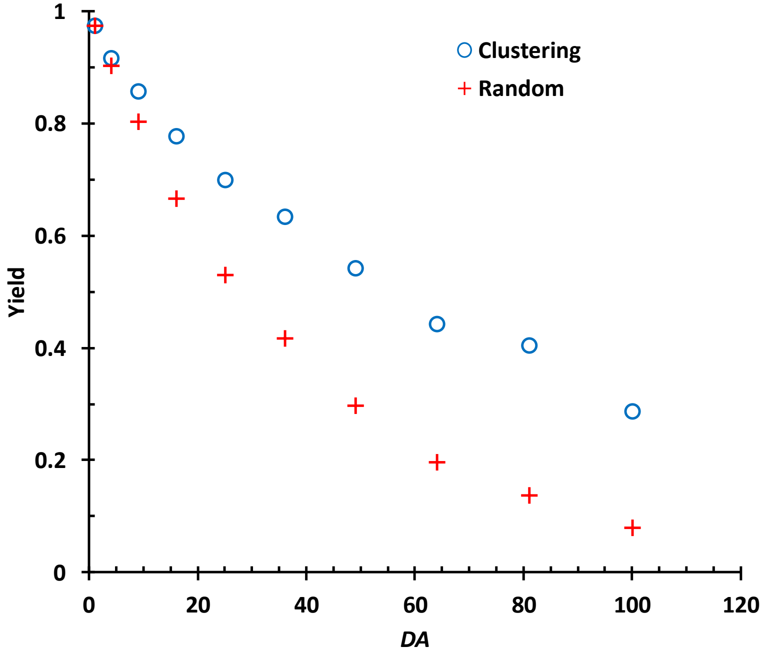


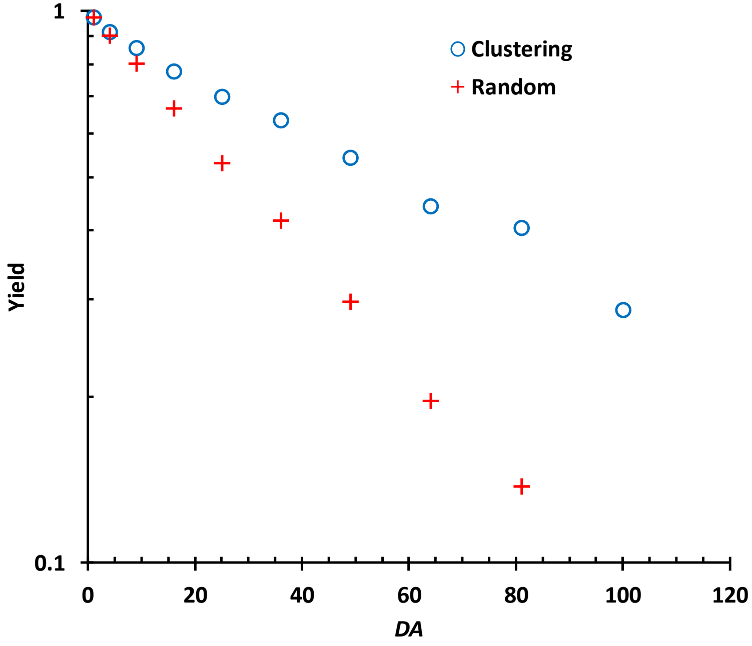
Clustering increases the yield of zero-defect dies.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Count	1120	248	95	57	46	17	5	7	3	1	0	1	0	0	0	0	0
🟥 Random Estimate	849	545	168	33	5	1	0	0	0	0	0	0	0	0	0	0	0

Effect of Clustering

- Window Method: Overlay pattern with 1x1, 2x2, 3x3, .. non-overlapping windows and count defect-free cells.
- Clustered defect patterns have higher yield!
- Business opportunity: For any given yield, with defect clustering a die may be larger and have more functions, giving a more competitive product.



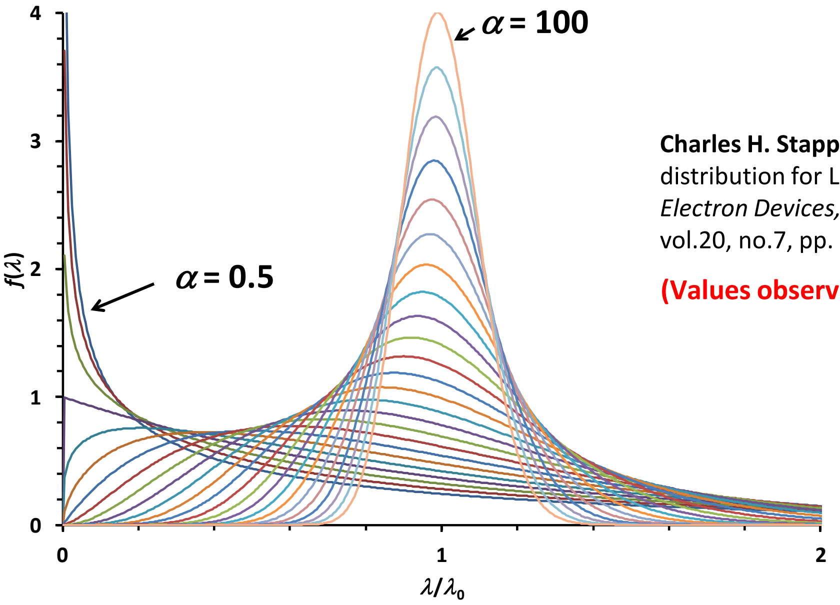


Murphy/Stapper Yield Model

- Murphy posited that defect density has a distribution so

$$Y = \exp(-AD) \quad \text{becomes} \quad Y = \int_0^\infty \exp(-AD)\, f(D)\, dD$$

- Stapper proposed the Gamma distribution for *f*(*D*)



Charles H. Stapper, "Defect density distribution for LSI yield calculations," *Electron Devices, IEEE Transactions on* , vol.20, no.7, pp. 655- 657, July 1973

(Values observed by Stapper: 0.2 < α < 100)

$$f(\lambda) = \frac{\alpha}{\Gamma(\alpha)\lambda_0}\left(\alpha\frac{\lambda}{\lambda_0}\right)^{\alpha-1}\exp\left(-\alpha\frac{\lambda}{\lambda_0}\right) = \mathrm{GAMMADIST}\left(\lambda, \alpha, \lambda_0/\alpha, \mathrm{FALSE}\right)$$

$$\mathrm{E}(\lambda) = \lambda_0 \qquad \mathrm{Var}(\lambda) = \frac{\lambda_0^2}{\alpha}$$

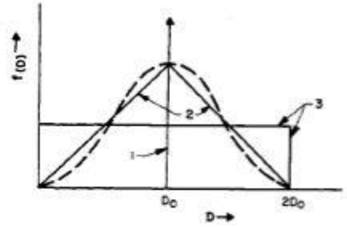
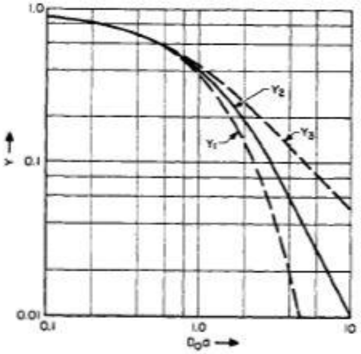


Fig. 1—Distribution functions.



$$Y_1 = e^{-D_0 a}$$

$$Y_2 = \left\{\frac{1-e^{-D_0 a}}{D_0 a}\right\}^2$$

$$Y_3 = \left\{\frac{1-e^{-2D_0 a}}{2D_0 a}\right\}$$

Fig. 2—Yield functions.

