ECE 510 Lecture 9
Silicon Rel Mechanisms, MLE

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Silicon Reliability Mechanisms

Semiconductor Reliability Mechanisms

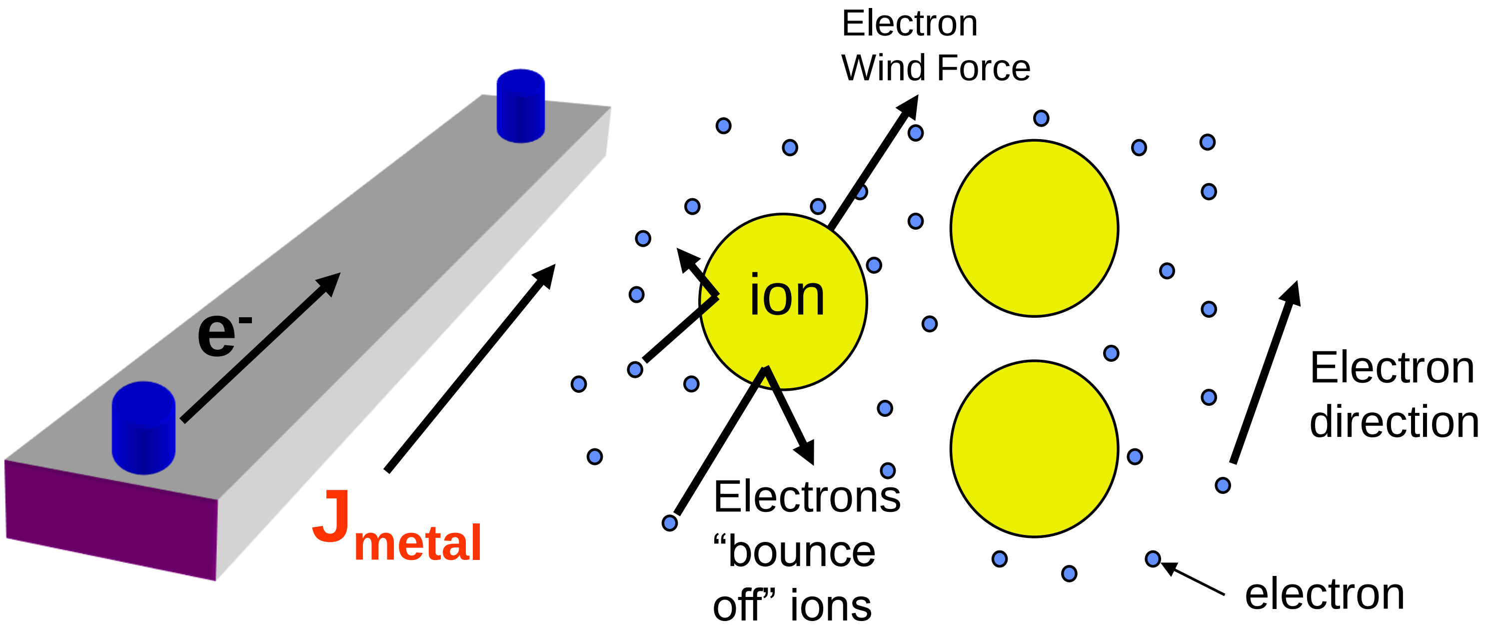
- Package
- Silicon
  - Infant Mortality
  - Constant fail rate
    - Soft errors
    - Environment (such as power fluctuations)
  - Wearout
    - Electromigration
    - Oxide wearout (TDDB)
    - Transistor degradation (NBTI or PBT)

Today's lecture

Electromigration

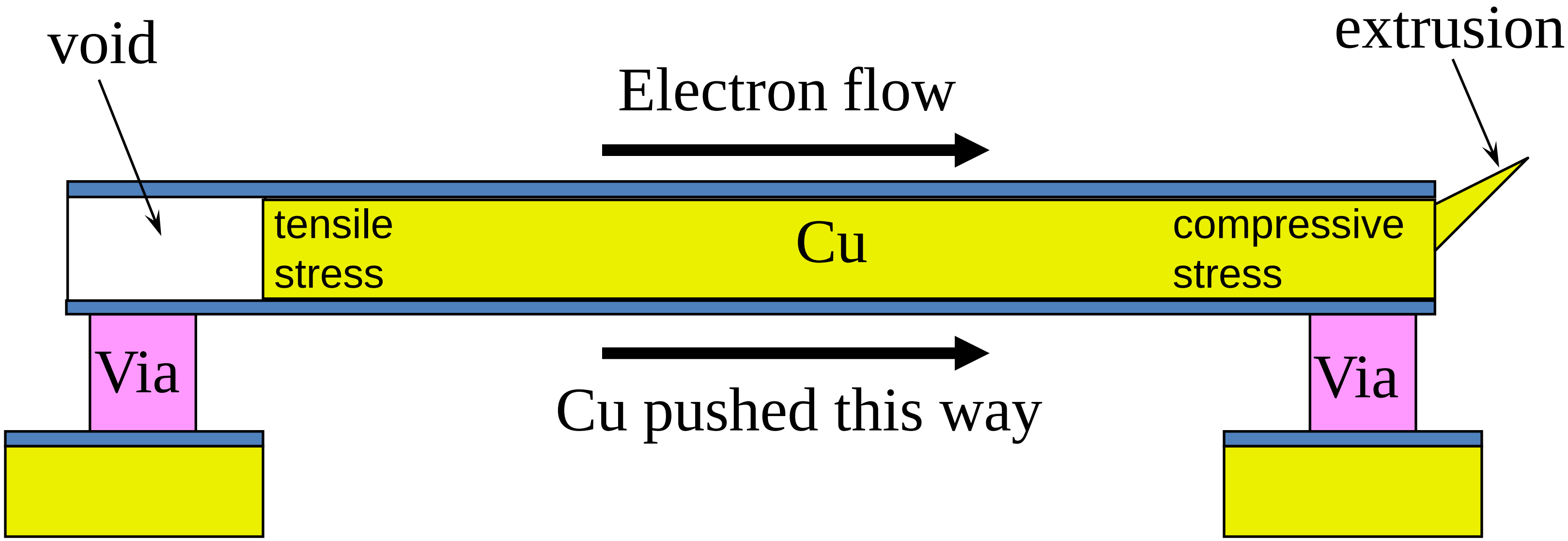
…where we will see how too much current through a small conductor can make it fail.

Electromigration



"Electron wind" from conduction current gradually pushes ions "down wind" into new positions in the lattice

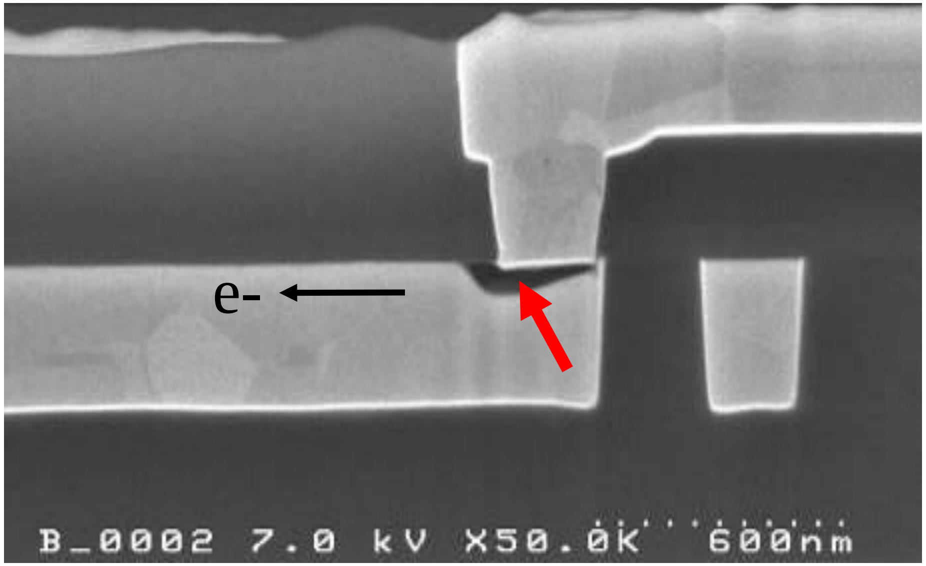
Causes Voids and Extrusions



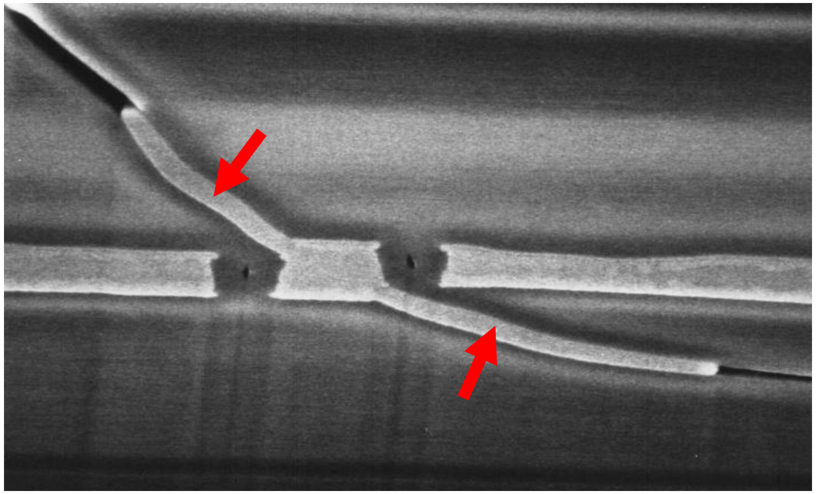
Electron wind pushes metal enough to cause voids on one end and extrusions on the other end

Voids and Extrusions

Example of a void

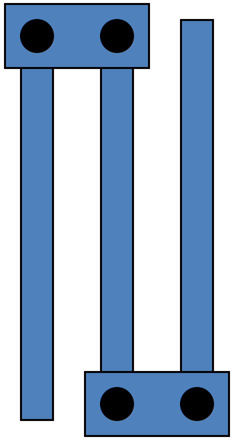


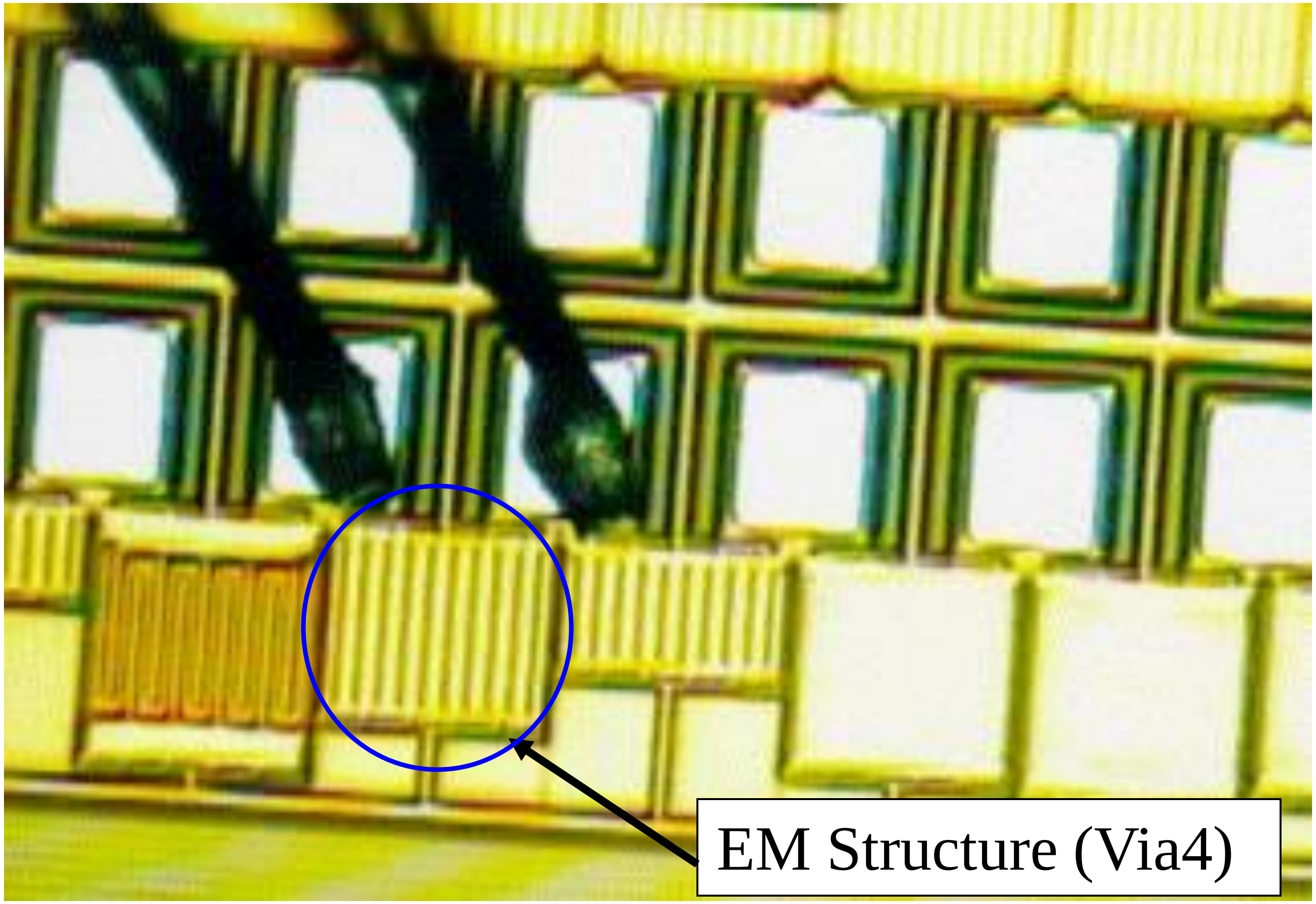
Example of extrusions



- Voids cause a rise in resistance
  - Dominant fail mode; reliable and easy to characterize
- Extrusions might cause shorts
  - Inconsistent and random

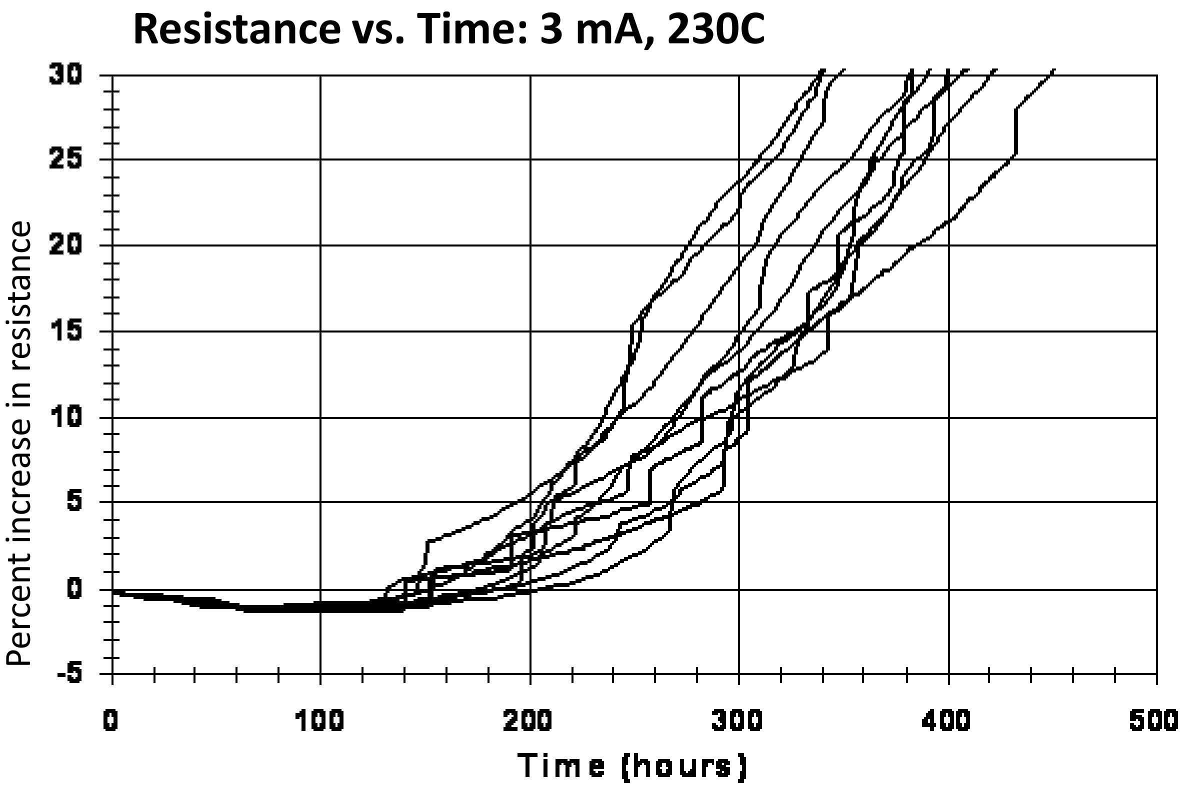
Characterizing EM





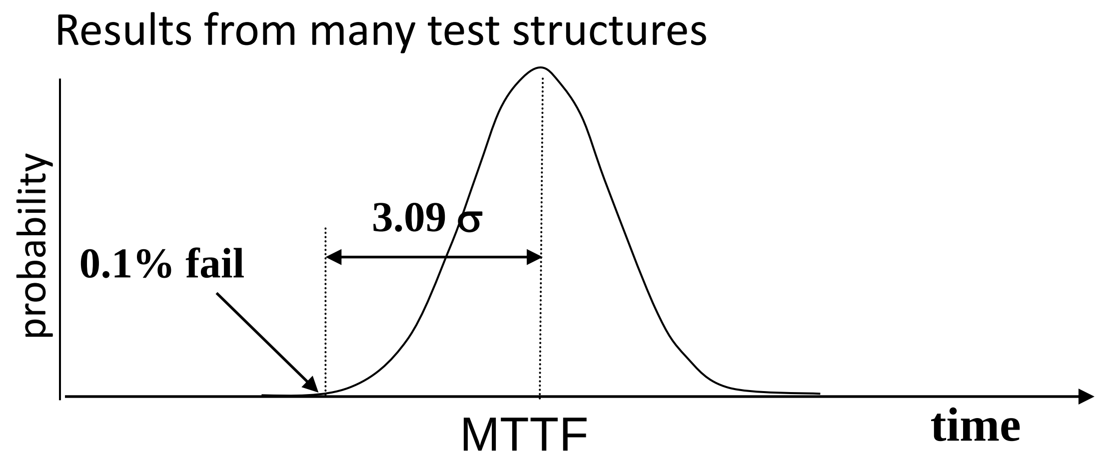
- Test structure to measure EM

Measuring Voids



- As voids form, resistance increases
- A threshold is chosen to define a fail

EM Model

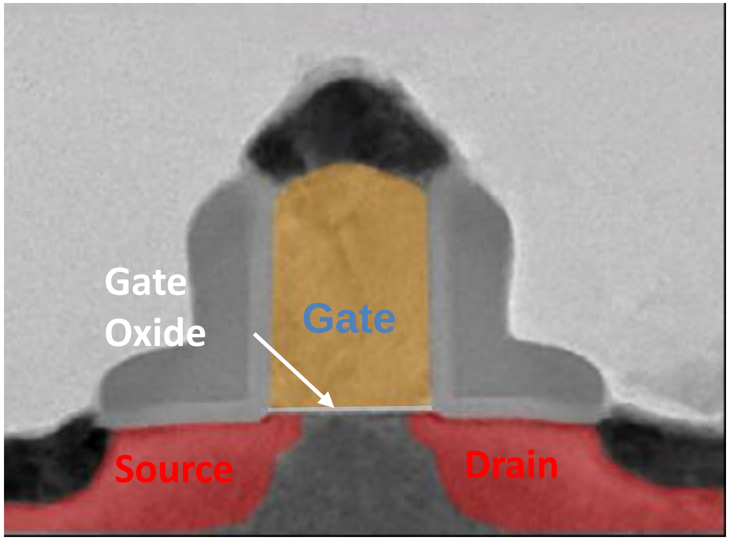


- Test structure data is used to calculate a max current (Imax) for which <0.1% fail at 7 yrs worst-case use
- This results in a design rule for Imax, which must be followed by all products using this technology

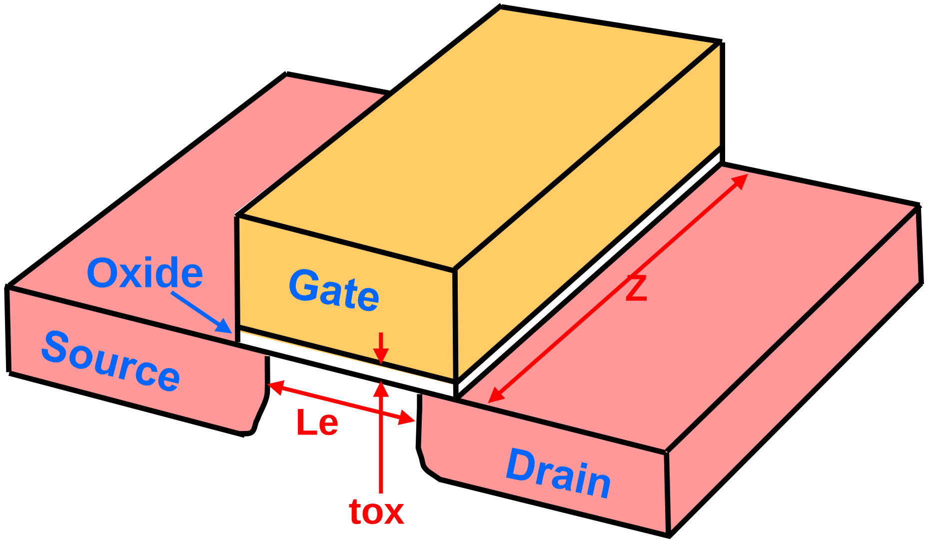
Gate Oxide

…where we will see how the ultra-thin and critically important gate oxide can fail in time.