B. T. Murphy, "Cost-size optima of monolithic integrated circuits," *Proceedings of the IEEE* , vol.52, no.12, pp. 1537- 1545, Dec. 1964

Yield Model with Clustering

- Probability that a die has exactly *n* defects is the Poisson distribution compounded by the Gamma distribution

$$P(N=n\,|\,\lambda_0,\alpha)=\int_0^\infty \frac{\lambda^n\exp(-\lambda)}{n!}f(\lambda)\,d\lambda=\frac{\Gamma(n+\alpha)}{n!\,\Gamma(\alpha)}p^n(1-p)^\alpha \text{ where } p=\frac{\lambda_0}{\alpha+\lambda_0}$$

$$\xrightarrow[\alpha\to\infty;n,\lambda_0\text{ finite}]{}\frac{\lambda_0^n\exp(-\lambda_0)}{n!}$$

  - Special case: probability that a die has exactly 0 defects is the yield

$$Y=\left(1+\frac{\lambda_0}{\alpha}\right)^{-\alpha}\xrightarrow[\alpha\to\infty;n,\lambda_0\text{ finite}]{}\exp(-\lambda_0)$$

- The cumulative negative binomial is yield if *n* defects are tolerated

$$P\left(N\le n\,|\,\lambda_0,\alpha\right)=\sum_{k=0}^{n}\frac{\Gamma(n+\alpha)}{n!\,\Gamma(\alpha)}p^n(1-p)^\alpha=1-B\left(\frac{\lambda_0}{\lambda_0+\alpha},n+1,\alpha\right)$$

  - Special case: *n* = 0 (no fault tolerance)

$$P\left(N=0\,|\,\lambda_0,\alpha\right)=1-B\left(\frac{\lambda_0}{\lambda_0+\alpha},1,\alpha\right)=\left(1+\frac{\lambda_0}{\alpha}\right)^{-\alpha}$$

Fault Tolerant Clustering Model

- Model parameters

$$\mathrm{E}(\lambda) = \lambda_0 = D_0 A$$

$$\mathrm{Var}(\lambda) = \frac{\lambda_0^2}{\alpha}$$

$$\text{Standard Error in } \lambda = \frac{\sqrt{\mathrm{Var}(\lambda)}}{\mathrm{E}(\lambda)} = \frac{1}{\sqrt{\alpha}}$$

n = Number of defects tolerated.

This is a one-page summary of fault-tolerance clustering yield model formulae.

- Yield formula; probability of $\leq n$ failures.

$$P(N \leq n \mid \lambda_0, \alpha) = Y = 1 - B\left(\frac{\lambda_0}{\lambda_0 + \alpha}, n+1, \alpha\right) = 1 - \mathrm{BETADIST}\left(\frac{\lambda_0}{\lambda_0 + \alpha}, n+1, \alpha\right)$$

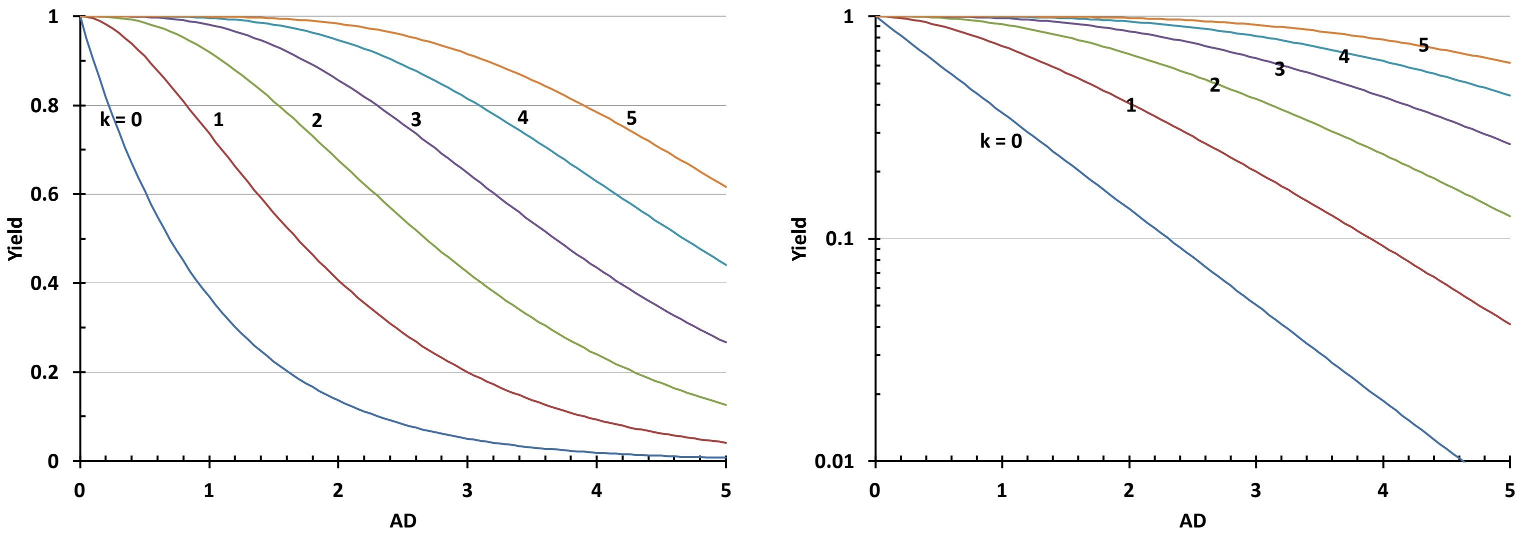
- Probability of exactly *n* failures

$$P(N = n \mid \lambda_0, \alpha) = \frac{\Gamma(n+\alpha)}{n!\Gamma(\alpha)}\left(\frac{\lambda_0}{\alpha + \lambda_0}\right)^n \left(\frac{\alpha}{\alpha + \lambda_0}\right)^\alpha = \mathrm{NEGBINOMDIST}\left(n, \alpha, \frac{\alpha}{\alpha + \lambda_0}\right)$$

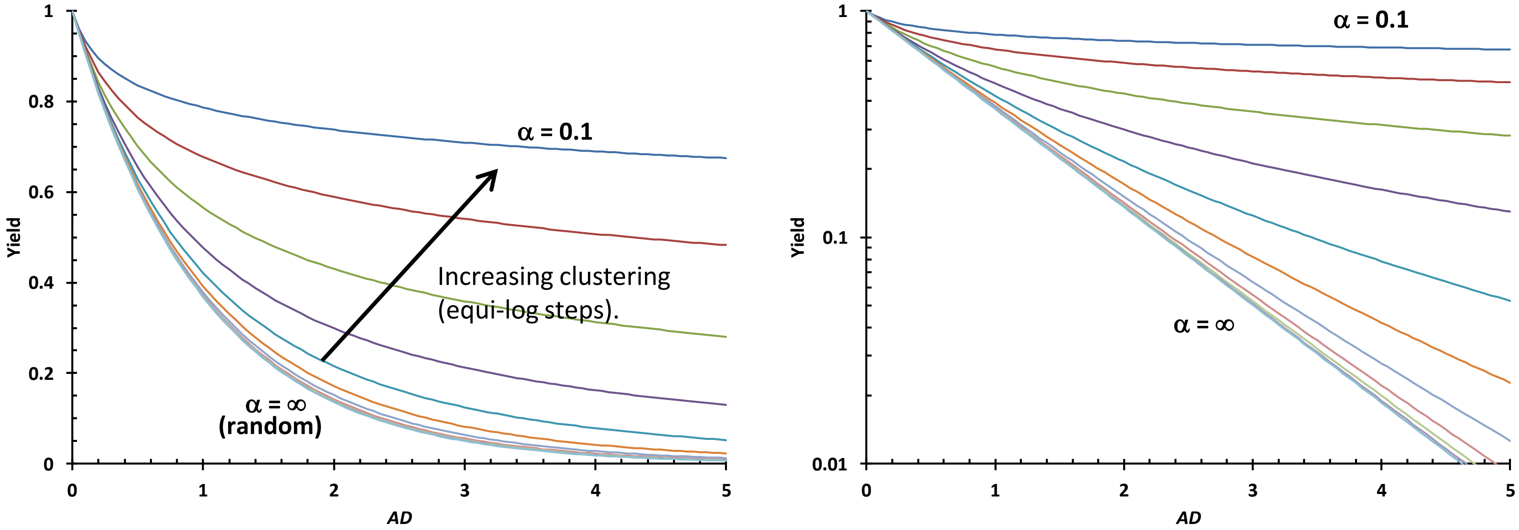
Excel's version of this function requires *n* and α to be integers, but real $\alpha \geq 0$ is meaningful in the theory. It is easy to write a user function for any real $\alpha \geq 0$, and integer *n*.

Fault Tolerant Clustering Model

- $\alpha = \infty$ No clustering (random)
- Various levels of fault tolerance (k).



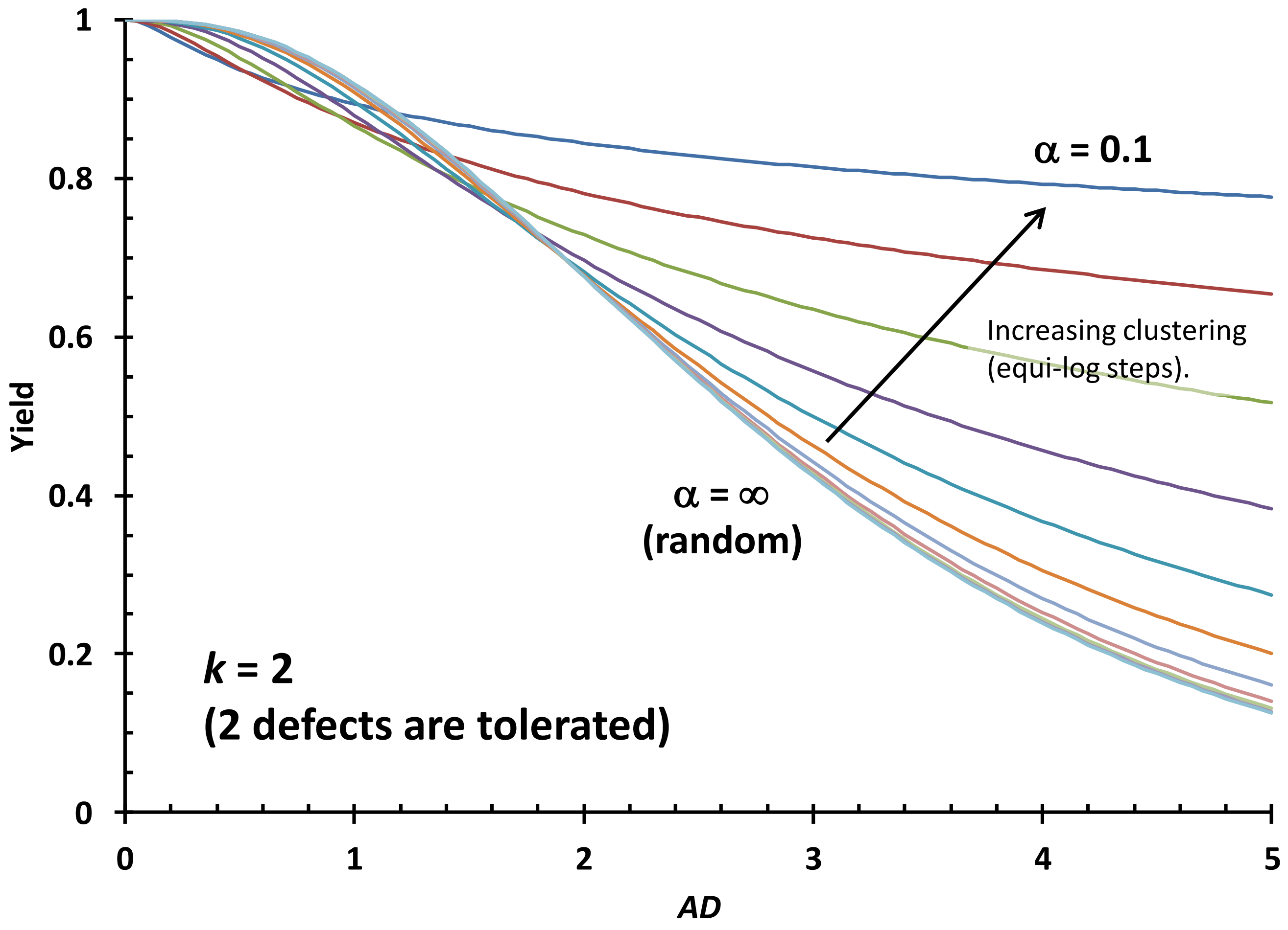
- Various levels of clustering.
- k = 0 (no fault tolerance).



Increasing clustering (equi-log steps).

Fault Tolerant Clustering Model

- What happens when fault tolerance and yield interact?
- With fault tolerance, yield can decrease with increasing clustering!



Limitation of Compound Model

- Defect density "compounding" models like this

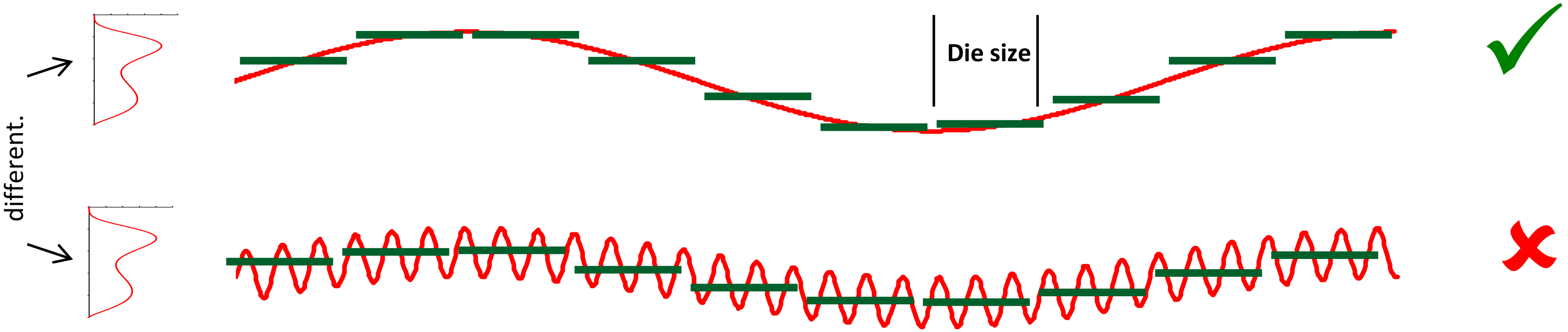
$$P(N=n\mid\lambda_0,\alpha)=\int_0^\infty \frac{\lambda^n\exp(-\lambda)}{n!}f(\lambda)\,d\lambda$$

are valid only when within-die spatial defect density variation is negligible.

- Compounding models are OK for spatial density variation from
  - Die-to-die
  - Wafer-to-wafer
  - Lot-to-lot
  - Factory-to-factory.