Background: Transistor Structure

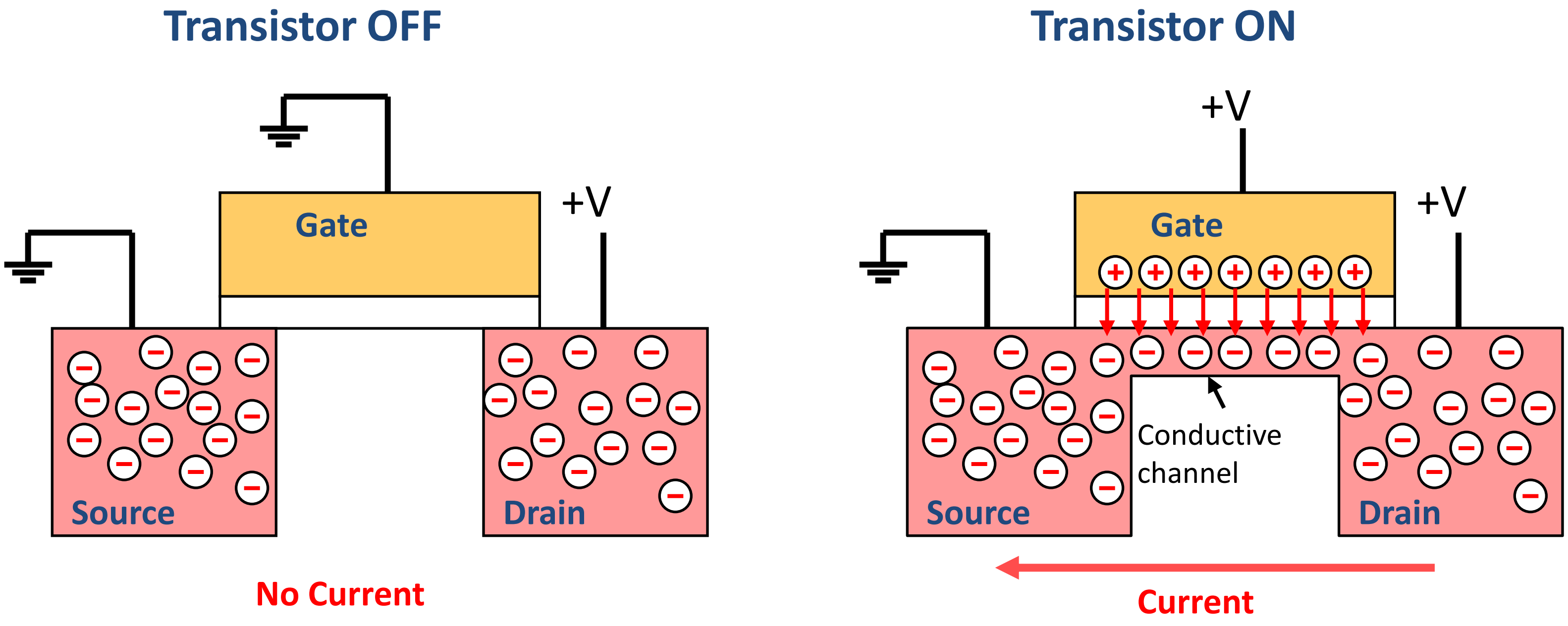


NMOS transistor



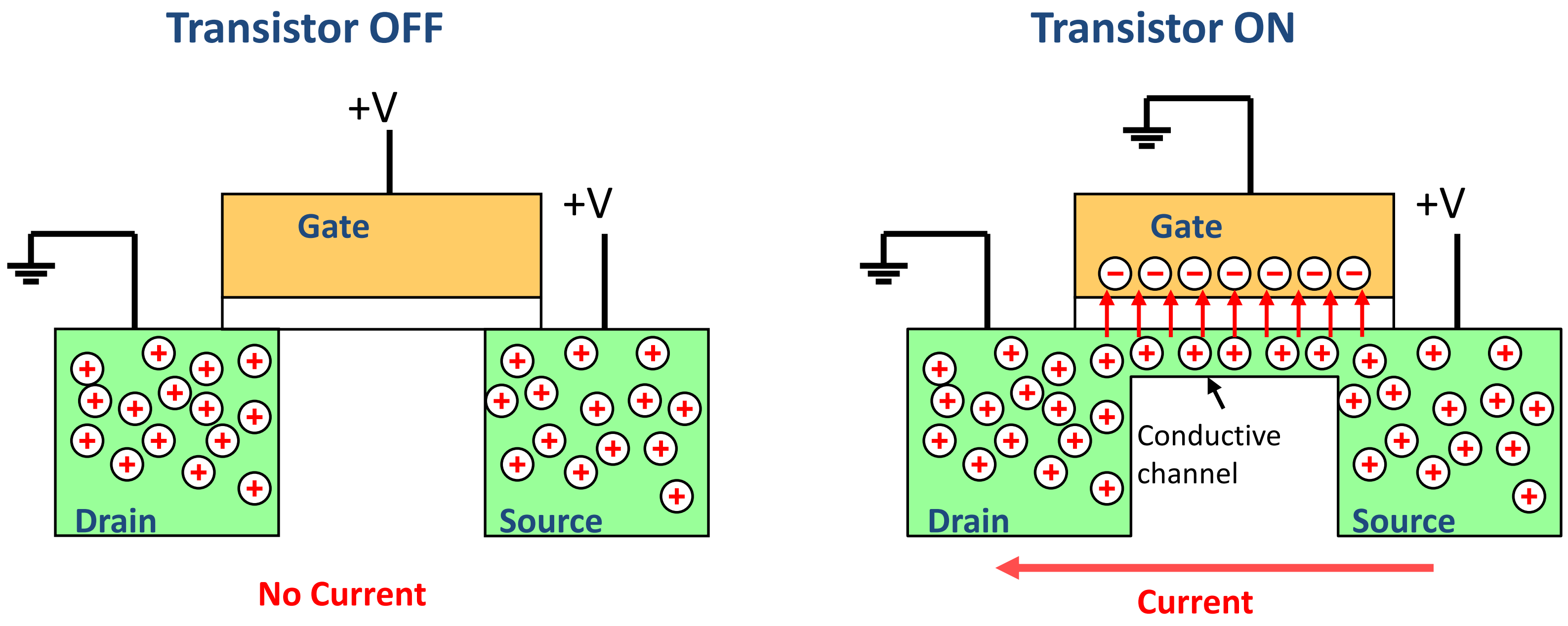
- Important dimensions of a transistor
  - Length of channel =Le
  - Width of transistor (and channel) = Z
  - Oxide thickness = tox

Background: Transistor Function



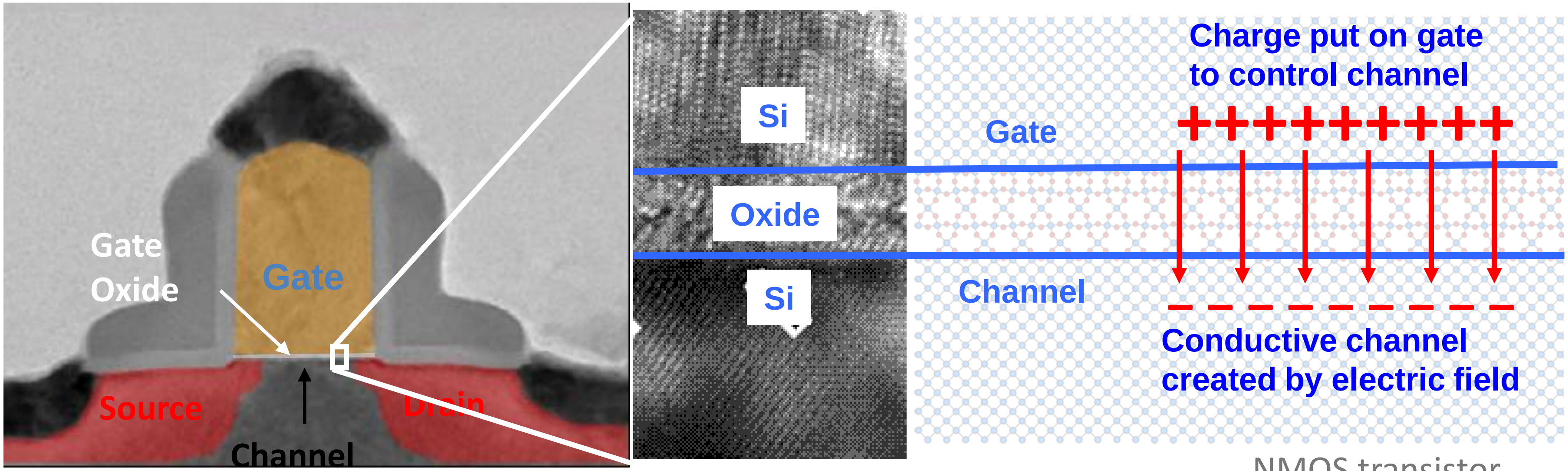
- When off, no channel, no current (except leakage)
- Turn transistor on with (+) charges on the gate
  - Electric field from (+) charges creates a conductive channel
- These are NMOS transistors

Background: PMOS Transistor



- PMOS works similarly to NMOS, but
  - The charge carriers are "holes" (missing electrons)
  - Transistor is ON when *negative* charge applied to gate

Gate Oxide



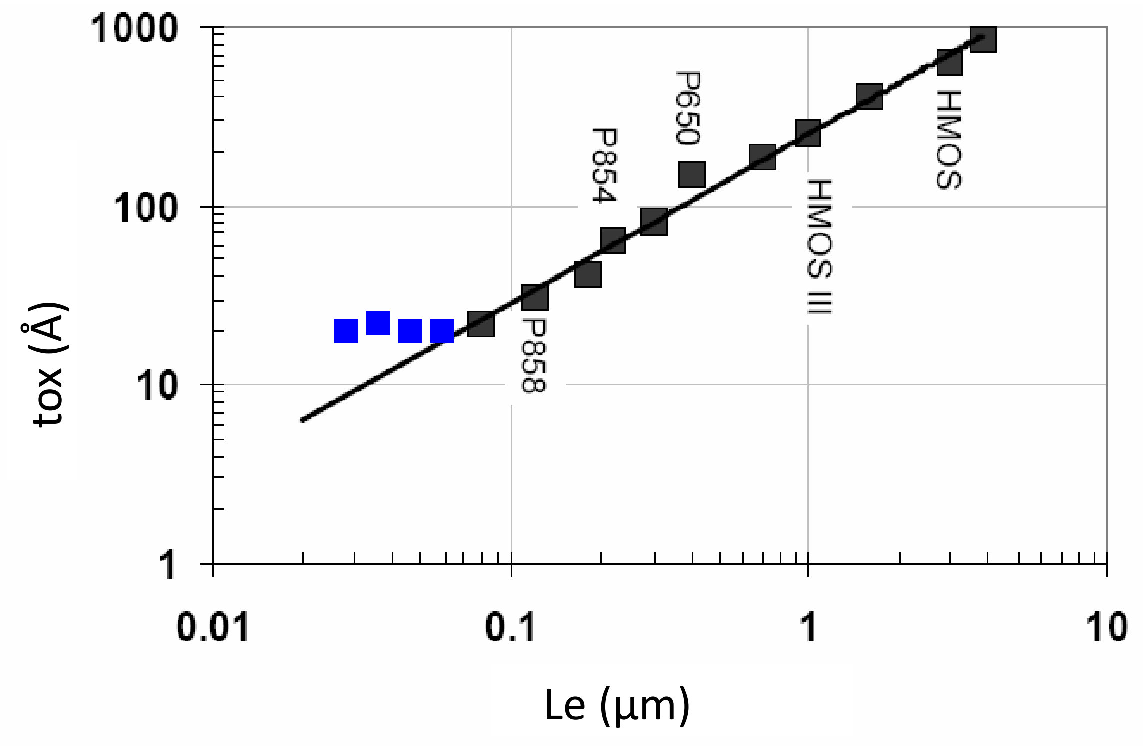
Charge put on gate to control channel

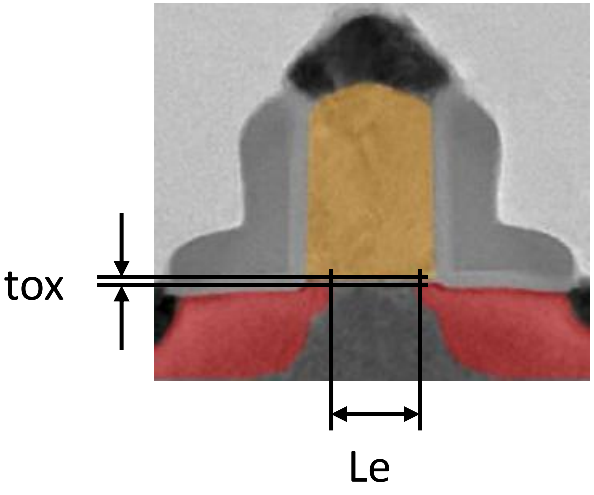
Conductive channel created by electric field

NMOS transistor

- Gate oxide is thin and critical
- Thinner oxide allows less charge on the gate to control the channel, and less charge means faster switching

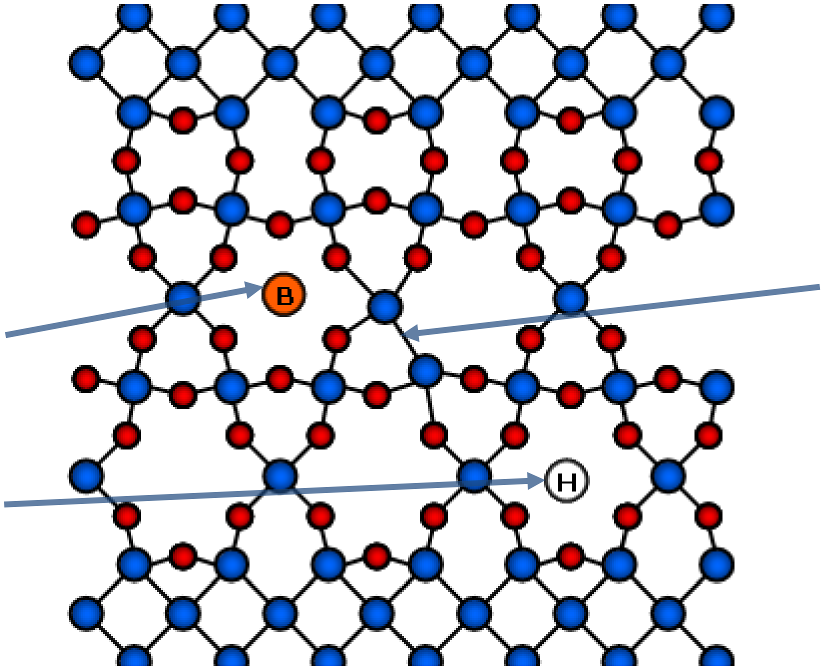
Transistor Scaling Trends





- Oxide thickness has leveled off around 20Å
  - Due to leakage and reliability
- Channel length continues to shrink

Oxide Degradation



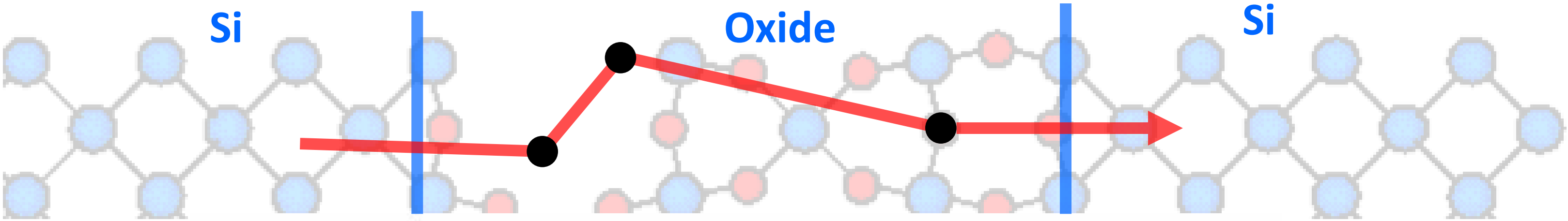
Stray dopant atom

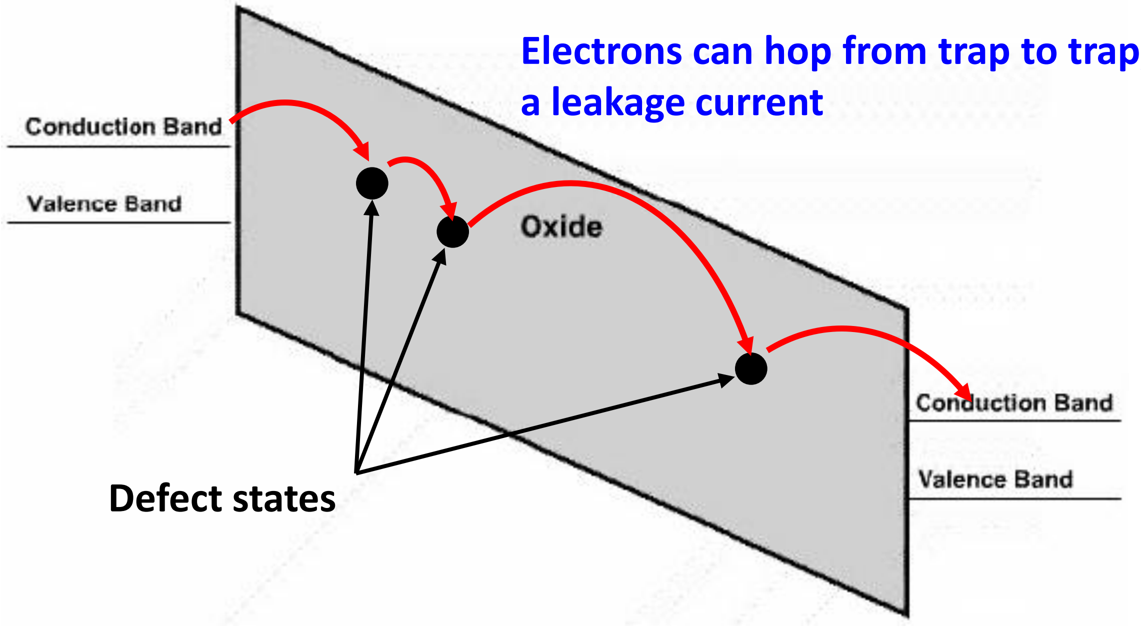
Weak Si-Si bond

Hydrogen atom

- Oxide degrades with time as
  - Impurities diffuse into it
  - Bonds change

Oxide Degradation



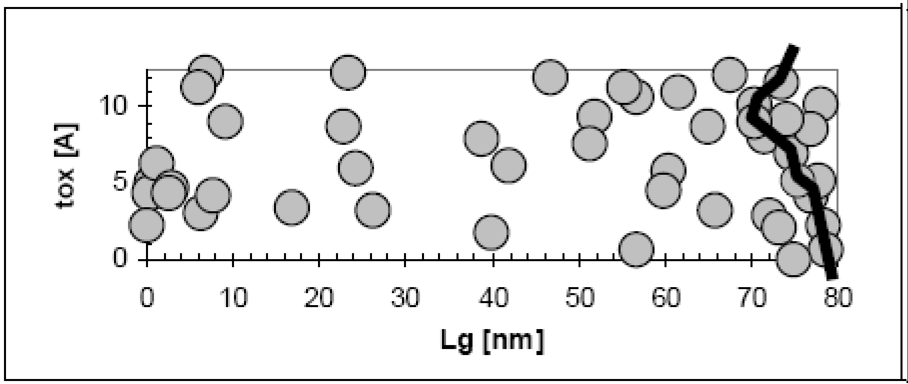


Electrons can hop from trap to trap, constituting a leakage current

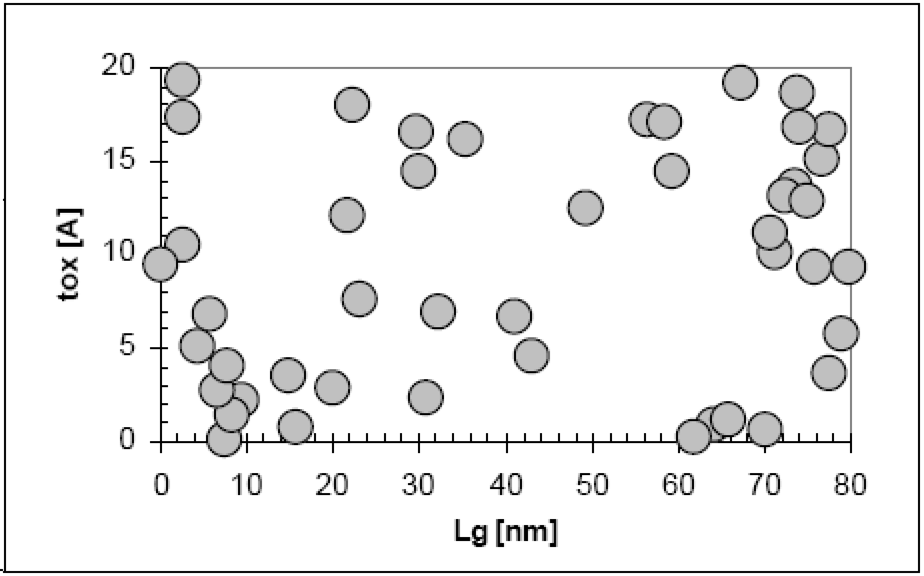
- Electrons can tunnel (“hop”) from defect to defect more easily than across the whole gate oxide layer