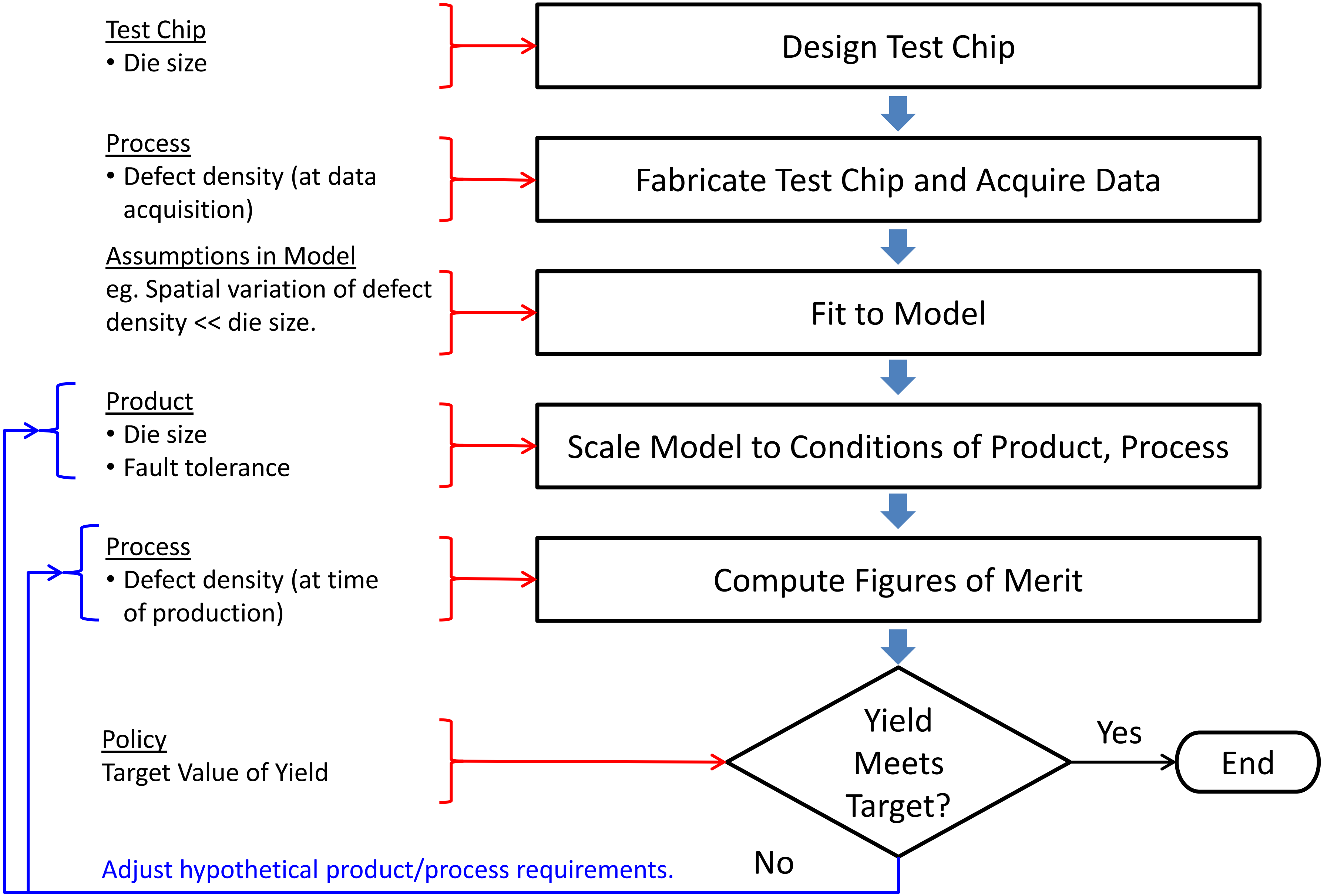
$f(\lambda)$

$f(\lambda)$ describes only the variation, not the spatial distribution.



These can be the same, even if the spatial variation is different.

Motivation: What’s it for?



Test Chip
- Die size

Design Test Chip

Process
- Defect density (at data acquisition)

Fabricate Test Chip and Acquire Data

Assumptions in Model
eg. Spatial variation of defect density << die size.

Fit to Model

Product
- Die size
- Fault tolerance

Scale Model to Conditions of Product, Process

Process
- Defect density (at time of production)

Compute Figures of Merit

Policy
Target Value of Yield

Yield Meets Target?

Adjust hypothetical product/process requirements.

Homework 13.1

- Yield data was aquired for a test chip with 1 cm^2 area and no fault tolerance enabled. The following windowing data was acquired:

$$\lambda = AD$$

Clustering

$$Y = 1 - B\left(\frac{\lambda}{\lambda+\alpha}, n+1, \alpha\right) = 1 - \mathrm{BETADIST}\left(\frac{\lambda}{\lambda+\alpha}, n+1, \alpha\right)$$

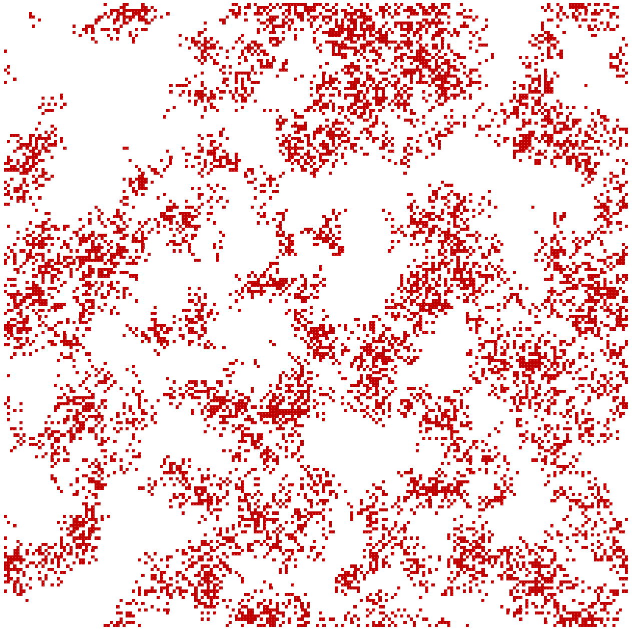
$$Y = 1 - B\left(\frac{\lambda}{\lambda+\alpha}, 1, \alpha\right) = \left(1+\frac{\lambda}{\alpha}\right)^{-\alpha} \qquad (n = 0,\ \text{no fault tolerance})$$

No Clustering

$$Y = \sum_{i=0}^{k} \frac{\lambda^i}{i!}\exp(-\lambda) = \mathrm{POISSON}(\lambda, k, \mathrm{TRUE})$$

$$Y = \exp(-\lambda) \qquad (n = 0,\ \text{no fault tolerance})$$

Window	Yield (%)
1x1	80.00
2x2	51.56
3x3	37.31
4x4	28.80
5x5	24.38
6x6	19.98
7x7	16.78
8x8	11.52
9x9	11.34
10x10	6.75

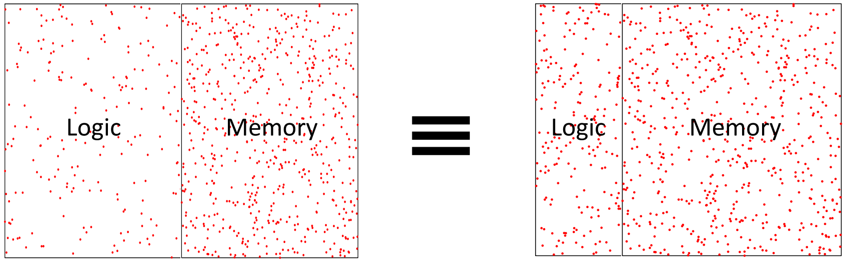


- Fit the data to a cluster yield model and thereby determine alpha and defect density of the process.
- Use the fitted model to calculate yields for products that can tolerate 0, 1,2, 3, 4, and 5 defects with die sizes ranging from 0.5 cm^2 to 4 cm^2 on the same process as the test chip (same defect density).

Critical Area Formulation

- Physical picture:
  - Defect densities for circuit blocks depend on the sensitivity of the block to defects (eg. memory vs logic).
  - Circuit block areas are the physical areas of the blocks.
- Critical area picture:
  - Defect densities for all blocks are the same reference density, $D_{\text{Reference}}$ determined by a standard measure of the fab process.
  - Areas of blocks are different from the physical areas of the circuit blocks.
- Benefit: Clear responsibility for parameters
  - Manufacturing owns measurement of defect density.
  - Design owns determination of critical areas by modeling.

$$A_{\text{Critical Area}} = \frac{D_{\text{Actual}}}{D_{\text{Reference}}} A_{\text{Physical}}$$



Defect densities greatly exaggerated!

$$\begin{aligned} Y &= Y_{\text{Logic}} Y_{\text{Memory}} \\ &= \exp\left(-D_{\text{Logic}} \times A_{\text{Logic(physical)}} - D_{\text{Memory}} \times A_{\text{Memory(physical)}}\right) \end{aligned}$$

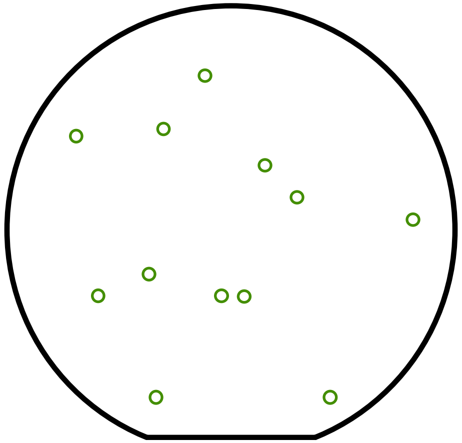
$$\begin{aligned} Y &= Y_{\text{Logic}} Y_{\text{Memory}} \\ &= \exp\left[-D_{\text{Reference}}\left(A'_{\text{Logic}} + A'_{\text{Memory}}\right)\right] \end{aligned}$$

Outline

- Introduction
- Models of Yield

● Models of Defect Reliability

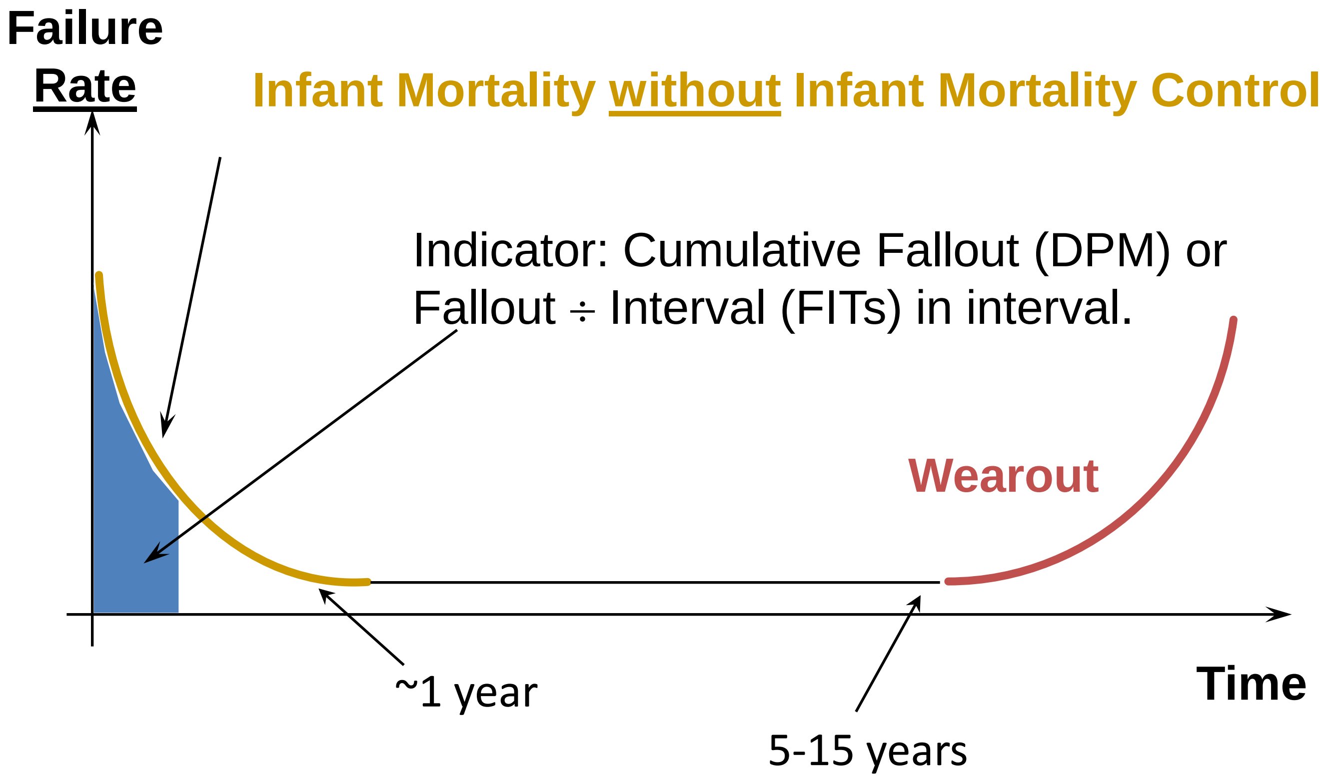
- Analysis and Synthesis of Lifetest and Burn In



Reliability Defect Density

Bathtub Curve

- Defects: A declining failure rate, affects early life.
- Materials, Design: Wearout, increasing failure rate, affects late life.



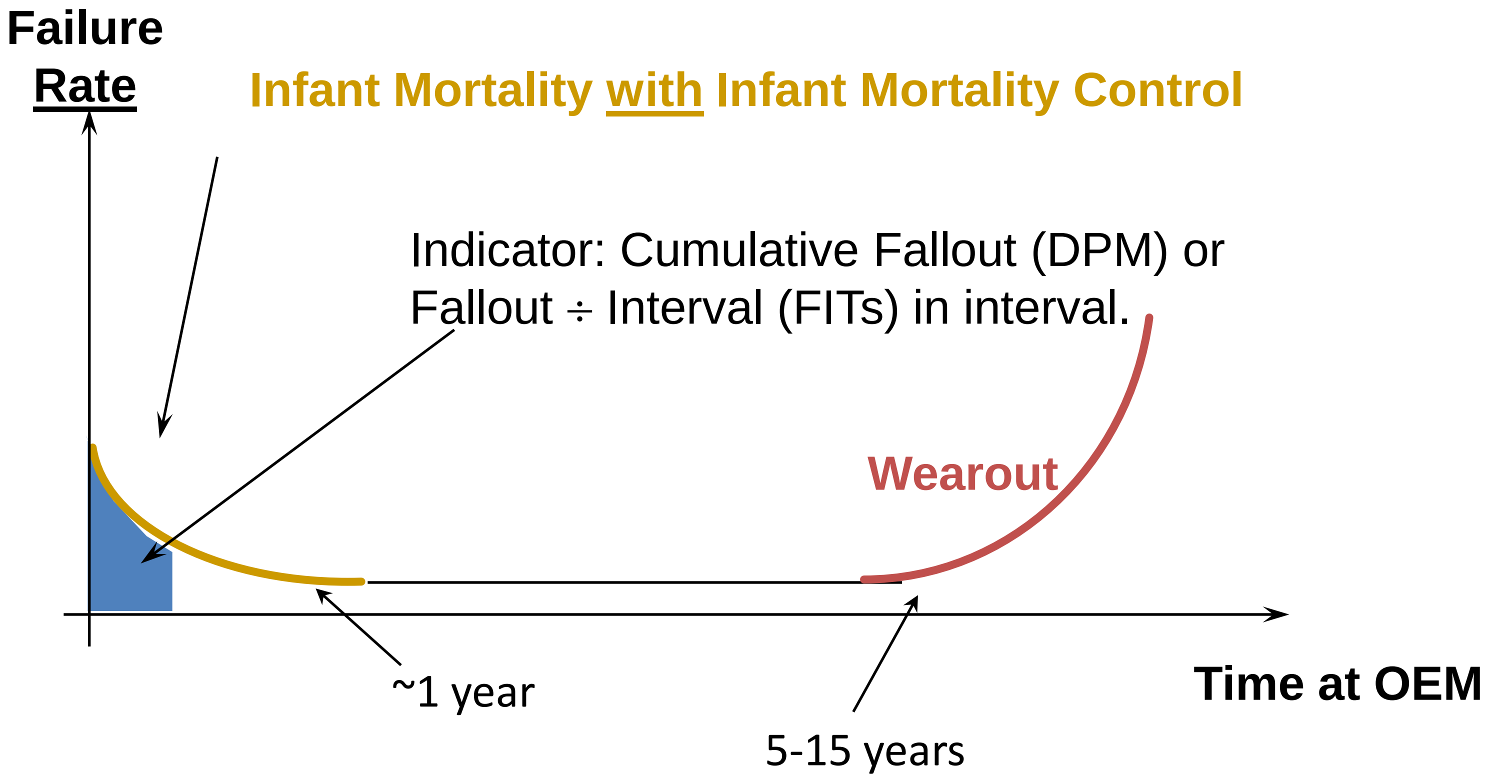
Infant Mortality without Infant Mortality Control

Indicator: Cumulative Fallout (DPM) or Fallout ÷ Interval (FITs) in interval.