Oxide Breakdown

Thin oxide, easier percolation paths

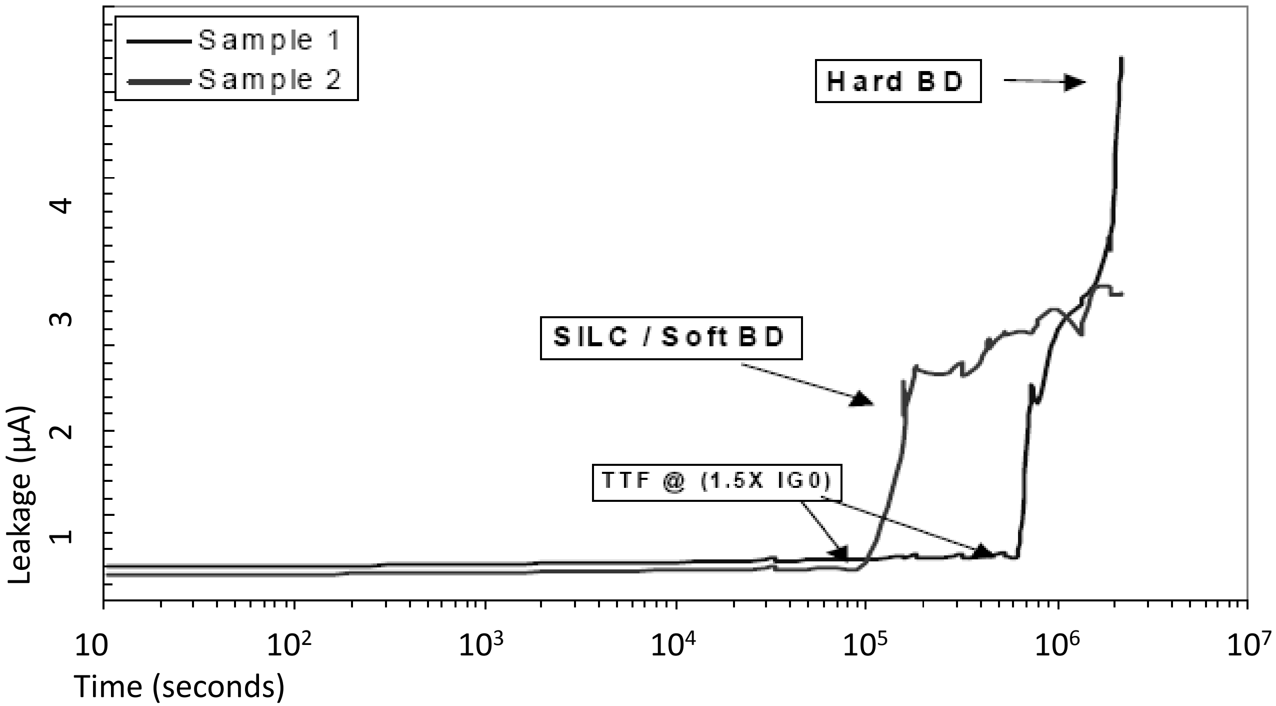


Thick oxide, more difficult to get percolation paths



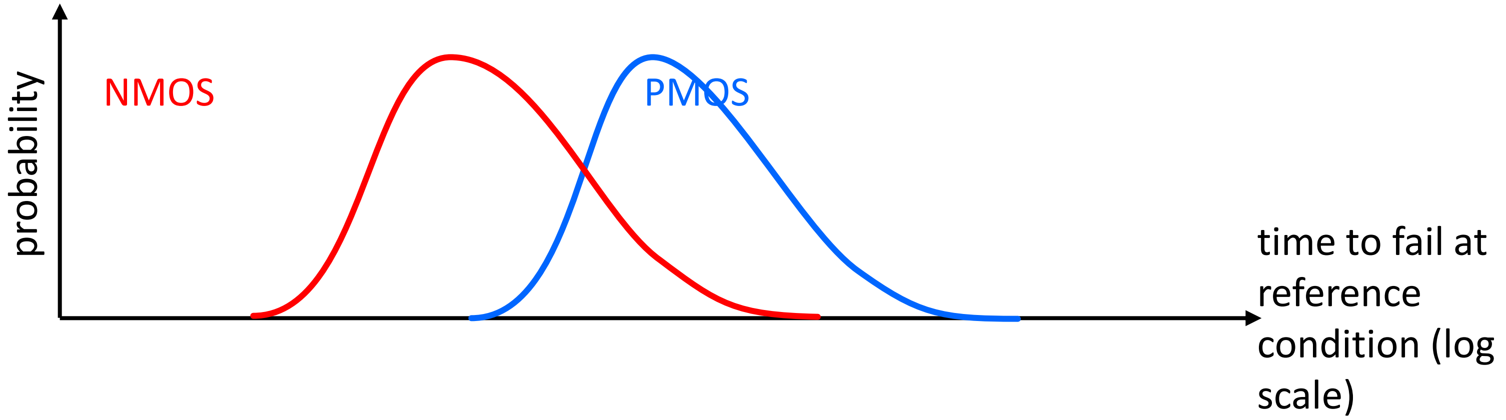
- Oxide leakage will go up dramatically when a fully connected percolation path forms

Soft Breakdown


Gate Oxide Leakage vs. Time (V stress=2.6V)

- The full percolation path makes a “soft” breakdown
  - Soft breakdown is considered a fail
- High current in the percolation path can change it to a “hard” breakdown

Oxide Fail Model

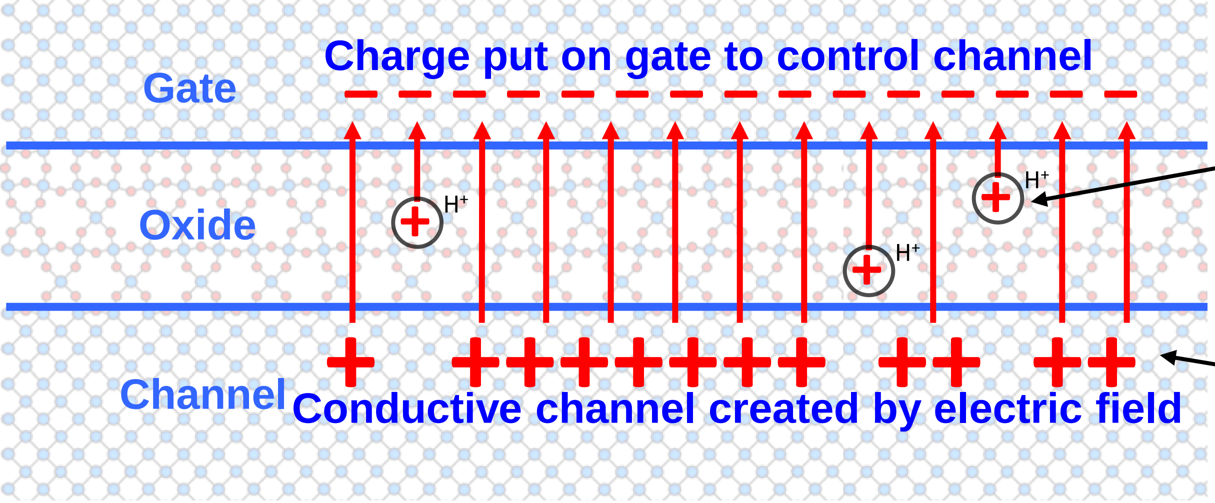


- Model is a distribution (Weibull) of times-to-fail
- Also determine V, T acceleration factors
- NMOS and PMOS size-scale differently
  - NMOS by oxide area A
    - $S_{prod} = S_{ref} \wedge (A_{prod}/A_{ref})$ (suggests fails are uniform)
  - PMOS by device width W
    - $S_{prod} = S_{ref} \wedge (W_{prod}/W_{ref})$ (suggests fails lie along edges)

Transistor Degradation

…where we will see how PMOS transistors degrade with stress.

PMOS Bias Temp



H^+ ions diffuse into the oxide, decrease the strength of the electric field at the channel, and decrease conductivity

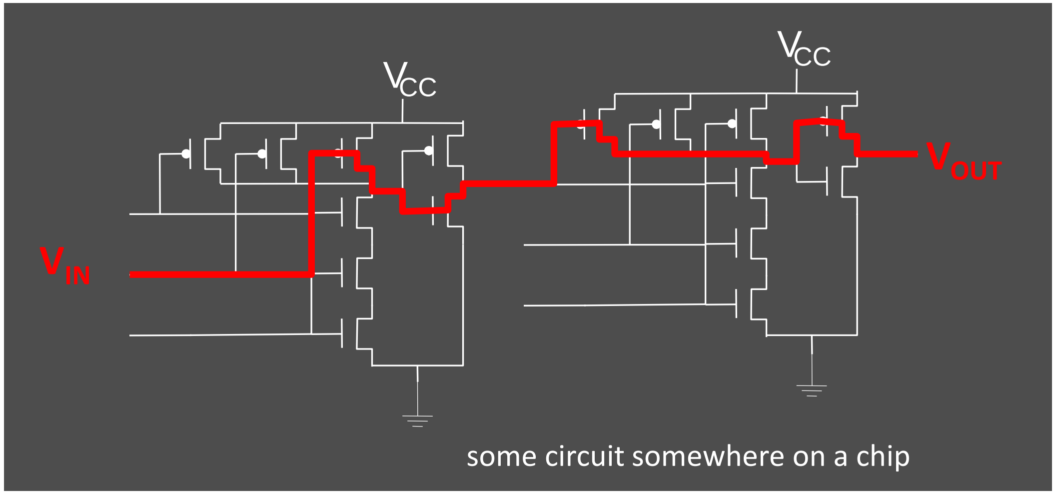
PMOS transistor

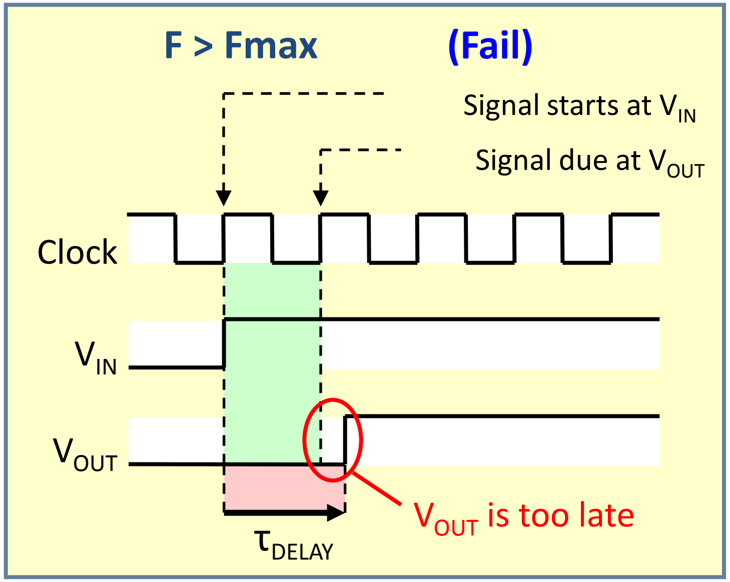
- PMOS negative bias / temperature instability
  - PBT or NBTI
  - Primarily affects PMOS transistors
  - Degrades device performance
- Primarily manifests in slower switching → Fmax degradation

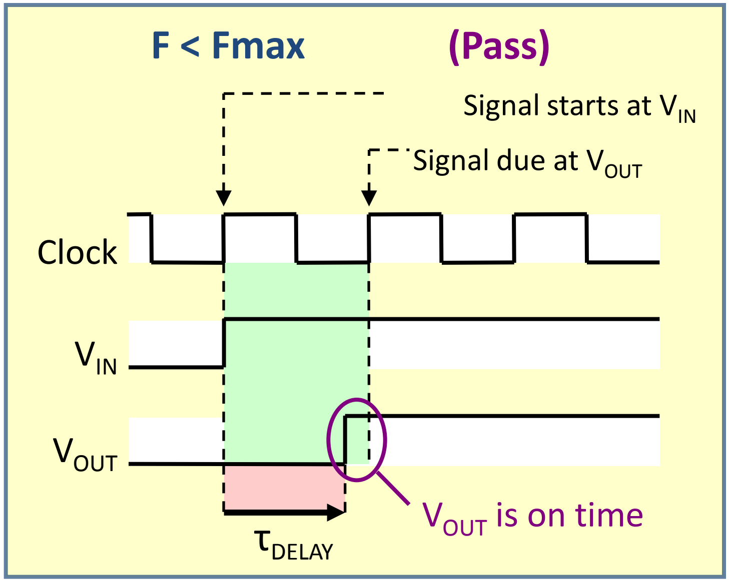
Fmax Degradation

…where we will see how degraded transistors cause a part to slow down, and sometimes fail.

Fmax



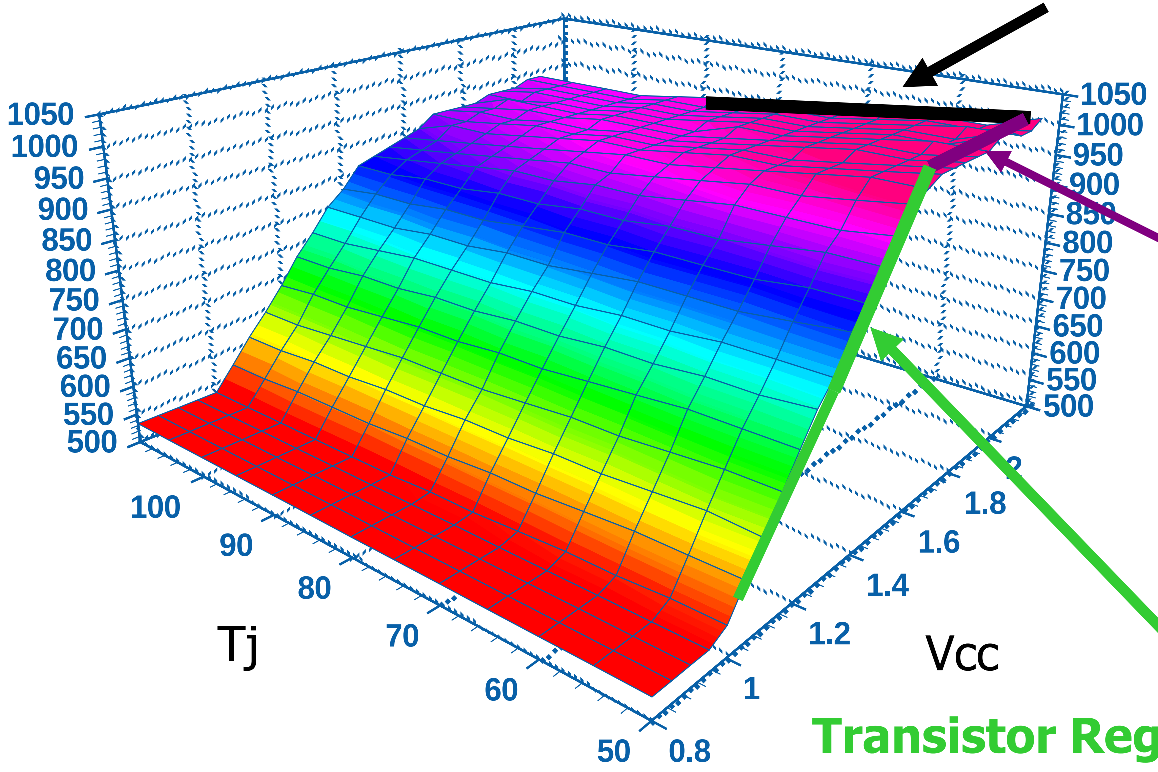




Fmax caused by a speed path

- Chip fails when some calculation is not ready in time
- Delays caused by
  - Transistor switching (higher V speeds them up)
  - Signal propagation

Fmax V & T Sensitivity

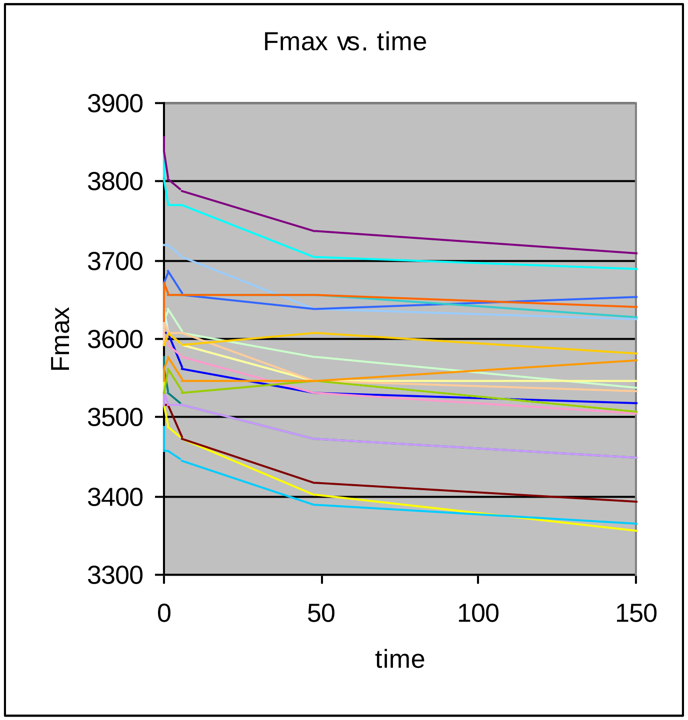


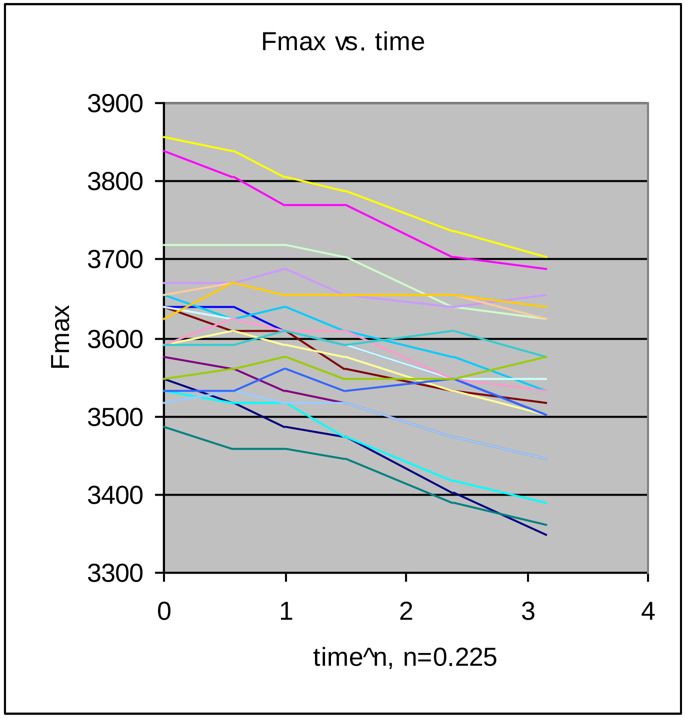
~ 1-2 MHz per °C

RC Regime:
~ 0.4 MHz per mV

Transistor Regime: ~ 1.0 MHz per mV

Fmax Behavior Over Time





- Fmax decreases by 0 to ~5% over the life of a part
- Roughly follows a power law
  - Transistor and IDV data follow very clean power laws
  - Products usually “pattern switch” giving many different degradation-vs.-time profiles