Typical Fallout w/o IMC: 2000 - 5000 DPM in 0-30d

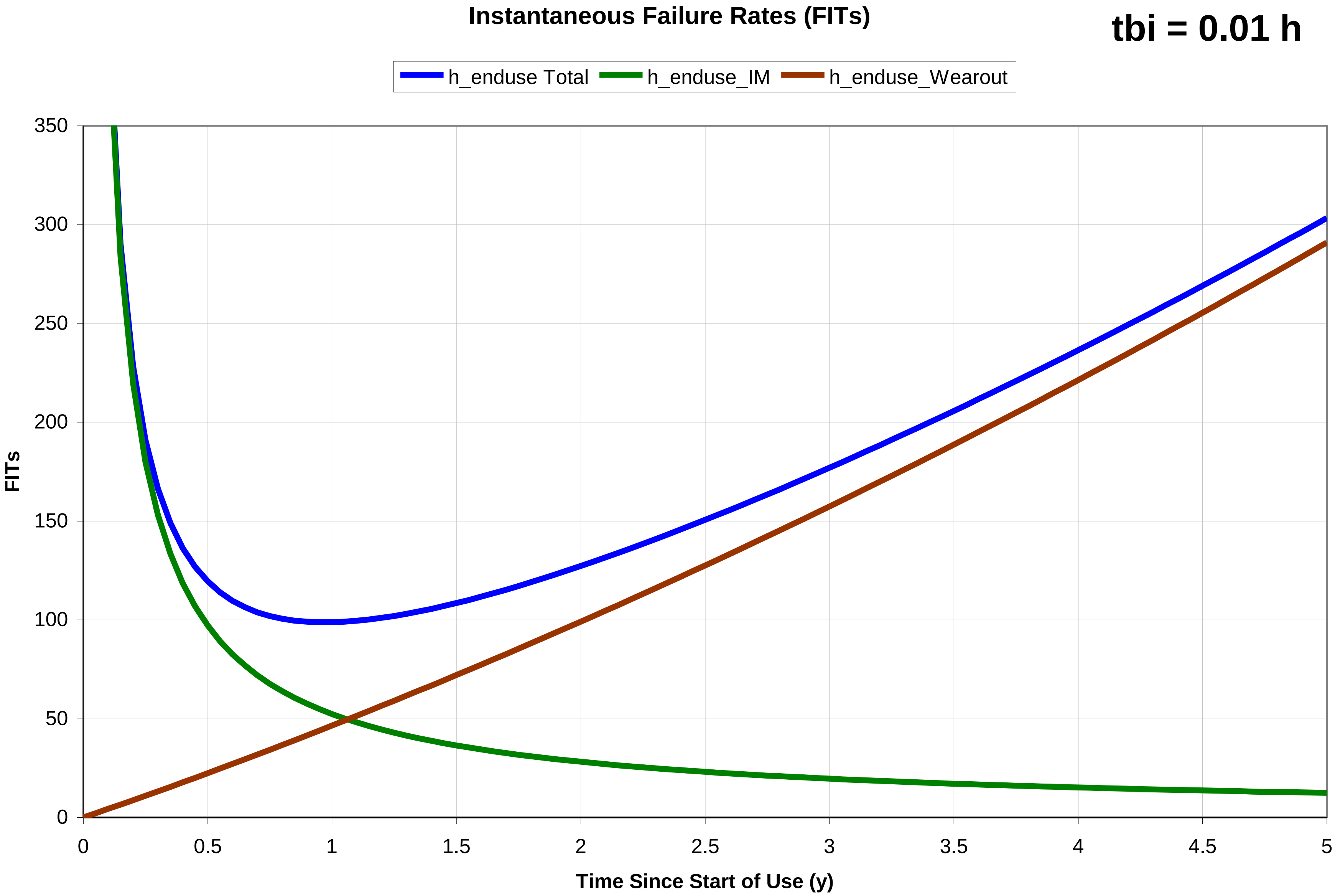
Customer-Perceived Bathtub Curve

- Use Infant Mortality Control (eg. Burn In) to reshape the bathtub fail rate curve as perceived by customers.