Fmax Degradation Model

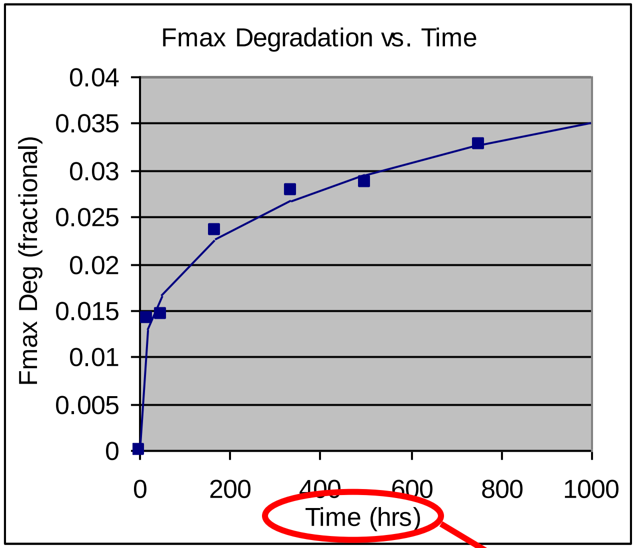
- We look at degradation fraction

$$\Delta = -\frac{Fmax(t) - Fmax(0)}{Fmax(0)}$$

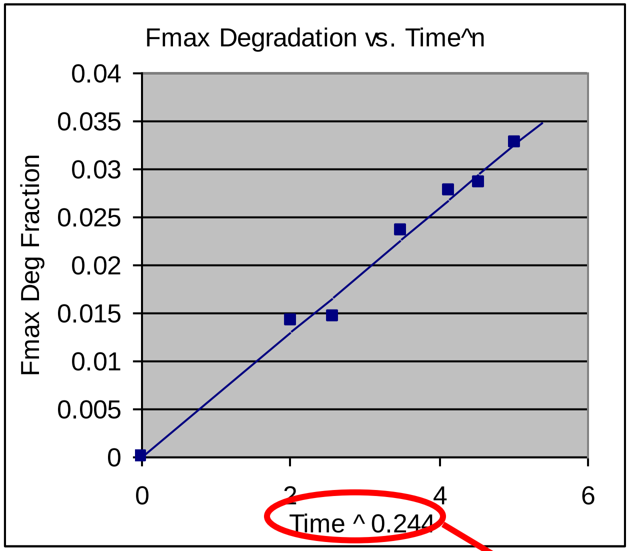
- Modeled using a power law

$$\Delta = A\,t^{n}$$

- Often plotted as Δ vs. t^n:



t

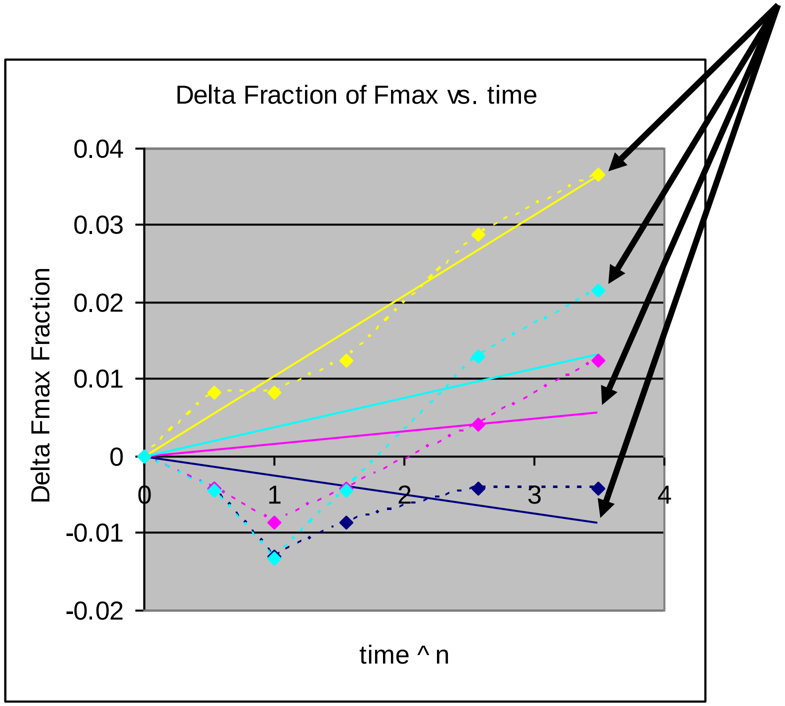


t^n

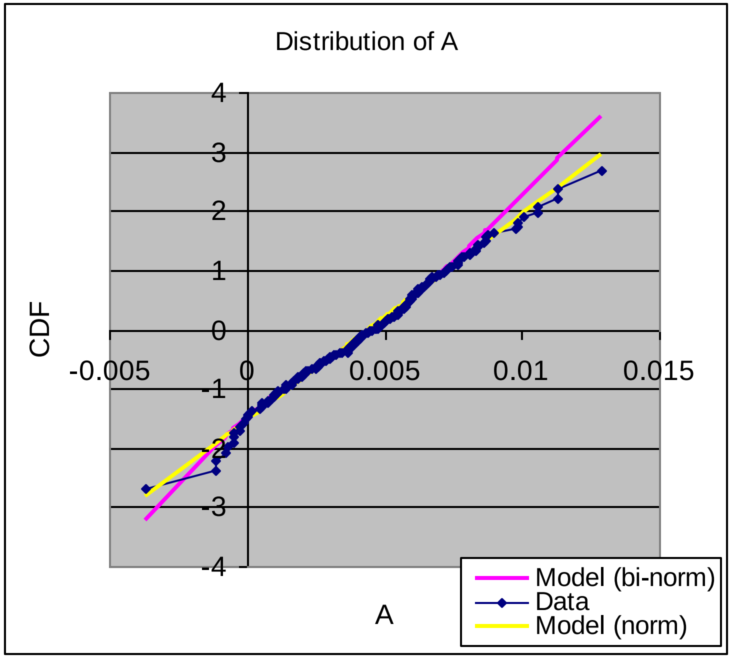
Fmax Degradation Model

One n for the whole population

One A for each unit

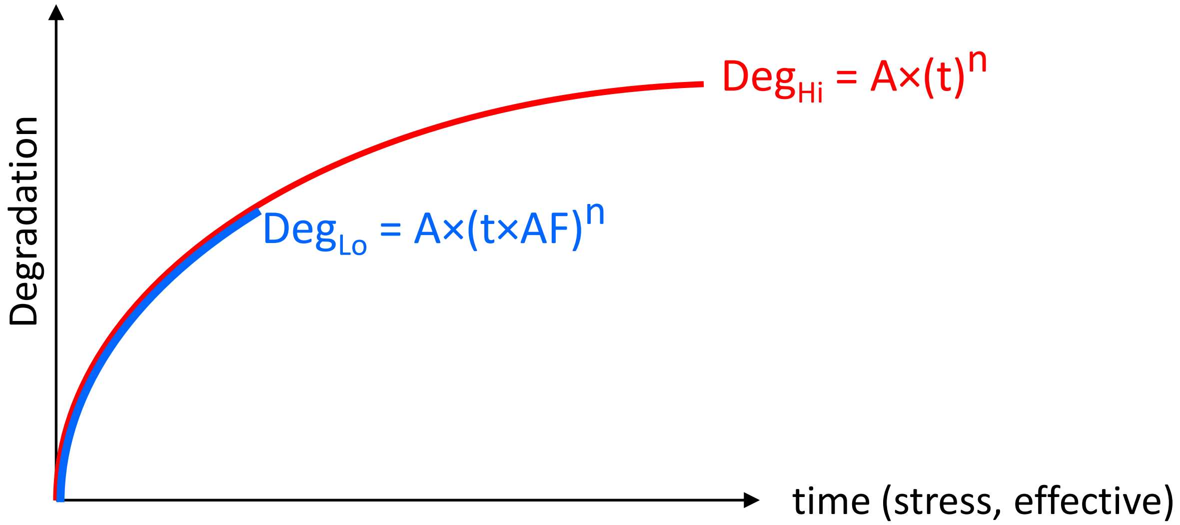


Gives a distribution of A:



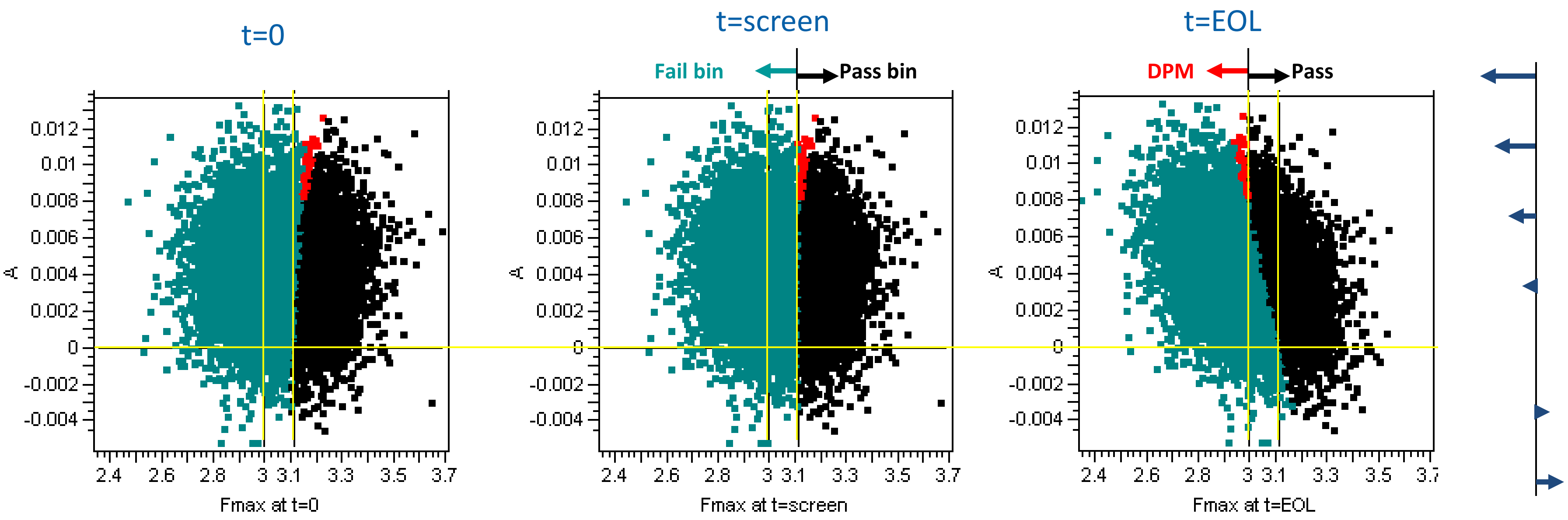
- This allows all units to be included and fit
  - Former method excluded units that showed improvement (even one data point)

Acceleration for Fmax Degradation



- Consider 2 matched units (same degradation)
- Stress them at 2 different conditions
  - Same clock time at lower stress = lower stress time
  - Call the high-stress the reference, then solve for AF

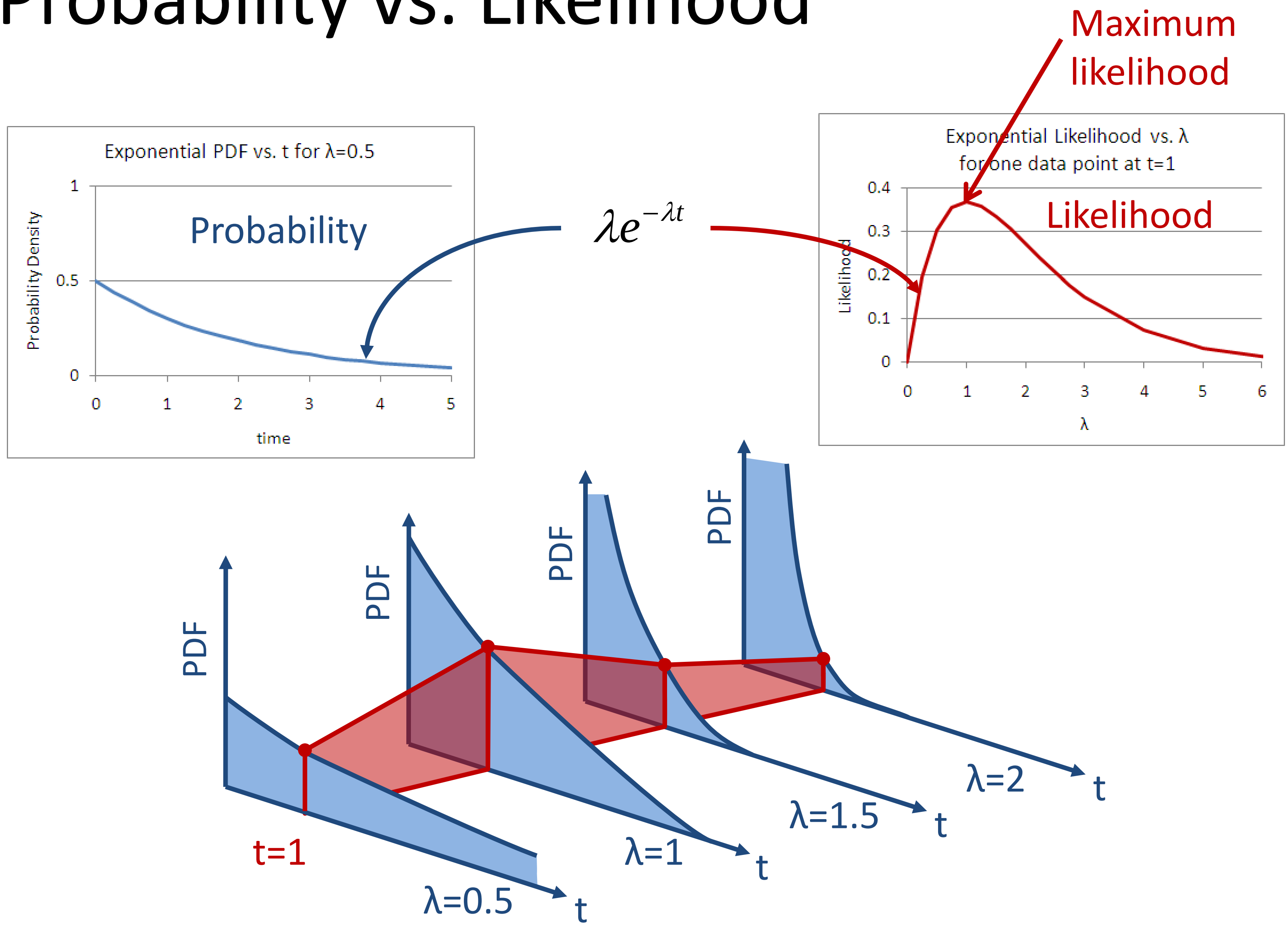
Use of Fmax Degradation Model



- Simplified view of Fmax calculation done in Cancun
  - t=0: Parts are “born” with initial Fmax and deg. constant
  - t=screen: Parts are binned
  - t=EOL: Parts are evaluated for failing Fmax at use

MLE Review

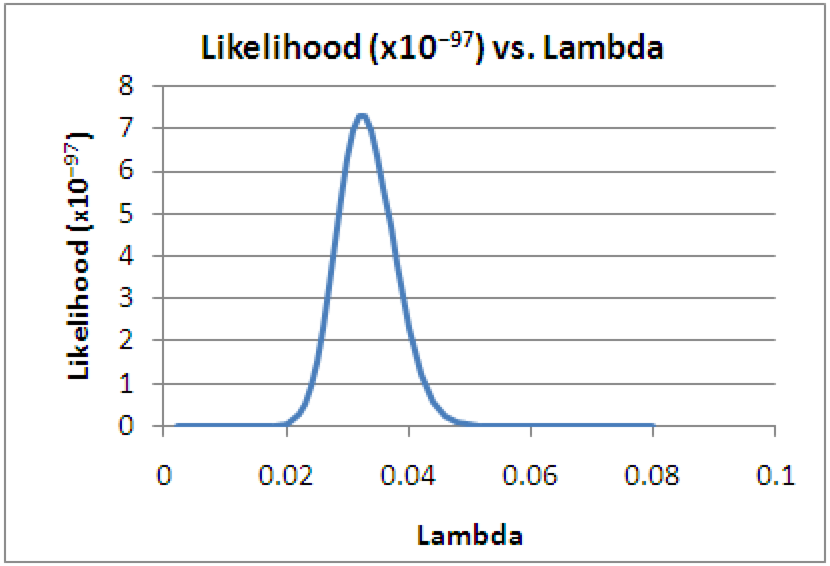
Probability vs. Likelihood

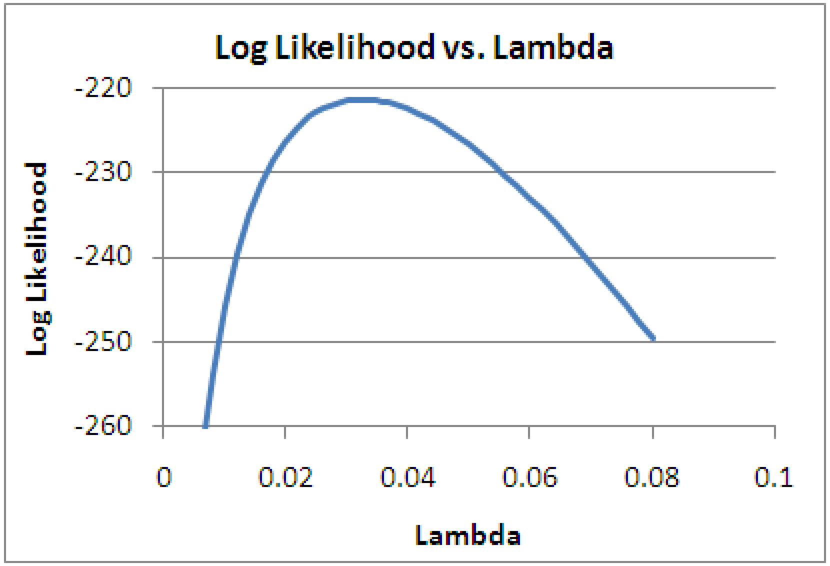


$\lambda e^{-\lambda t}$

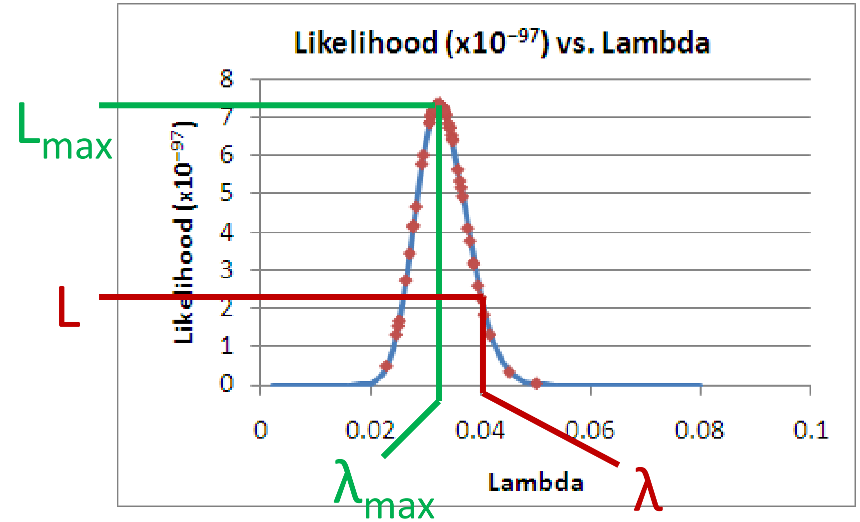
Graphs of Likelihood vs. Lambda

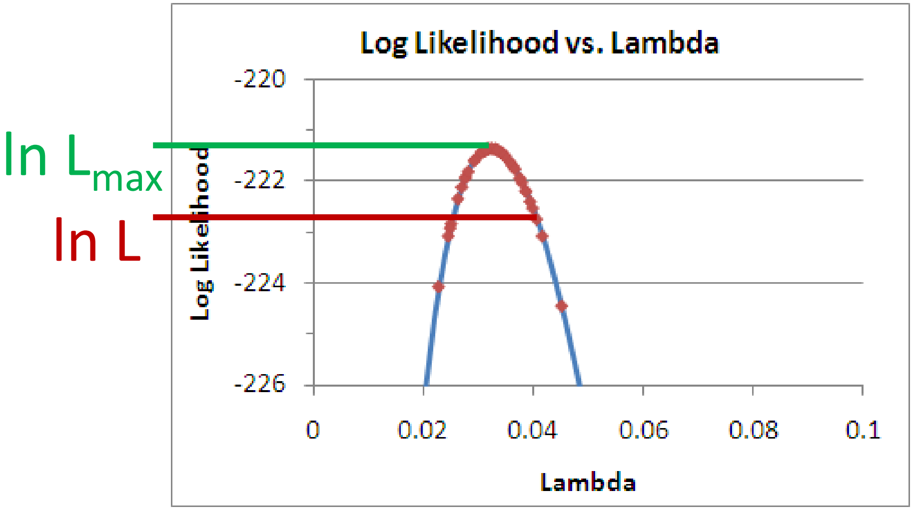
Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			



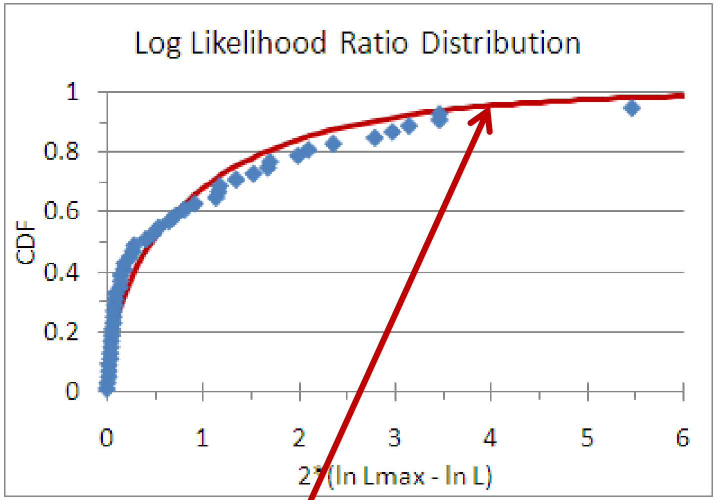


Likelihood Ratio Lambda Uncertainty





$$\ln\left(\frac{L_{max}}{L}\right)^2 = 2\times\left(\ln L_{\max} - \ln L\right)$$

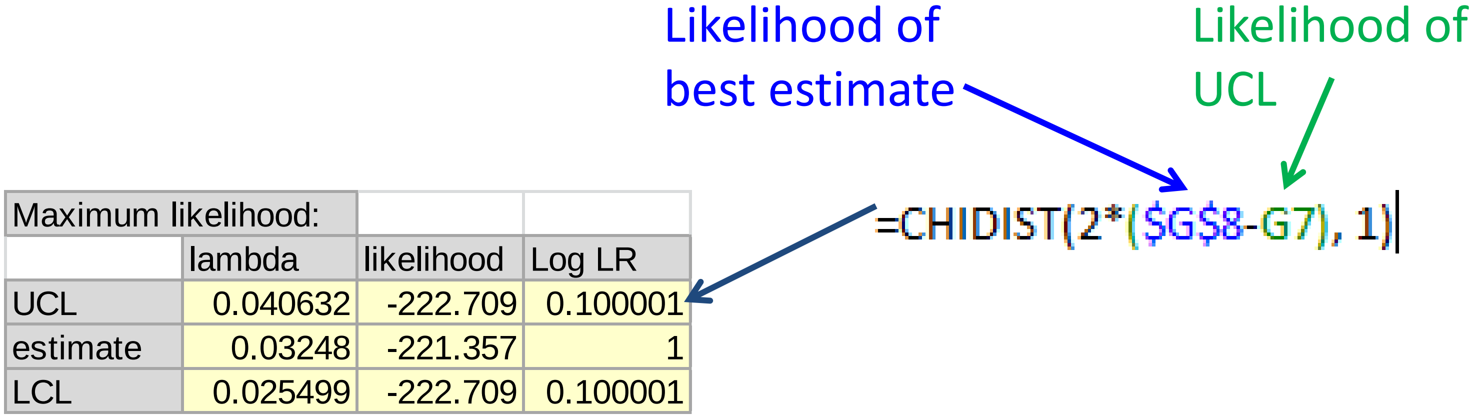


1–CHIDIST(Log LR, 1)

Number of parameters in model (=1 for exponential)

Exercise 8.3c

- Calculate UCL and LCL for lambda:
  - Calculate Log LR for each (below)
  - Choose lambda for each to set Log LR = 0.1
    - Do by hand first, then use Solver to fine-tune