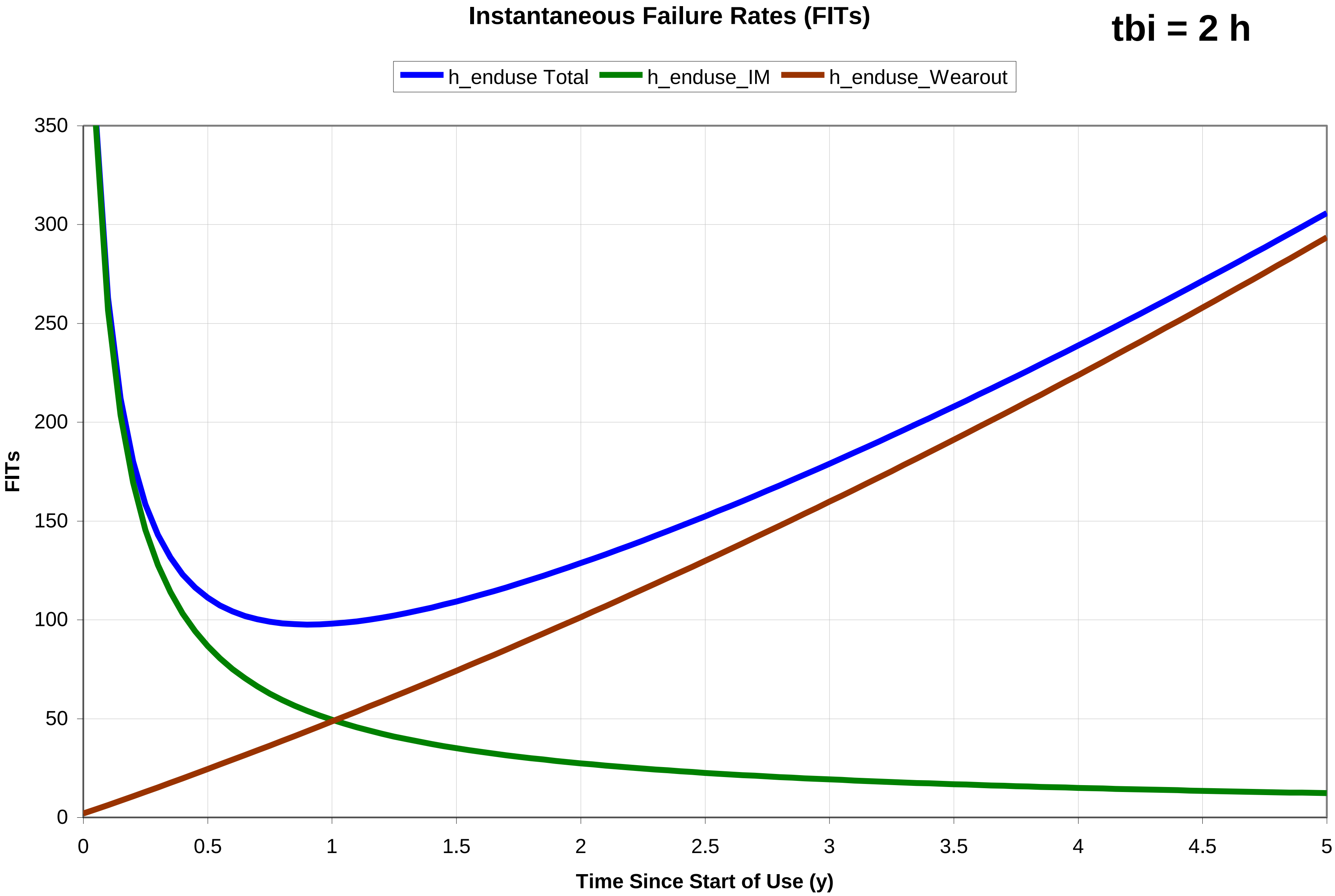


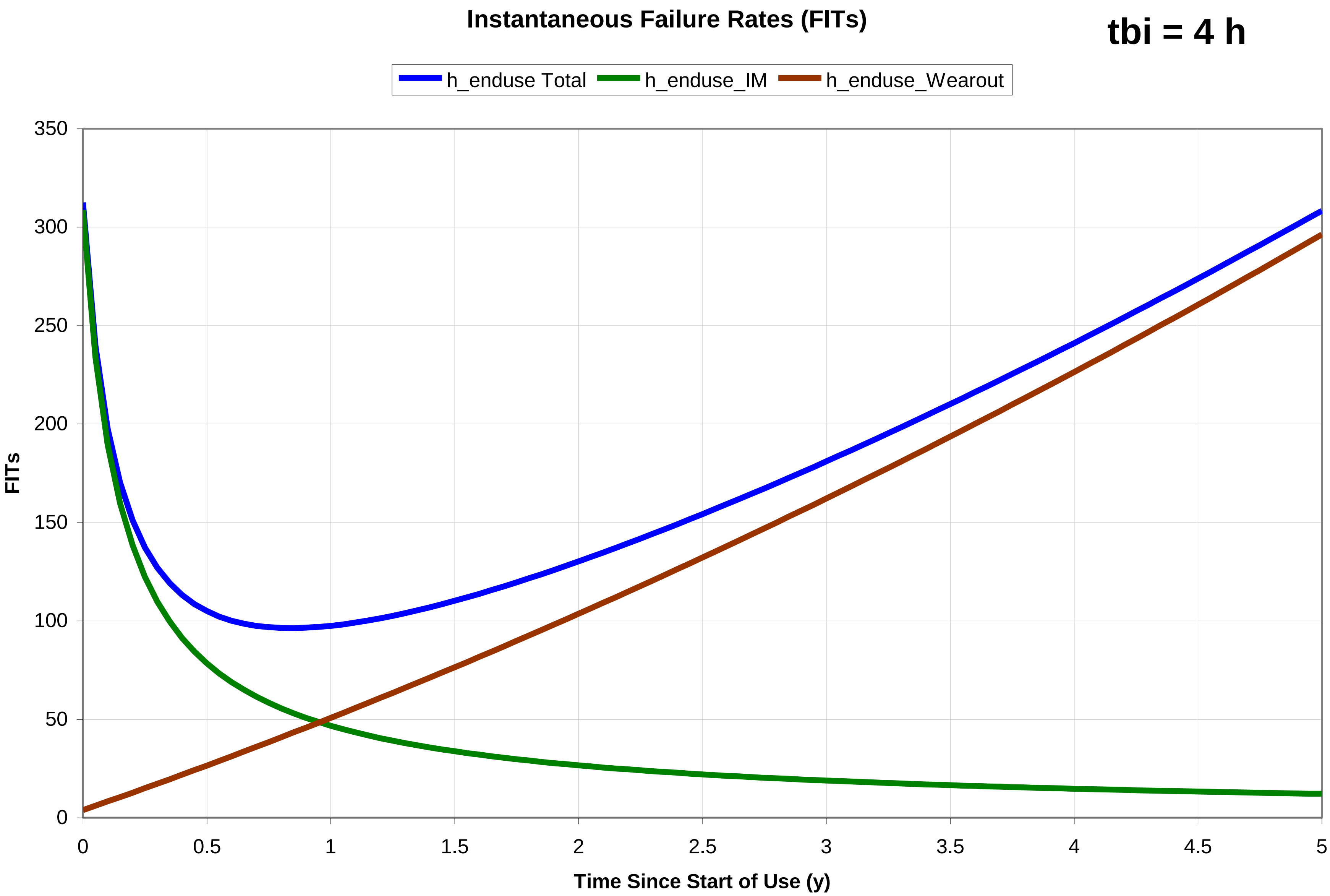
Typical Goals: 100 -1000 DPM 0-30d; 200 - 400 FITs 0-1y

tbi = 0.01 h


Instantaneous Failure Rates (FITs)
h_enduse Total
h_enduse_IM
h_enduse_Wearout
FITs
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time Since Start of Use (y)

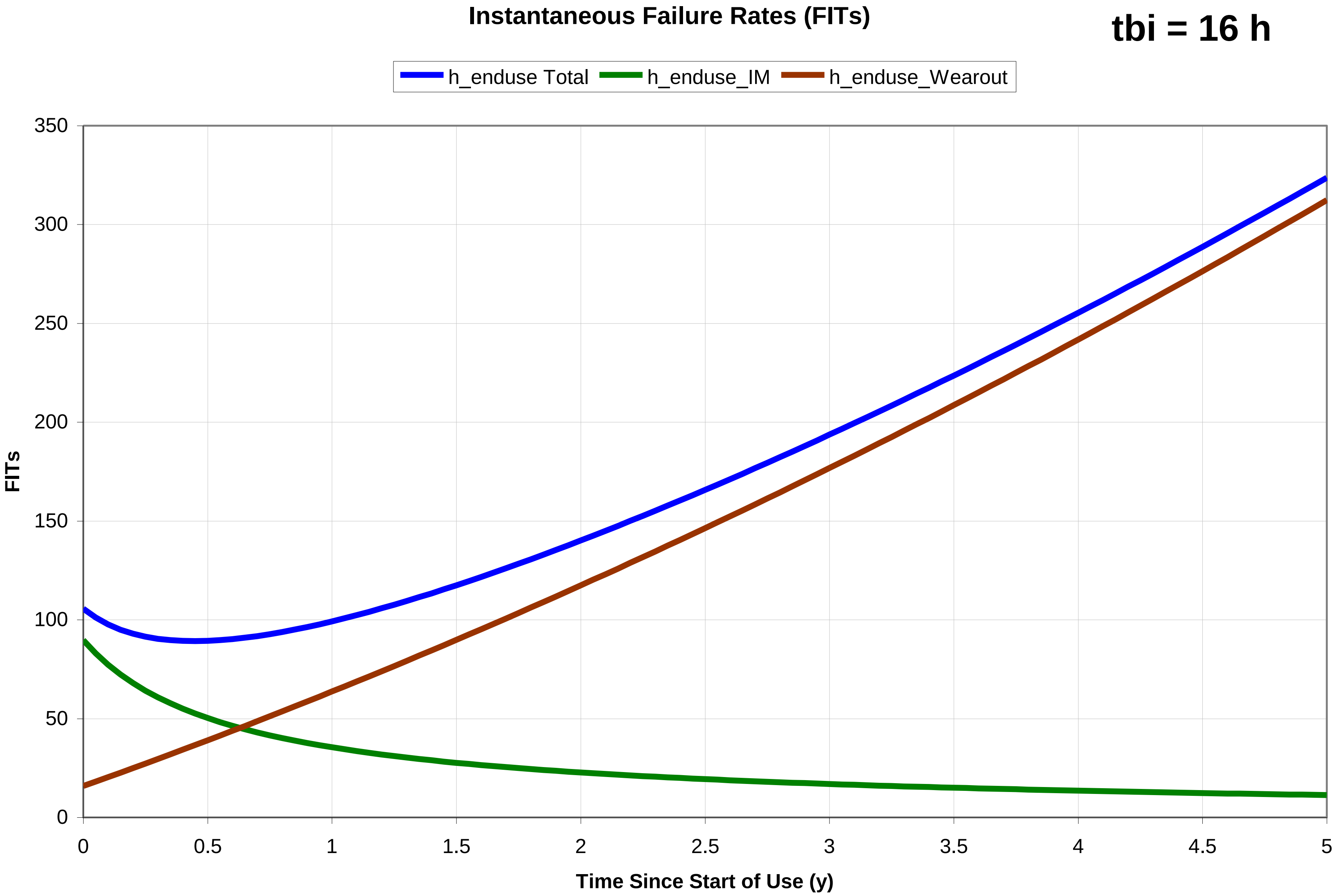
tbi = 2 h


Instantaneous Failure Rates (FITs)
h_enduse Total
h_enduse_IM
h_enduse_Wearout
FITs
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time Since Start of Use (y)


Instantaneous Failure Rates (FITs)
h_enduse Total
h_enduse_IM
h_enduse_Wearout
FITs
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time Since Start of Use (y)

tbi = 4 h

tbi = 16 h

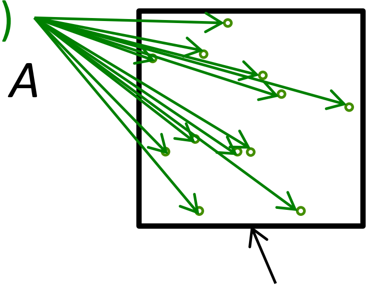

Instantaneous Failure Rates (FITs)
h_enduse Total
h_enduse_IM
h_enduse_Wearout
FITs
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time Since Start of Use (y)

Defect Model of Reliability

- Each defect has a "time-to-fail" – the run time at a specific "reference condition" of temperature, voltage etc. before failure.
- The fraction of defects with time-to-fail less than *t* is the defect survival function, *s*(*t*).
- The number of reliability defects on a die is $\lambda_{rel} = D_{rel} \times A$
- Survival function of chip is

$$S(t)=s(t)^{\lambda_{rel}}=\exp\left[\lambda_{rel}\ln s(t)\right]=\exp\left[-\lambda_{rel}H(t)\right]$$

where *H*(*t*) is the cumulative hazard of the rel defects.



$$S(t)=\underbrace{s(t)\times s(t)\times ..}_{\text{Number of rel defects..}}$$

- Example. If defects have a Weibull life distribution

$$s(t)=\exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right],\qquad H(t)=\left(\frac{t}{\alpha}\right)^{\beta}$$

What if the defects have some other distribution? See NIST gallery of distributions.

then the survival function of the chip is also Weibull but with different α.

special to Weibull

$$S(t)=\exp\left[-D_{rel}A\left(\frac{t}{\alpha}\right)^{\beta}\right]=\exp\left[-\left(\frac{t}{\alpha'}\right)^{\beta}\right]\qquad \alpha'=\frac{\alpha}{(D_{rel}A)^{1/\beta}}$$

C. Glenn Shirley "A Defect Model of Reliability," invited Tutorial at 33rd Annual International Reliability Physics Symposium, including supplemental paper. Las Vegas, Nevada, pp. 3.1 - 3.56, 1995.

Scaling from Test Chip to Product

- Wanted: The survival function of a product from the survival function of a test chip, such as an SRAM, to avoid life-testing the product.
- Survival function of both product and SRAM (test chip, tc) is affected by the same latent reliability defects.
- But the numbers of defects are different because
  - Critical areas are different due to different “physical area” and design layout.
  - Latent reliability defect densities are different because of Fab process variation at time of production of test chip and product.

Different numbers of latent reliability defects.

$$S_{tc}(t) = s(t)^{D_{rel,tc} A_{tc}}$$

$$S_p(t) = s(t)^{D_{rel,p} A_p}$$

Same latent reliability defects

Eliminate *s*(*t*), a property of individual defects.

$$S_p(t) = S_{tc}(t)^{\frac{D_{rel,p} A_p}{D_{rel,tc} A_{tc}}}$$

Scaling from Test Chip to Product, ct’d

- Problem: D_{rel} is hard to measure.
- But remember..
  - The same kinds of defects that degrade yield, degrade “infant mortality” perceived by end users.
- So it is plausible that the latent reliability defect density varies in proportion to the “killer” defect density as the Fab yield varies.
  - This is the “special K” concept that Bill Roesch mentioned..

$$\text{If} \quad \frac{D_{rel,p}}{D_{yield,p}} = \frac{D_{rel,tc}}{D_{yield,tc}} = \boxed{\kappa} \qquad \text{then} \quad \frac{D_{rel,p}}{D_{rel,tc}} = \frac{D_{yield,p}}{D_{yield,tc}}$$

  - Typically $\kappa \approx 0.01$ (1%)
- So

$$S_p(t) = S_{tc}(t)^{\frac{D_{rel,p}A_p}{D_{rel,tc}A_{tc}}} = S_{tc}(t)^{\frac{D_{yield,p}A_p}{D_{yield,tc}A_{tc}}}$$

- The last expression is useful because D_{yield} is much easier to measure than D_{rel}.

At this point, we can forget about D_{rel} because the models for relating test chip (aka SRAM, aka “ref”) only involve D_{yield}

Scaling from Test Chip to Product, ct'd

- So we now have a way to scale the SRAM (test chip) model to the product considering:
  - Different die areas between SRAM and product.
  - Yield (killer) defect densities different at the time of production of the SRAM and the time of production of the product.
  - The product operating at different temperature and voltage from the SRAM.

Area/Defect Density Scaling

- The scaling factor of a product (area A) produced on a process (with defect density D) to a reference such as a test vehicle is

Special V

$$\nu_{p|ref} = \frac{D_p A_p}{D_{ref} A_{ref}}$$

D_p and D_{ref} are killer defect densities (ie D_{yield}) measured at Sort.

- Example. Suppose test chip life data at a specific temperature and voltage is fitted to a Weibull distribution to produce a reference model. The product's survival function at the reference temperature and voltage is

$$S_{tc}(t) = \exp\left[-\left(\frac{t}{\alpha_{tc}}\right)^{\beta}\right]$$

$$S_p(t) = \exp\left[-\nu_{p|tc}\left(\frac{t}{\alpha_{tc}}\right)^{\beta}\right] = \exp\left[-\left(\frac{t}{\alpha_p}\right)^{\beta}\right] \quad \text{where} \quad \alpha_p = \alpha_{tc}\nu_{p|tc}^{-1/\beta}$$

- That is, the product survival function is also Weibull with the same shape β, but with different α.

$$\ln\alpha_p = \ln\alpha_{tc} - \frac{1}{\beta}\ln\nu_{p|tc}$$

$$\beta_p = \beta_{tc}$$

Acceleration

- A commonly used burn in acceleration model is

$$A_{21} = \exp\left\{\frac{Q}{k}\left[\frac{1}{T_1} - \frac{1}{T_2}\right] + C(V_2 - V_1)\right\}$$

- Where
  - T_2, V_2, T_1, V_1 are operating temperatures (in deg K) and voltages at conditions 2 and 1, respectively.
  - $k = 8.61 \times 10^{-5}$ eV/K is Boltzmann's constant.
  - Q (eV) is the thermal activation energy.
  - C (volts^{-1}) is the voltage acceleration constant.
- Example: What is the acceleration of burn in relative to Use for the following conditions:

	T (C)	V
BI Condition	135	4.6
Use Condition	85	3.3

Acceleration Model Parameters

Q	0.3	eV
C	2.6	/V

Ans: 96.7

Burn In

- Now we put together defect density/area scaling and acceleration to write the complete model.
- The survival function of a product which has undergone t_{bi} hours at burn in conditions followed by t_{use} hours in use is

$$S_p(t_{use}, t_{bi}) = \left[S_{ref}\left(\overbrace{AF_{bi|ref}\, t_{bi} + AF_{use|ref}\, t_{use}}^{\text{Equivalent time at ref condition.}} \right) \right]^{\nu_{p|ref}} \qquad \nu_{p|ref} = \frac{D_p A_p}{D_{ref} A_{ref}}$$

- The customer-perceived survival function is conditioned on surviving the test screen following burn in

$$S_p\left(t_{use} \mid t_{bi}\right) = \frac{S_p\left(AF_{bi|ref}\, t_{bi} + AF_{use|ref}\, t_{use}\right)}{S_p\left(AF_{bi|ref}\, t_{bi}\right)} = \left[\frac{S_{ref}\left(AF_{bi|ref}\, t_{bi} + AF_{use|ref}\, t_{use}\right)}{S_{ref}\left(AF_{bi|ref}\, t_{bi}\right)} \right]^{\nu_{p|ref}}$$

- From the product survival function any figure of merit can be calculated.
  - Fractions failing (DPM) between two times, expressed in DPPM.
  - Average failure rates (AFR) between two times, expressed in Fits.