Likelihood of best estimate

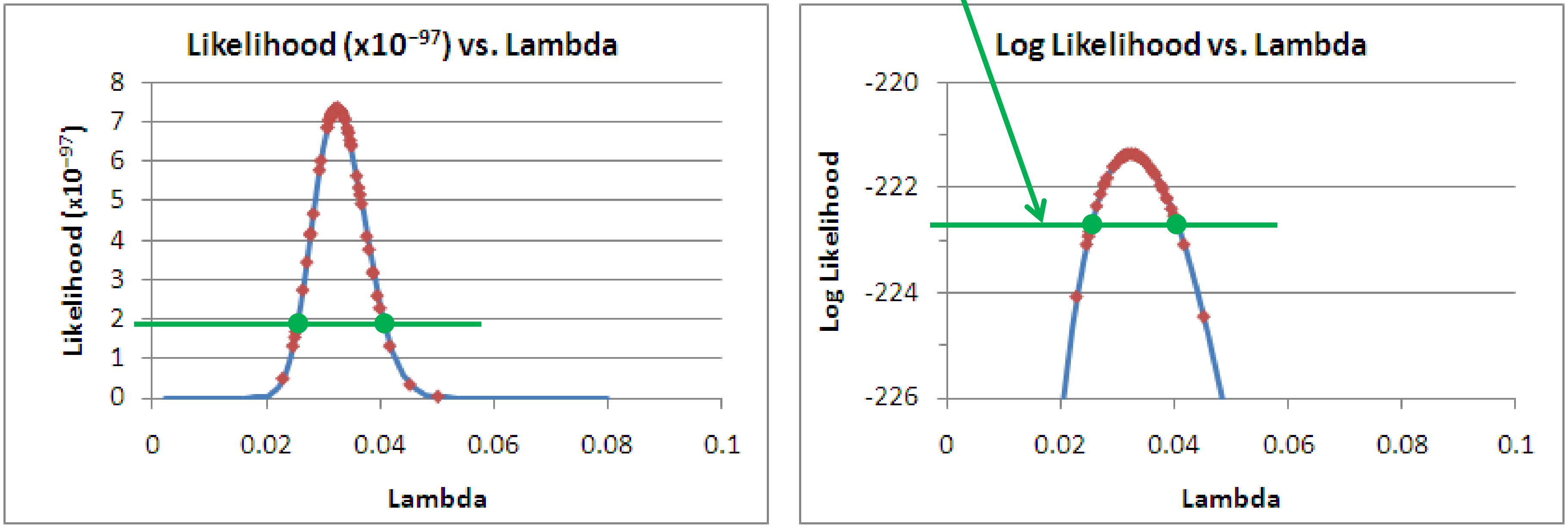
Likelihood of UCL

`=CHIDIST(2*($G$8-G7), 1)`

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001

Solution 8.3c

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001



Analytic Lambda Uncertainty

for 90% CL

Note N+1 vs. N

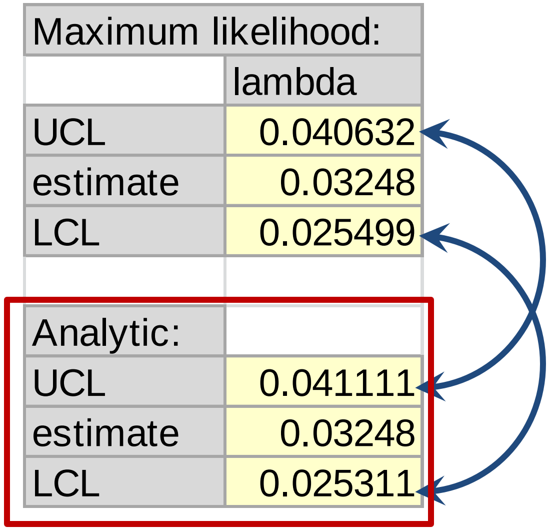
$$\lambda_{UCL} = \lambda_{BE} \frac{\mathrm{CHIINV}(5\%,\, 2(N+1))}{2N}$$

$$\lambda_{LCL} = \lambda_{BE} \frac{\mathrm{CHIINV}(95\%,\, 2N)}{2N}$$

Exercise 8.3d

- Calculate lambda UCL and LCL analytically

Solution 8.3d



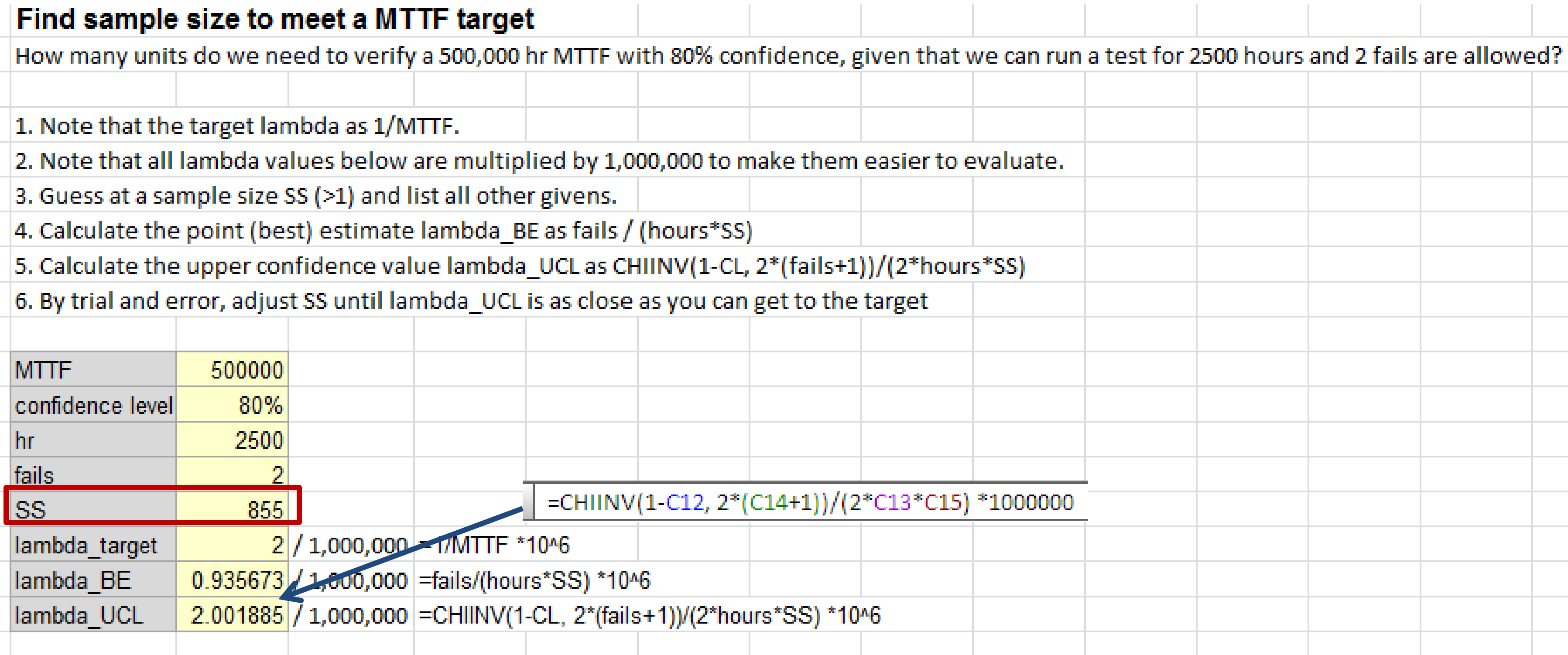
Maximum likelihood:	
	lambda
UCL	0.040632
estimate	0.03248
LCL	0.025499

Analytic:	
UCL	0.041111
estimate	0.03248
LCL	0.025311

Exercise 8.4

- This is Tobias & Trindade problem 3.1
- How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?
- Hint: you can do this by trial and error. Calculate the UCL on λ as a function of sample size SS and then adjust SS until the UCL equals the target λ.

Solution 8.4



Find sample size to meet a MTTF target

How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?

1. Note that the target lambda as 1/MTTF.
2. Note that all lambda values below are multiplied by 1,000,000 to make them easier to evaluate.
3. Guess at a sample size SS (>1) and list all other givens.
4. Calculate the point (best) estimate lambda_BE as fails / (hours*SS)
5. Calculate the upper confidence value lambda_UCL as CHIINV(1-CL, 2*(fails+1))/(2*hours*SS)
6. By trial and error, adjust SS until lambda_UCL is as close as you can get to the target

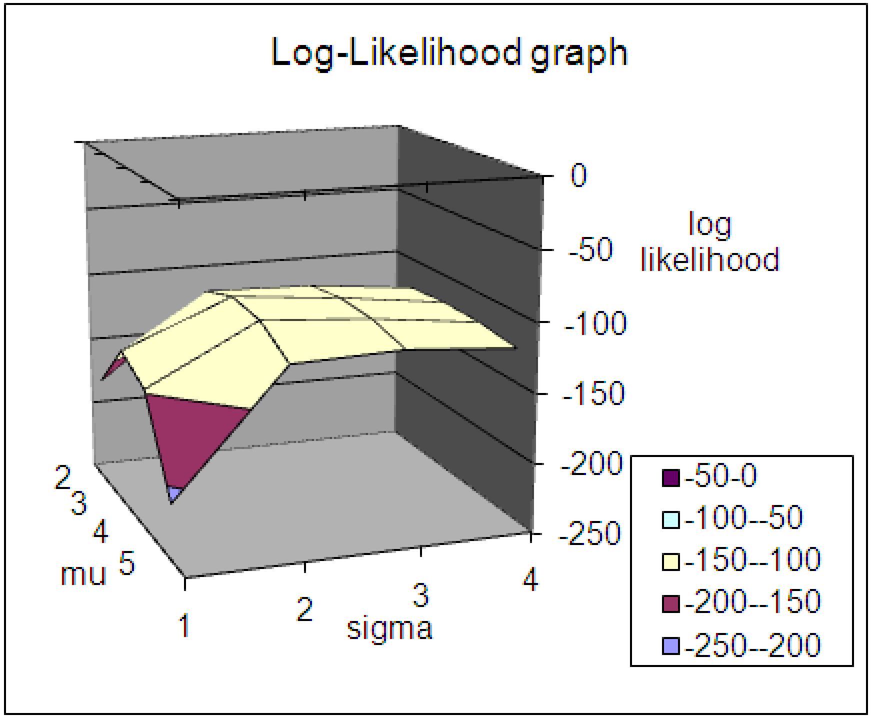
MTTF	500000		
confidence level	80%		
hr	2500		
fails	2		
SS	855		
lambda_target	2	/ 1,000,000	= 1/MTTF *10^6
lambda_BE	0.935673	/ 1,000,000	=fails/(hours*SS) *10^6
lambda_UCL	2.001885	/ 1,000,000	=CHIINV(1-CL, 2*(fails+1))/(2*hours*SS) *10^6

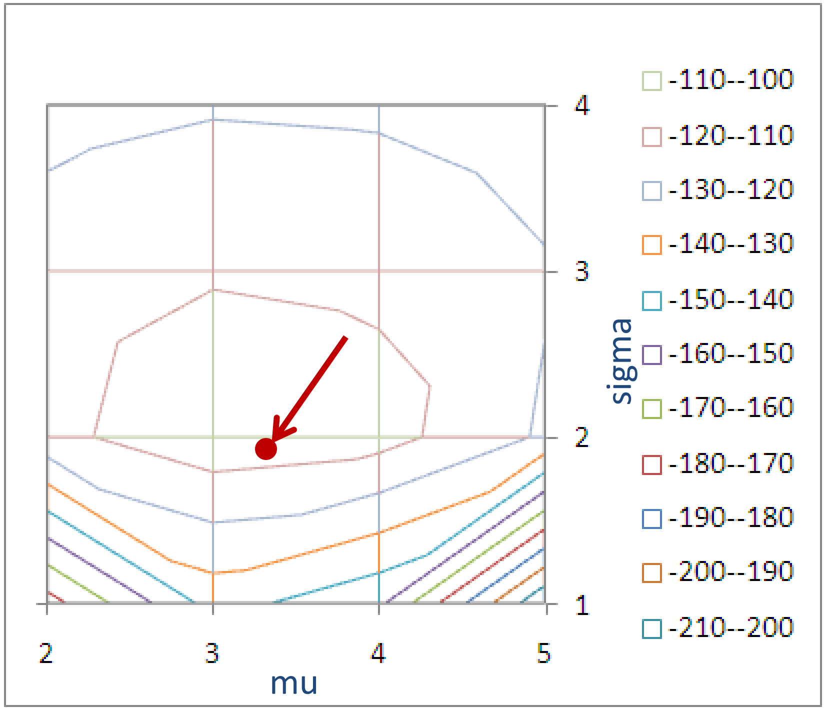
=CHIINV(1-C12, 2*(C14+1))/(2*C13*C15) *1000000

Normal Distribution MLE and Analytic

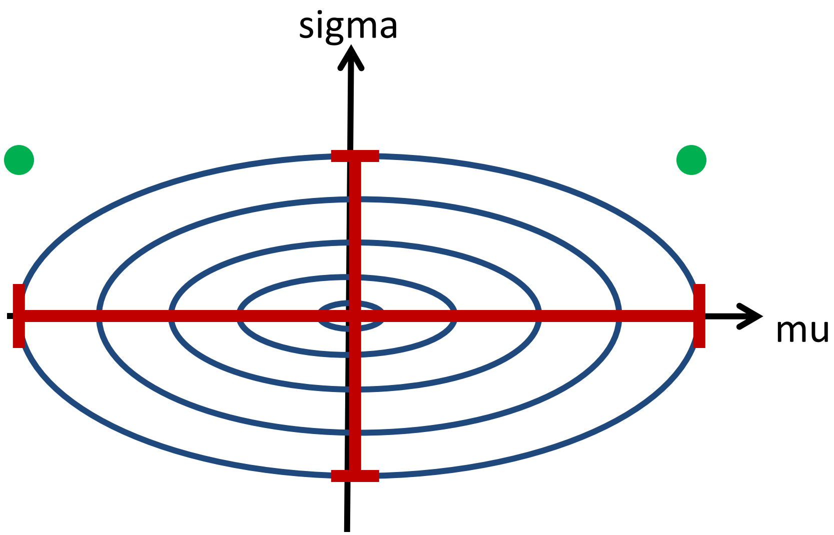
MLE for the Normal

$$L_i = \ln\left(\text{NORMDIST}\left(\text{data}_i, \overset{\downarrow}{\mu}, \overset{\downarrow}{\sigma}, \text{false}\right)\right)$$

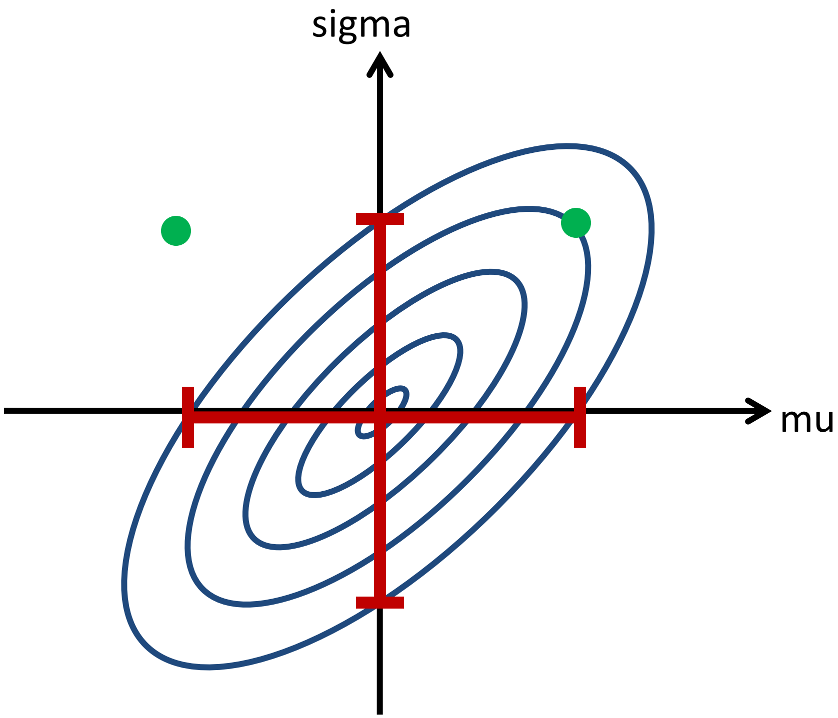




2D MLE



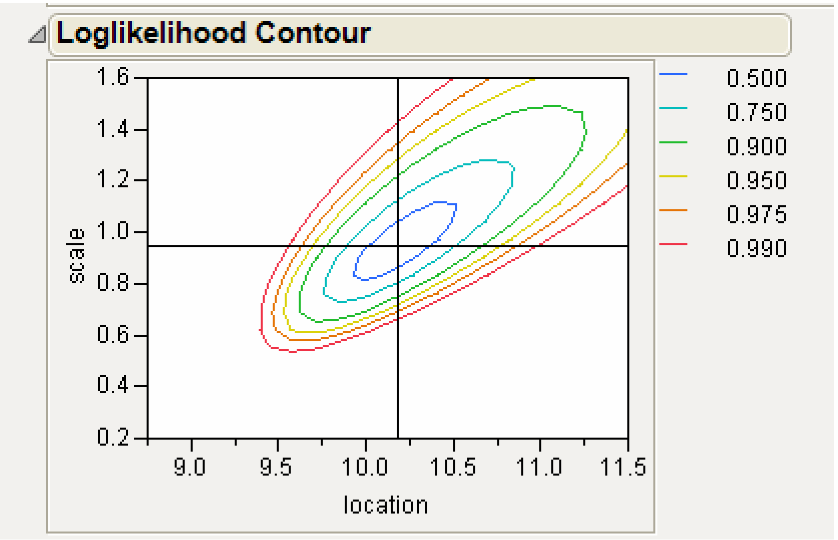
Uncorrelated



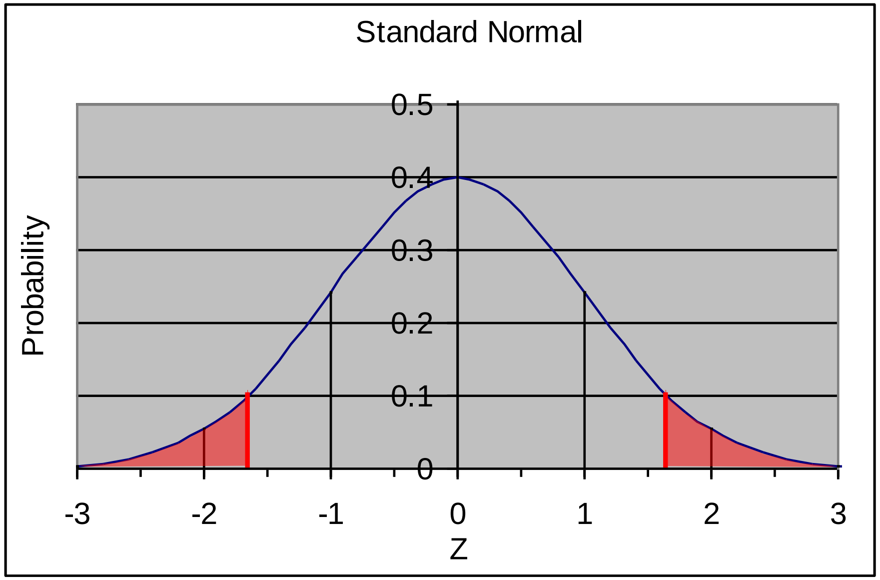
Correlated

Described by covariance matrix

JMP MLE Correlations



Analytic Uncertainty of Mean



Z-statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$\bar{X}$ = sample mean

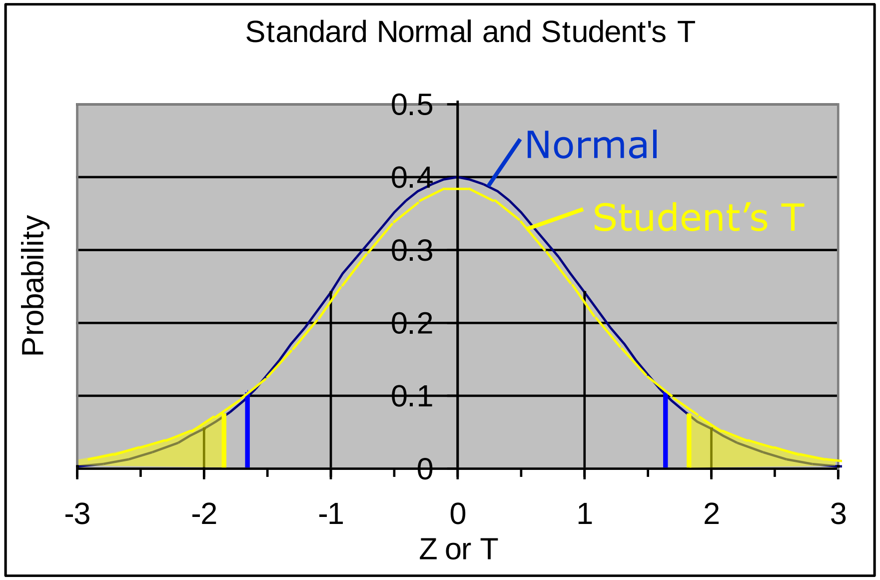
μ = population mean

σ = population stdev

n = sample size

- Using a *Z-statistic* is exactly equivalent to the previous slides
  - Gives error in $\bar{X}$ in units of $\sigma/\sqrt{n}$
- (Note that we need the true population standard deviation)

Analytic Uncertainty of Mean



T-statistic:

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

$\overline{X}$ = sample mean

μ = population mean

S = sample stdev

n = sample size

- Similar to *Z-statistic* but
  - Gives error in $\bar{X}$ in units of $S/\sqrt{n}$
- Preferred over *Z* because true σ is usually not known
- Calculate μ range from:

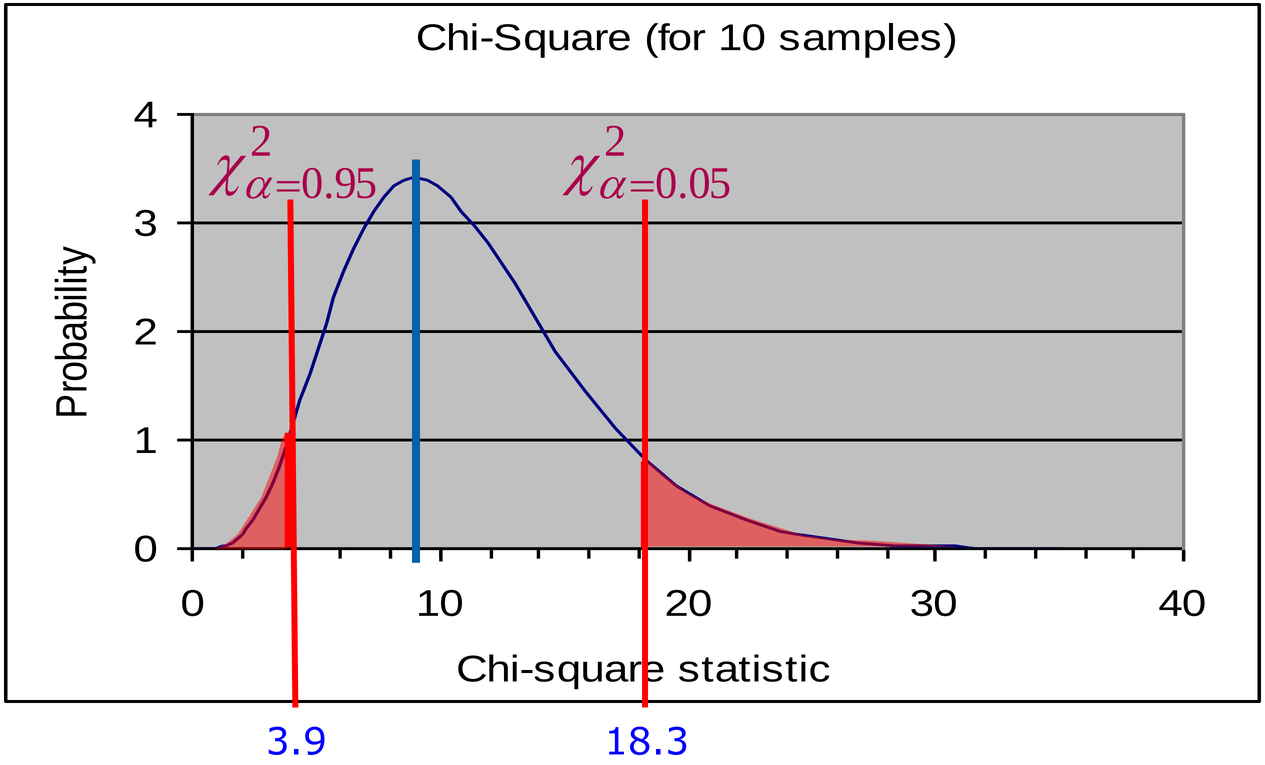
$$\bar{X} - \frac{t_{\%,n-1}\,S}{\sqrt{n}} \le \mu \le \bar{X} + \frac{t_{\%,n-1}\,S}{\sqrt{n}}$$

% = percentile

(e.g., 0.95 for 95% point for UCL)

T.INV(%, n–1)

Analytic Uncertainty of Standard Deviation



Chi^2 statistic:

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

S = sample stdev

σ = population stdev

n = sample size

- Gives error as ratio of sample variance / pop variance (times n-1)
- Distribution of chi^2 statistic follows a chi^2 distribution
- Calculate σ^2 range from: $$\frac{(n-1)\,S^2}{\chi^2_{\%,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{\%,n-1}}$$

% = percentile

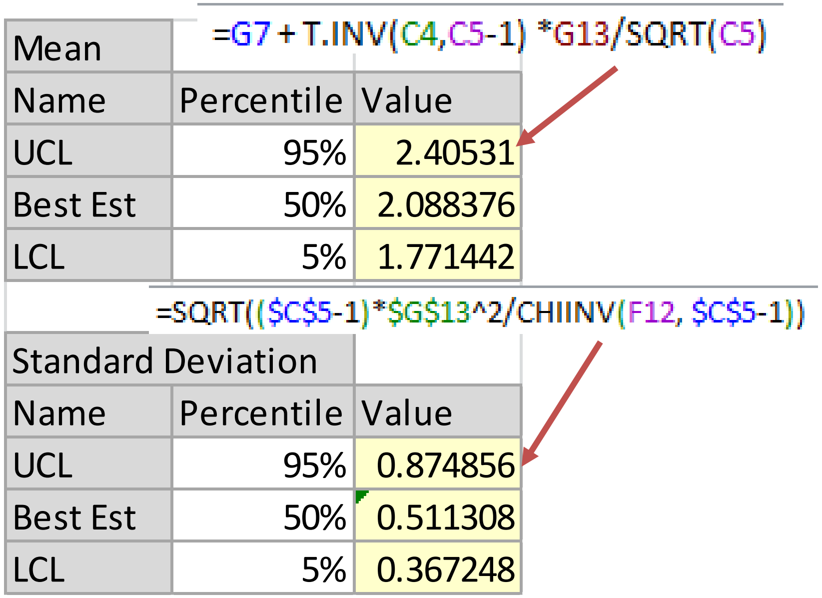
(e.g., 0.95 for 95% point for UCL)

CHIINV(%, n–1)

Exercise 9.1a

- For the 9 data points given, extract mu, sigma, and their 95% confidence intervals.

Solution 9.1a

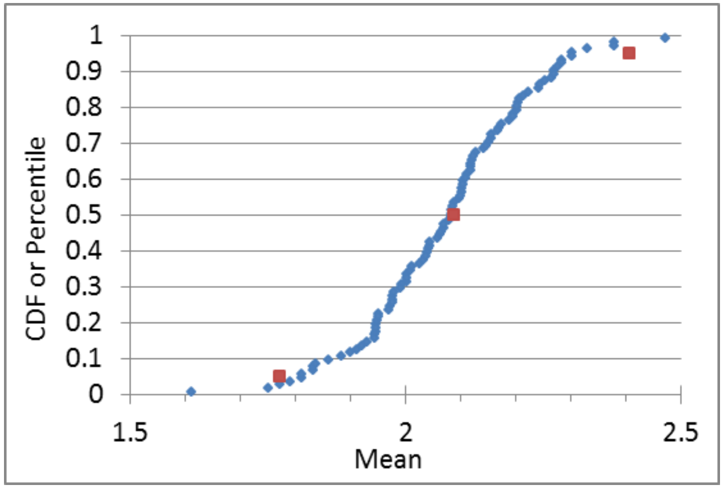


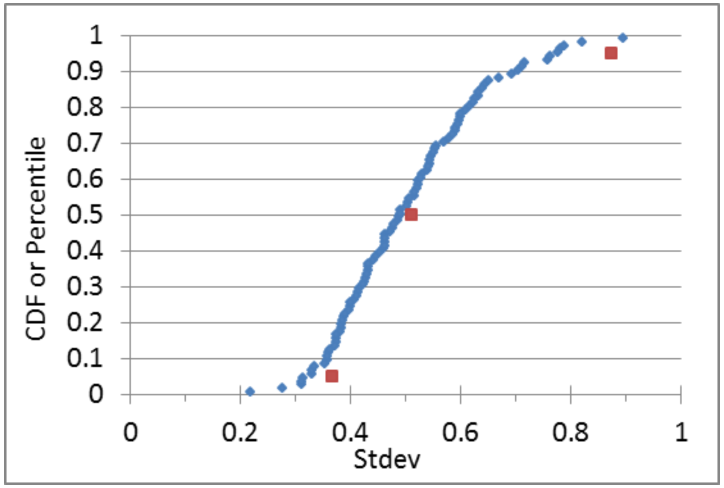
=G7 + T.INV(C4,C5-1) *G13/SQRT(C5)

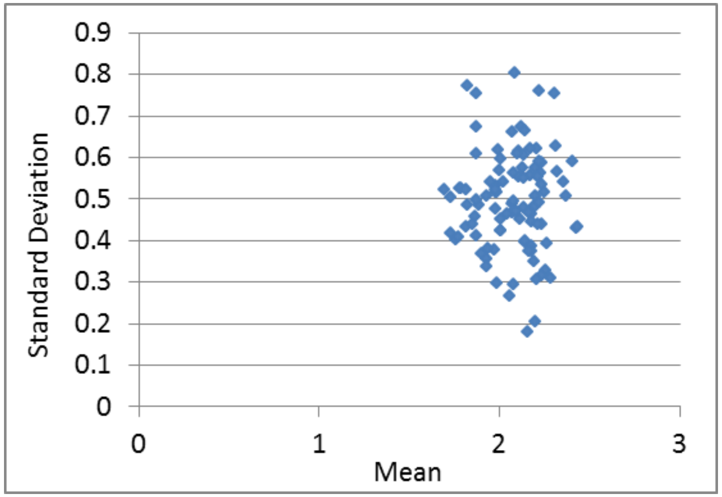
Mean		
Name	Percentile	Value
UCL	95%	2.40531
Best Est	50%	2.088376
LCL	5%	1.771442

=SQRT((C5-1)*G13^2/CHIINV(F12, C5-1))

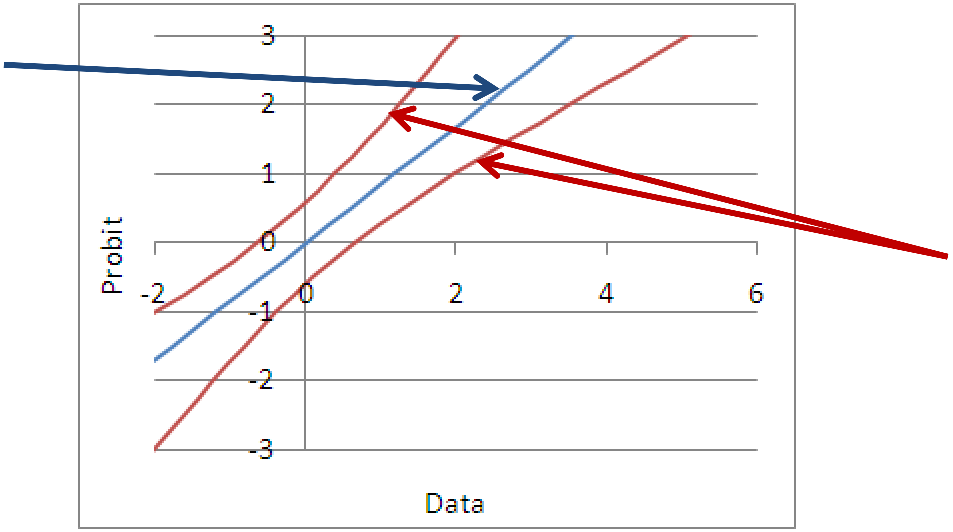
Standard Deviation		
Name	Percentile	Value
UCL	95%	0.874856
Best Est	50%	0.511308
LCL	5%	0.367248







Normal Distribution Uncertainties



Best estimate

Confidence limits

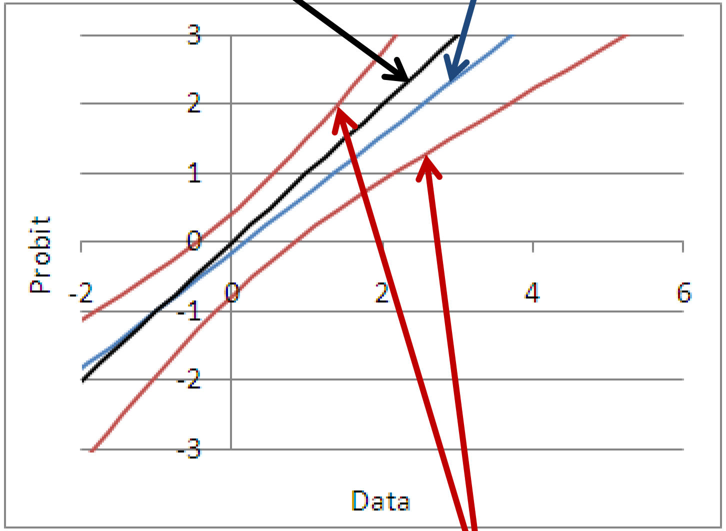
	Best estimate	Uncertainty
Mean μ	μ = AVERAGE(data)	m = NORMSINV(CL) * σ / SQRT(N)
Stdev σ	σ = STDEV(data)	s = σ * (SQRT(CHIINV(1-CL, N-1) / (N-1)) - 1)
Percentile	μ + z*σ	UCL = $\mu + z*\sigma + \mathrm{SQRT}(m^2 + z^2*s^2)$ LCL = $\mu + z*\sigma - \mathrm{SQRT}(m^2 + z^2*s^2)$

CL = confidence level (e.g., 95%)

z = probit value at which to evaluate distribution (e.g., -2)

N = number of samples in data set

Exercise 9.1b

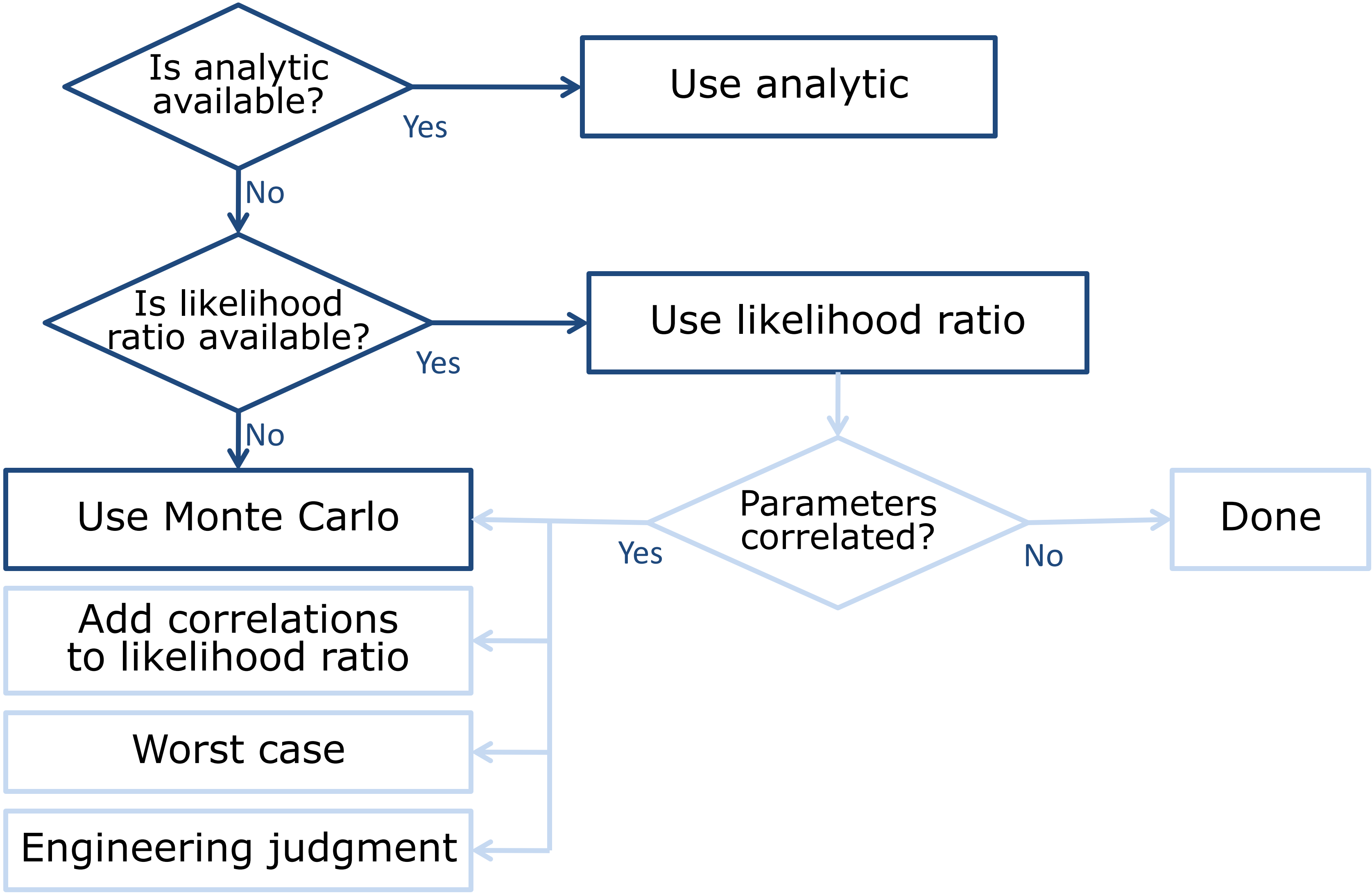


Theory

Best estimate

Confidence limits

Calculation Method Flowchart



Is analytic available?

Yes

Use analytic

No

Is likelihood ratio available?

Yes

Use likelihood ratio

No

Use Monte Carlo

Parameters correlated?

Yes

No

Done

Add correlations to likelihood ratio

Worst case

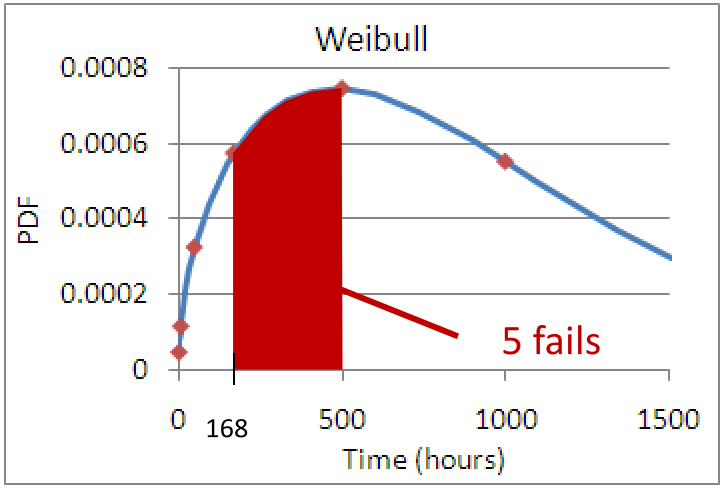
Engineering judgment

Weibull MLE with Readout Data

MLE

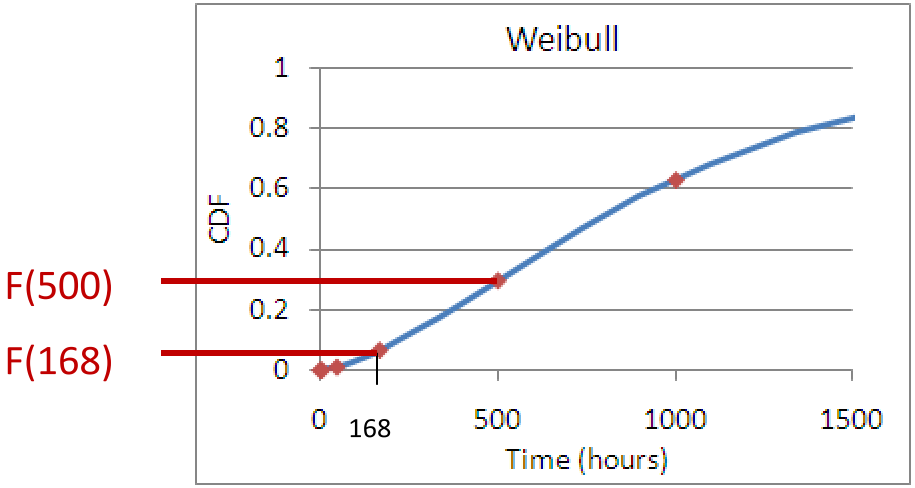
- Likelihood for each point
  - For exact values (exact times to fail), use the PDF
  - For ranges (failed between two readout times), use CDF delta
  - Multiply all together (or add logs)
- Use
  - Choose a model functional form with adjustable parameters
  - Adjust the parameters to maximize the likelihood

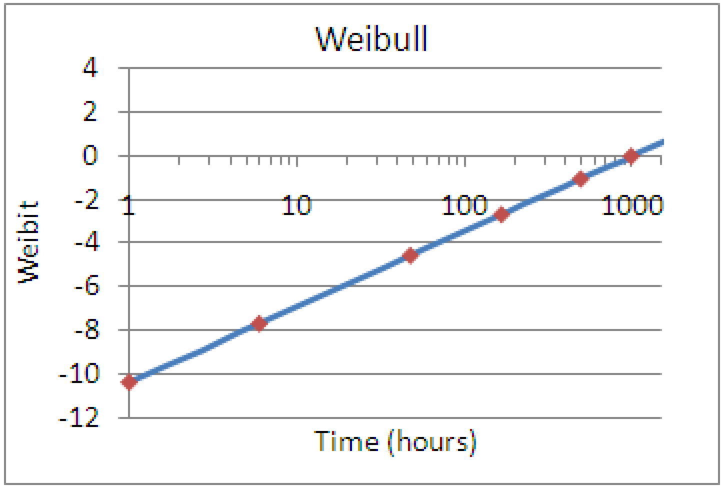
Weibull Readout Data



$$LIK = \left[F(500) - F(168)\right]^5$$

$$L = 5 \cdot \ln\left[F(500) - F(168)\right]$$



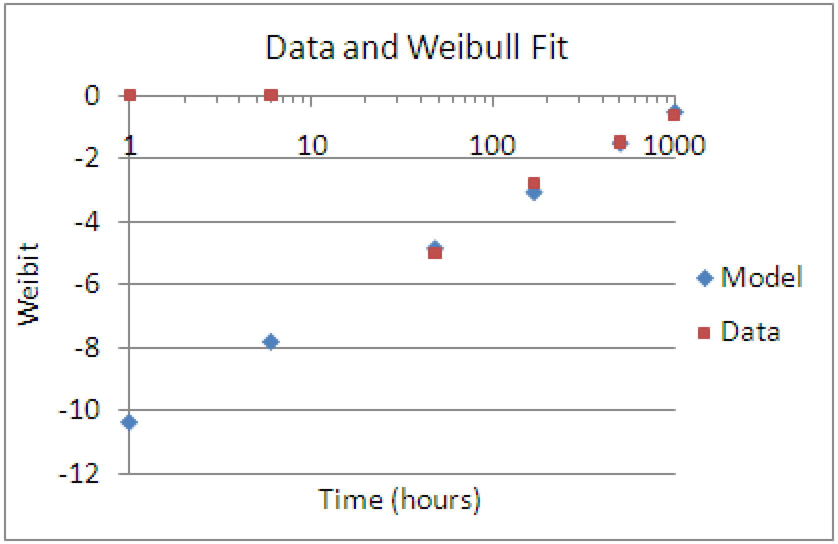


MLE for Weibull

Vary these to maximize this

shape	1.2
lifetime	1500
SS	300

time	fails	model F	L
0		0	
1	0	0.000154	0
6	0	0.001325	0
48	2	0.015948	-8.45037
168	16	0.069736	-46.763
500	43	0.234771	-77.4687
1000	63	0.459218	-94.1294
survivors	176	0.540782	-108.194
		Ltotal	-335.006



$$L=\sum_{r=1}^{R}\left[\boxed{n_r}\cdot\boxed{\ln\left\{F\left(t_r\right)-F\left(t_{r-1}\right)\right\}}+\underbrace{d_r\cdot\ln S\left(t_r\right)}\right]$$

$$+\boxed{s_R}\cdot\boxed{\ln S\left(t_R\right)}$$

$$\boxed{F(t)=1-\exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\}}$$

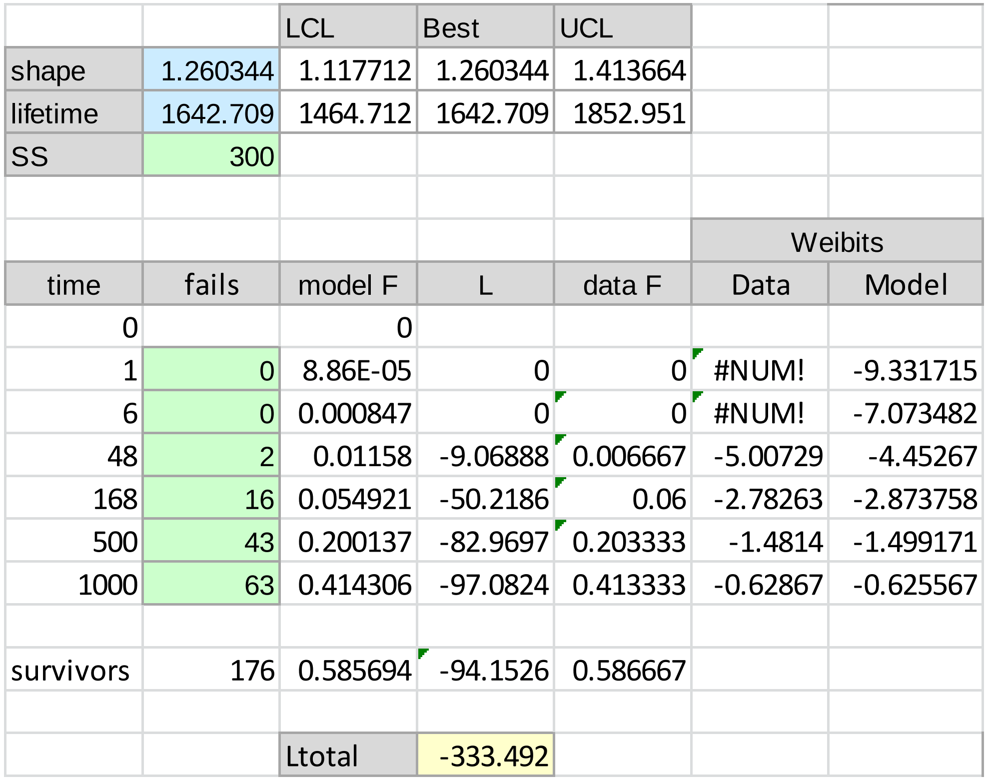
$$\boxed{S(t)=\exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\}}$$

Exercise 9.2a

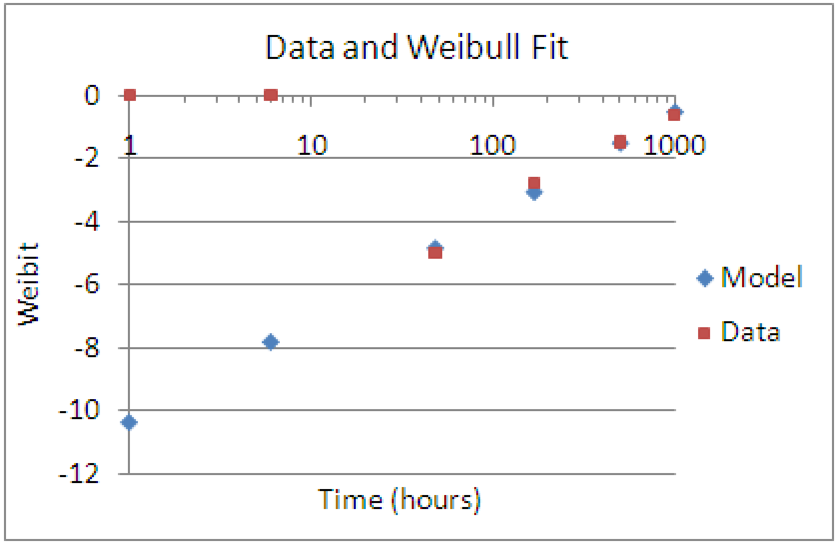
- Use MLE to determine Weibull fit parameters for the readout data given below and on the Ex16 tab.
- Also, find (separate) 90% confidence intervals for each parameter using the likelihood ratio technique. (That is the confidence where the LR=0.1.)

time	fails
0	
1	0
6	0
48	2
168	16
500	43
1000	63

Solution 8.6a



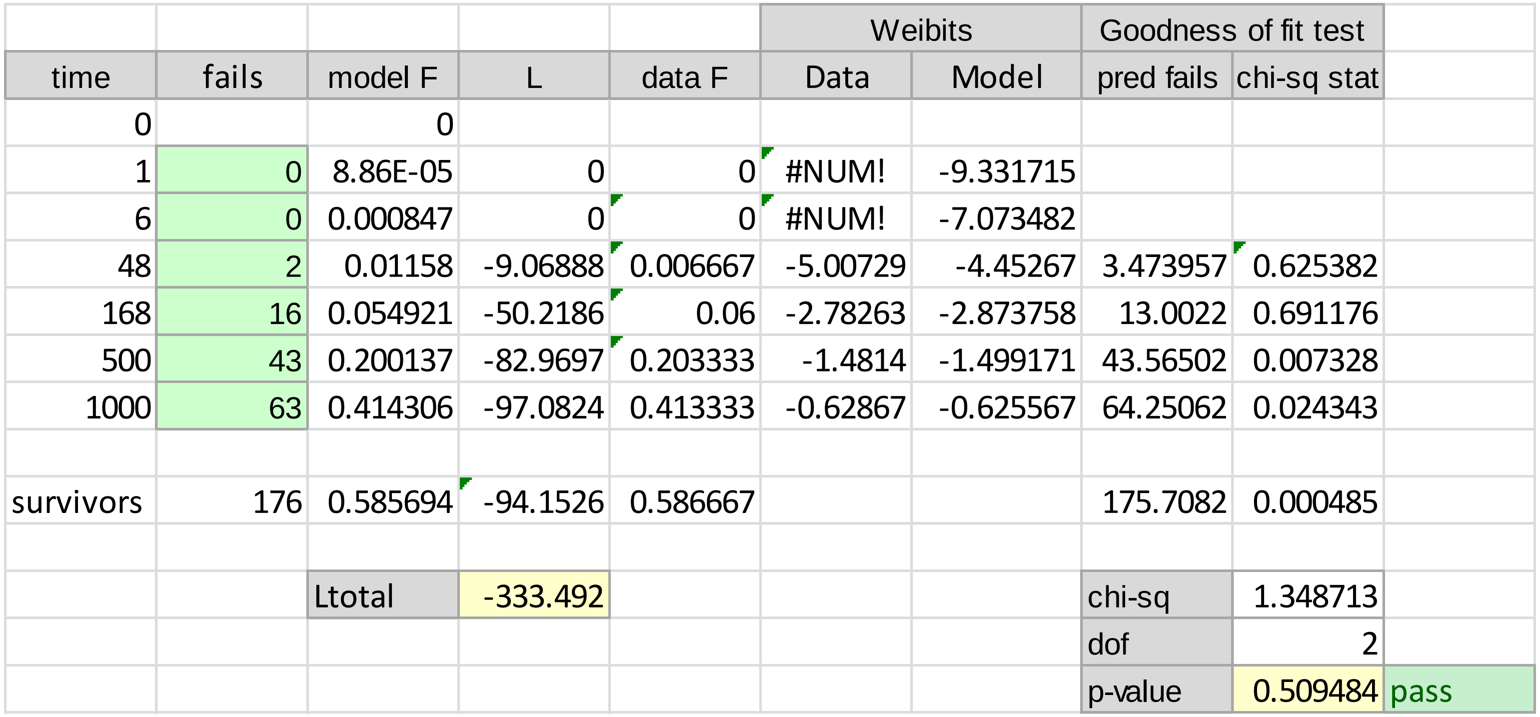
		LCL	Best	UCL		
shape	1.260344	1.117712	1.260344	1.413664		
lifetime	1642.709	1464.712	1642.709	1852.951		
SS	300					
					Weibits	
time	fails	model F	L	data F	Data	Model
0		0				
1	0	8.86E-05	0	0	#NUM!	-9.331715
6	0	0.000847	0	0	#NUM!	-7.073482
48	2	0.01158	-9.06888	0.006667	-5.00729	-4.45267
168	16	0.054921	-50.2186	0.06	-2.78263	-2.873758
500	43	0.200137	-82.9697	0.203333	-1.4814	-1.499171
1000	63	0.414306	-97.0824	0.413333	-0.62867	-0.625567
survivors	176	0.585694	-94.1526	0.586667		
		Ltotal	-333.492			



Exercise 9.2b

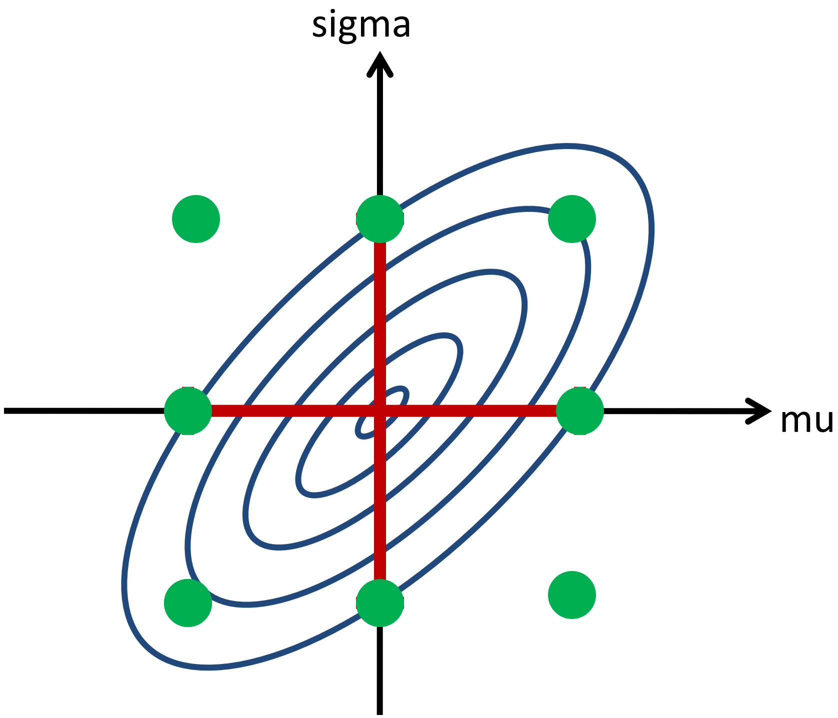
- Add a chi-square goodness-of-fit test to your fit from part (a). Recall that the bins should have more than about 5 fails each, so you will need to combine the first few readouts into 1 bin. So we do things the same way, combine the first 3 readouts into 1 bin, even though they only have 2 total fails.

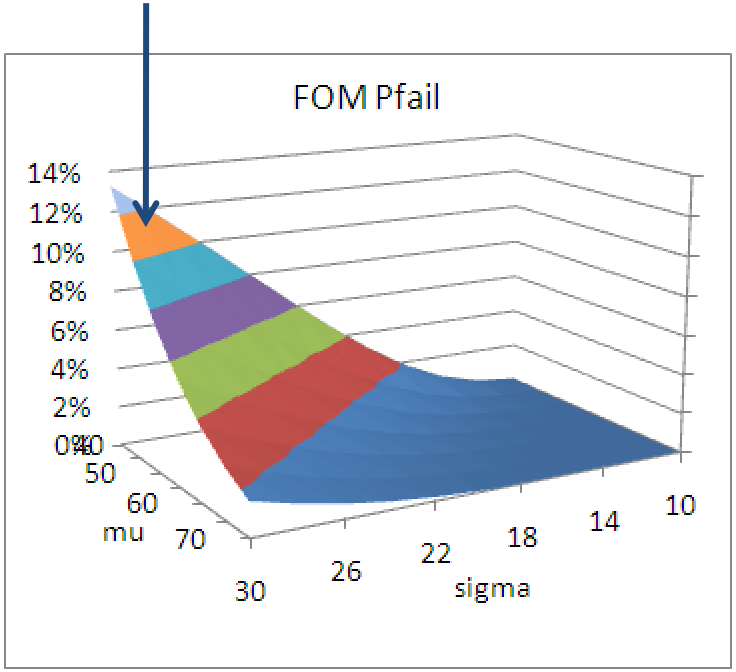
Solution 9.2b



					Weibits		Goodness of fit test		
time	fails	model F	L	data F	Data	Model	pred fails	chi-sq stat	
0		0							
1	0	8.86E-05	0	0	#NUM!	-9.331715			
6	0	0.000847	0	0	#NUM!	-7.073482			
48	2	0.01158	-9.06888	0.006667	-5.00729	-4.45267	3.473957	0.625382	
168	16	0.054921	-50.2186	0.06	-2.78263	-2.873758	13.0022	0.691176	
500	43	0.200137	-82.9697	0.203333	-1.4814	-1.499171	43.56502	0.007328	
1000	63	0.414306	-97.0824	0.413333	-0.62867	-0.625567	64.25062	0.024343	
survivors	176	0.585694	-94.1526	0.586667			175.7082	0.000485	
		Ltotal	-333.492				chi-sq	1.348713	
							dof	2	
							p-value	0.509484	pass

Confidence and Figures of Merit





Exercise 9.2c

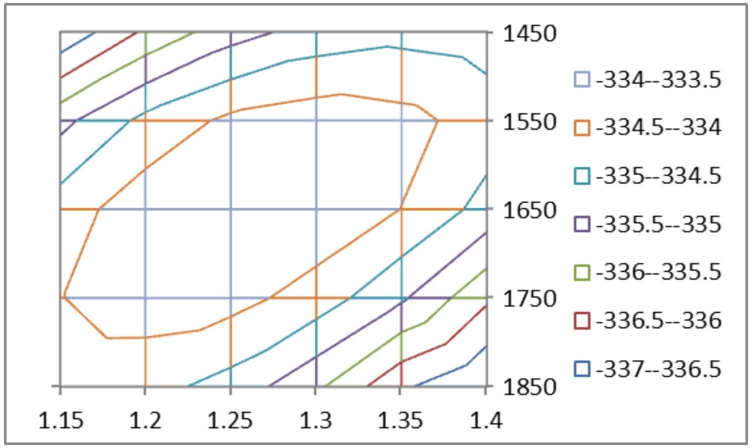
- Use the pfail at 2000 hours as the FOM for the Weibull model of part (a). Evaluate this pfail as a function of the shape and lifetime parameters.
- Try various corners of the “space” of shape and lifetime values and find the worst case corner. Report these values as your worst case shape and lifetime values.

Solution

		LCL	Best	UCL		Worst case		FOM time	2000
shape	1.413664	1.117712	1.260344	1.413664		1.4136642		FOM pfail	0.788438
lifetime	1464.712	1464.712	1642.709	1852.951		1464.7121			0.722
SS	300								

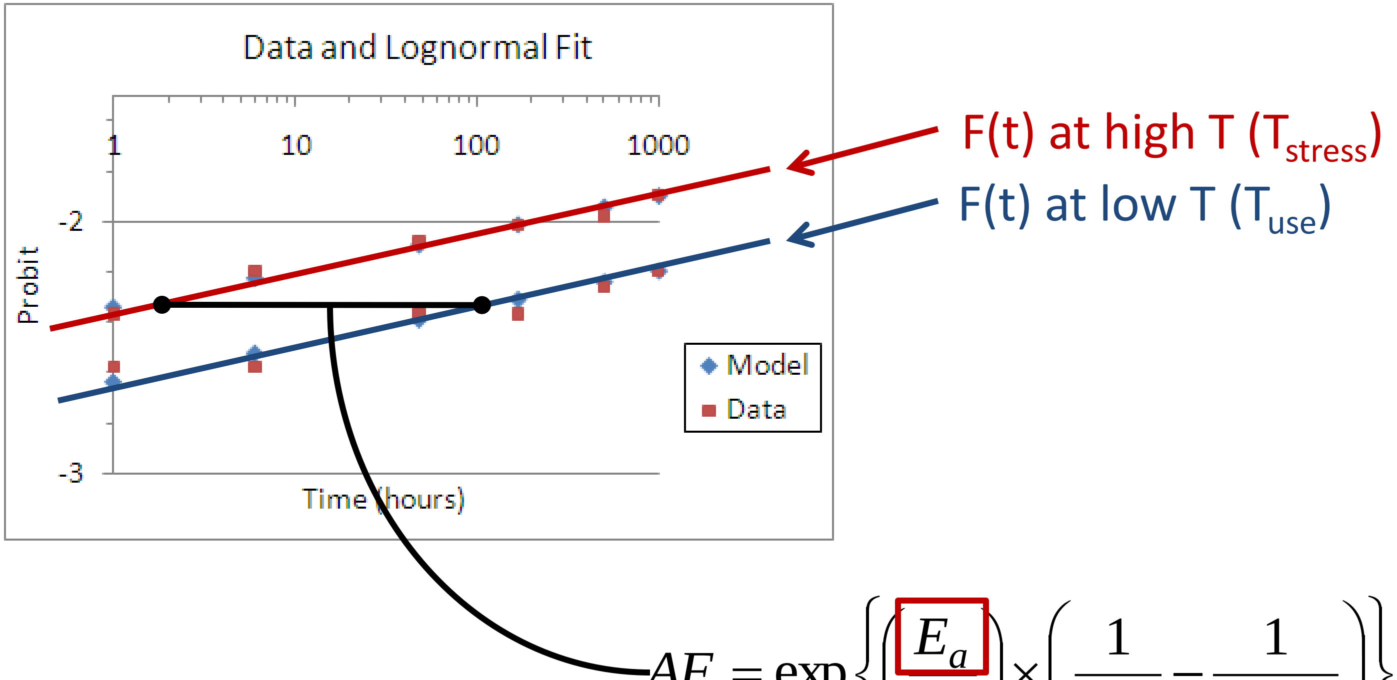
Ltotal	-334.69
best L	-333.492
LR p-value	0.121738

	1.15	1.2	1.25	1.3	1.35	1.4
1450	-336.92	-335.908	-335.208	-334.798	-334.656	-334.764
1550	-335.151	-334.357	-333.889	-333.724	-333.84	-334.218
1650	-334.247	-333.702	-333.497	-333.606	-334.008	-334.684
1750	-334.016	-333.741	-333.817	-334.22	-334.926	-335.915
1850	-334.314	-334.321	-334.691	-335.397	-336.417	-337.728



Acceleration Calculations

Acceleration



F(t) at high T (T_{stress})

F(t) at low T (T_{use})

$$AF = \exp\left\{\left(\frac{\boxed{E_a}}{k}\right) \times \left(\frac{1}{T_{use}} - \frac{1}{T_{stress}}\right)\right\}$$

Acceleration MLE

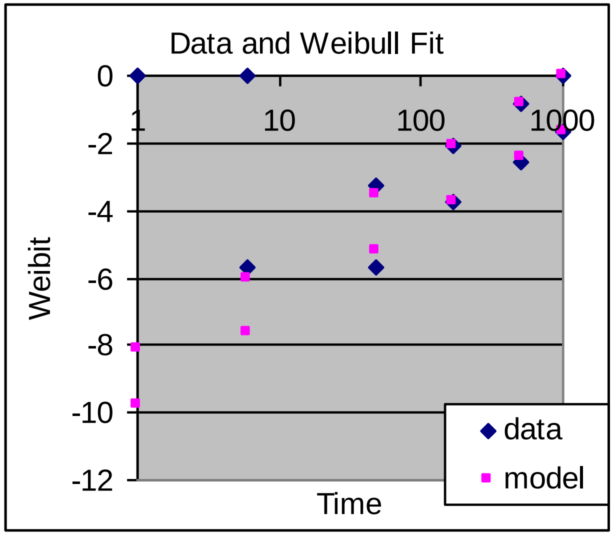
Now vary
these 3

			shape	1.176941		
			lifetime	4036.87		
			Ea	0.797264		
			Tref	80		
			SS	300		
					model	-614.108115
	T	time_clock	fails	time_eff	F	log-likelihood
leg1	80	0		0	0	
	80	1	0	1	5.7E-05	0
	80	6	0	6	0.000469	0
	80	48	1	48	0.005413	-5.3096733
	80	168	6	168	0.023432	-24.0978516
	80	500	15	500	0.08203	-42.5559058
	80	1000	31	1000	0.175945	-73.3263981
leg2	100				0	
	100	1	0	4.072326	0.000298	0
	100	6	1	24.43396	0.002449	-6.14171667
	100	48	10	195.4716	0.02794	-36.6941477
	100	168	24	684.1508	0.116439	-58.1942923
	100	500	72	2036.163	0.360369	-101.583041
	100	1000	84	4072.326	0.635907	-108.27857
leg1	80		survivors	247	0.824055	-47.7988956
leg2	100		survivors	109	0.364093	-110.127623

$$AF = \exp\left\{\left(\frac{\boxed{E_a}}{k}\right)\times\left(\frac{1}{\boxed{T_{ref}}+273}-\frac{1}{\boxed{T}+273}\right)\right\}$$

$$t_{eff} = AF\times t_{clock}$$

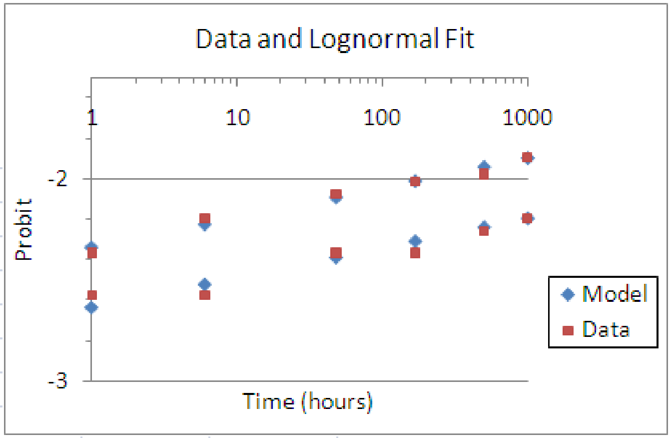
$$F(t)=1-\exp\left\{-\left(\frac{\boxed{t_{eff}}}{\alpha}\right)^{\beta}\right\}$$

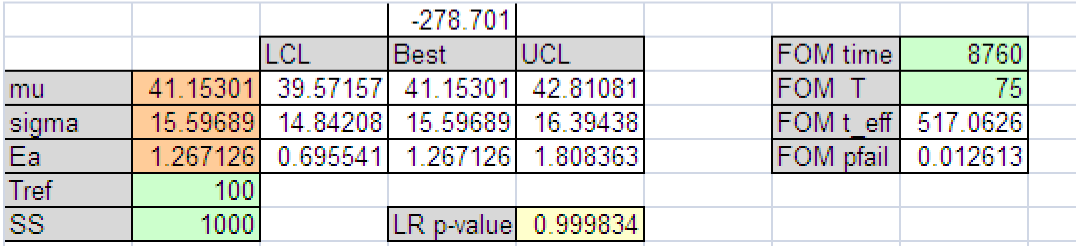


Exercise 9.3

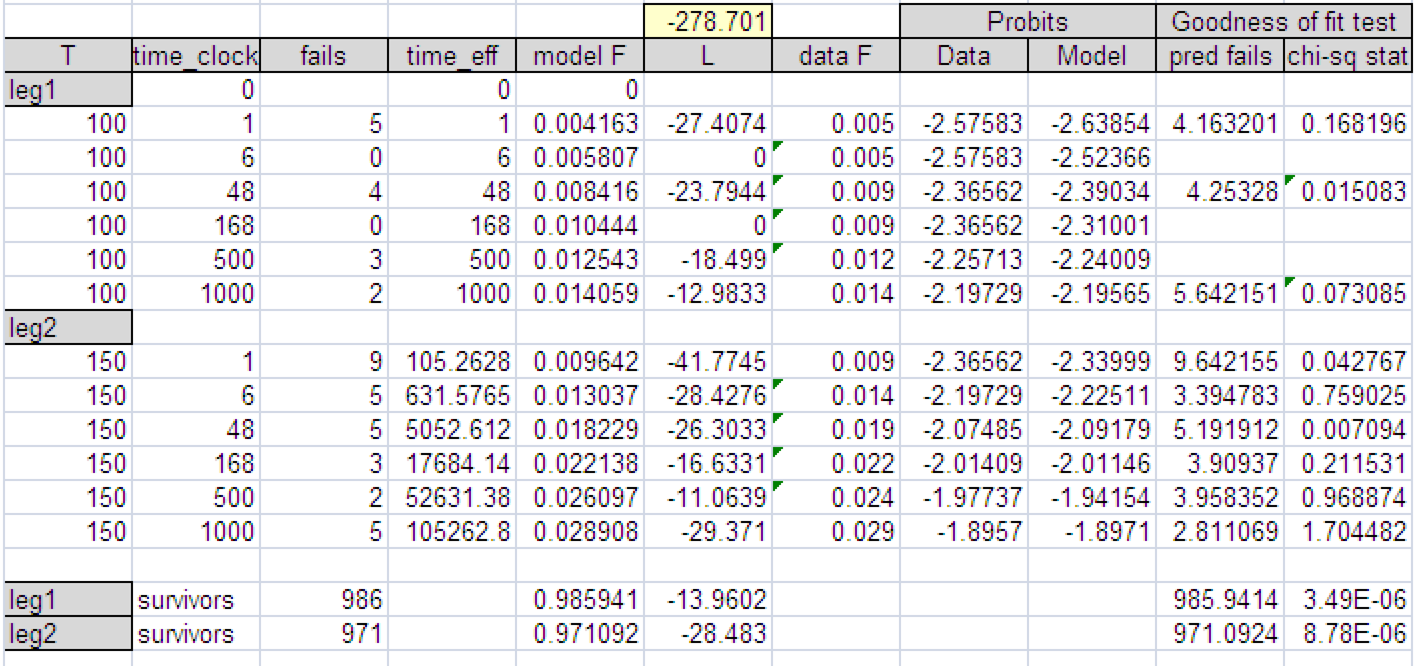
- Do an MLE to get the 3 parameters (mu, sigma, and Ea) for a lognormal model of times to fail with an Arrhenius temperature acceleration for the given data.
- Also do a goodness-of-fit test to see if the lognormal is a good fit to the data.
- Also determine 90% confidence limits on all 3 parameters (separately).
- Use a FOM of 1 year at T=75 and find what the conservative combination of confidence limits is.

Solution 9.3

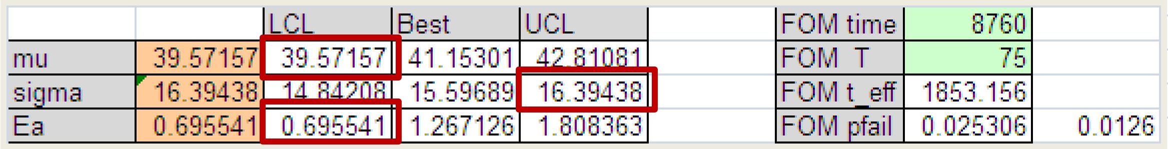




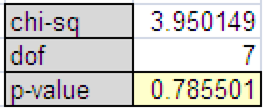
			-278.701				
		LCL	Best	UCL		FOM time	8760
mu	41.15301	39.57157	41.15301	42.81081		FOM T	75
sigma	15.59689	14.84208	15.59689	16.39438		FOM t_eff	517.0626
Ea	1.267126	0.695541	1.267126	1.808363		FOM pfail	0.012613
Tref	100						
SS	1000		LR p-value	0.999834			



					-278.701		Probits		Goodness of fit test	
T	time_clock	fails	time_eff	model F	L	data F	Data	Model	pred fails	chi-sq stat
leg1	0		0	0						
100	1	5	1	0.004163	-27.4074	0.005	-2.57583	-2.63854	4.163201	0.168196
100	6	0	6	0.005807	0	0.005	-2.57583	-2.52366		
100	48	4	48	0.008416	-23.7944	0.009	-2.36562	-2.39034	4.25328	0.015083
100	168	0	168	0.010444	0	0.009	-2.36562	-2.31001		
100	500	3	500	0.012543	-18.499	0.012	-2.25713	-2.24009		
100	1000	2	1000	0.014059	-12.9833	0.014	-2.19729	-2.19565	5.642151	0.073085
leg2										
150	1	9	105.2628	0.009642	-41.7745	0.009	-2.36562	-2.33999	9.642155	0.042767
150	6	5	631.5765	0.013037	-28.4276	0.014	-2.19729	-2.22511	3.394783	0.759025
150	48	5	5052.612	0.018229	-26.3033	0.019	-2.07485	-2.09179	5.191912	0.007094
150	168	3	17684.14	0.022138	-16.6331	0.022	-2.01409	-2.01146	3.90937	0.211531
150	500	2	52631.38	0.026097	-11.0639	0.024	-1.97737	-1.94154	3.958352	0.968874
150	1000	5	105262.8	0.028908	-29.371	0.029	-1.8957	-1.8971	2.811069	1.704482
leg1	survivors	986		0.985941	-13.9602				985.9414	3.49E-06
leg2	survivors	971		0.971092	-28.483				971.0924	8.78E-06

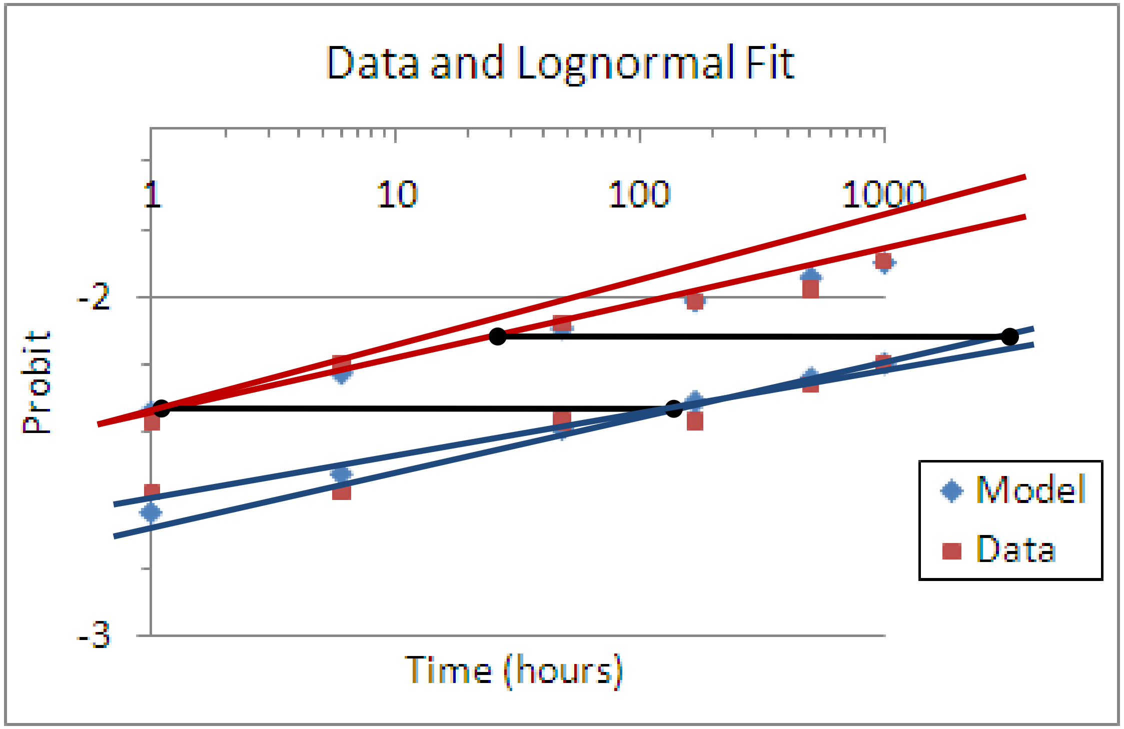


		LCL	Best	UCL		FOM time	8760	
mu	39.57157	39.57157	41.15301	42.81081		FOM T	75	
sigma	16.39438	14.84208	15.59689	16.39438		FOM t_eff	1853.156	
Ea	0.695541	0.695541	1.267126	1.808363		FOM pfail	0.025306	0.0126



chi-sq	3.950149
dof	7
p-value	0.785501

Is Acceleration Valid?



Likelihood Ratio Test

Likelihood ratio statistic:

$$LR = 2 \times (\ln L_1 - \ln L_2)$$

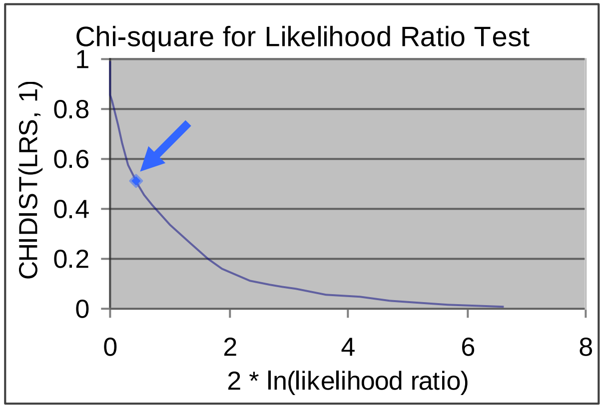
Chi-square distributed:

p-value

CHIDIST(LR, ΔDoF)

= CHIDIST(0.43, 1) = 0.51 (likely)

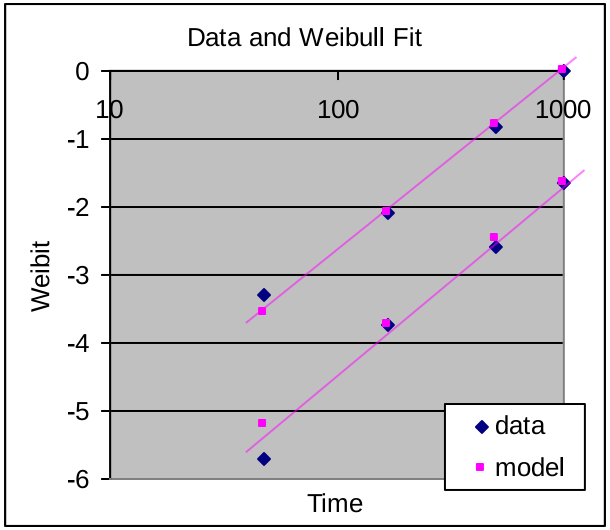
Acceleration is valid



Case 1: One Distribution

shape	1.176943
lifetime	4036.862
Ea	0.797262

log L_1= -614.108

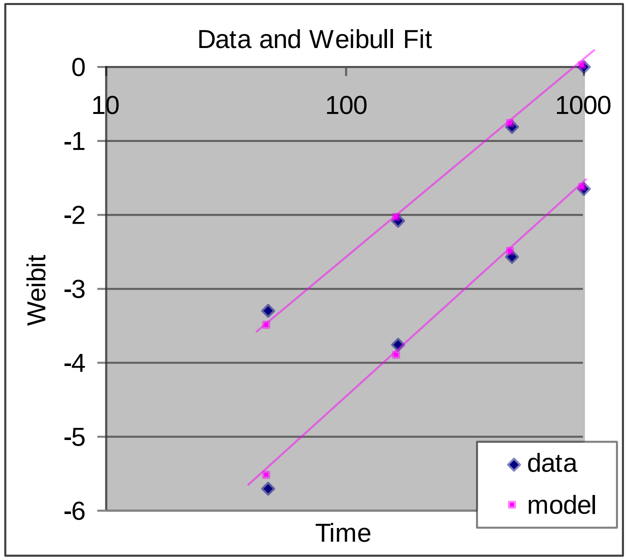


Case 2: Separate Distributions

leg1	shape	1.281392
	lifetime	3592.847
leg2	shape	1.154944
	lifetime	4071.605

-192.912	Leg1
-420.981	Leg2

log L_2= -613.893



Likelihood Ratio Test

Likelihood ratio statistic:

$$LR = 2 \times (\ln L_1 - \ln L_2)$$

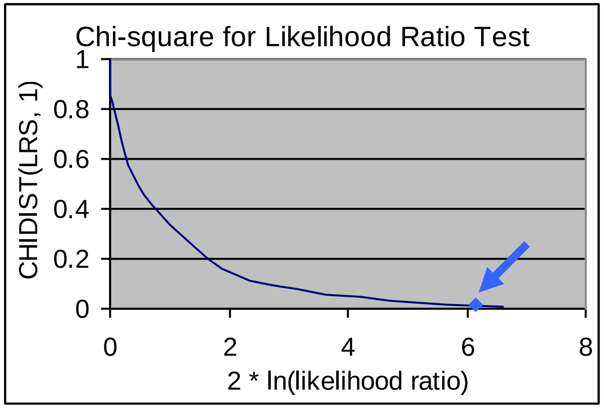
Chi-square distributed:

CHIDIST(LR, ΔDoF)

= CHIDIST(6.17, 1) = 0.013 (not likely)

p-value

Acceleration is ***not*** valid



Case 1: One Distribution

shape	1.198326
lifetime	3344.311
Ea	0.475108

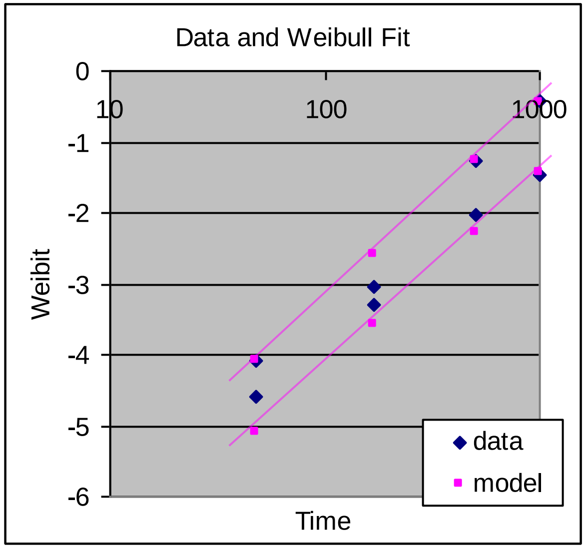
$\log \Gamma_1 =$ -580.91

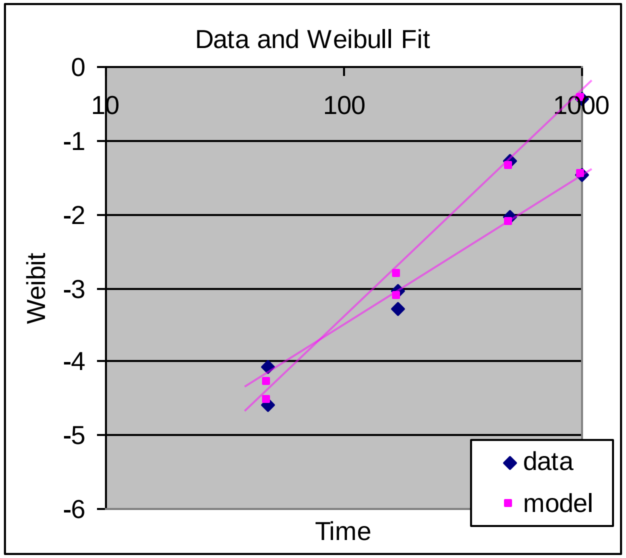
Case 2: Separate Distributions

leg1	shape	0.933166
	lifetime	4782.319
leg2	shape	1.350774
	lifetime	5607.301

-226.744	Leg1
-351.081	Leg2

$\log \Gamma_2 =$ -577.826





Exercise 9.3b

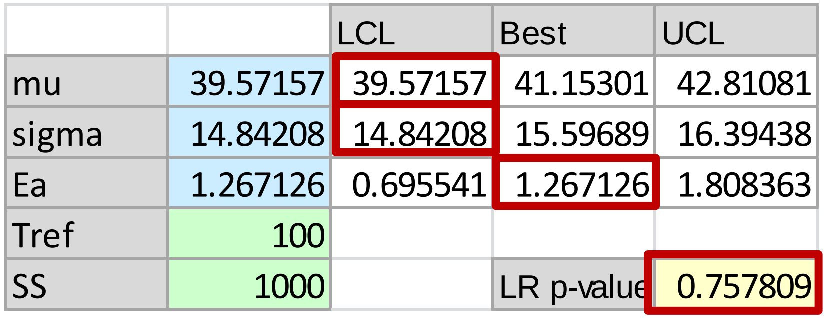
- Do a likelihood ratio test to see if the acceleration concept is valid for the Ex 9.3 data set. You will have to use 2 separate likelihood sums and two sets of mu and sigma parameters with no Ea, and compare that to what you have for part (a).

Solution 9.3b

LR test of using an acceleration:				
	L (together)	L (separate)	mu	sigma
leg 1	-96.644233	-96.620776	43.59891	16.69311
leg 2	-182.05632	-182.04515	40.20612	15.10995
sum	-278.70056	-278.66593		
LR p-value	0.79241468			

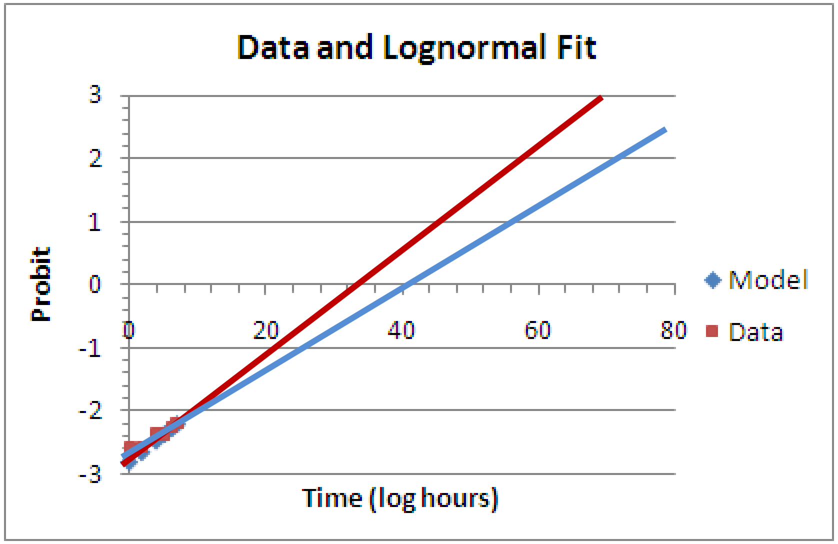
p-value > 0.05, so acceleration is valid

When to use Monte Carlo?



		LCL	Best	UCL
mu	39.57157	39.57157	41.15301	42.81081
sigma	14.84208	14.84208	15.59689	16.39438
Ea	1.267126	0.695541	1.267126	1.808363
Tref	100			
SS	1000		LR p-value	0.757809

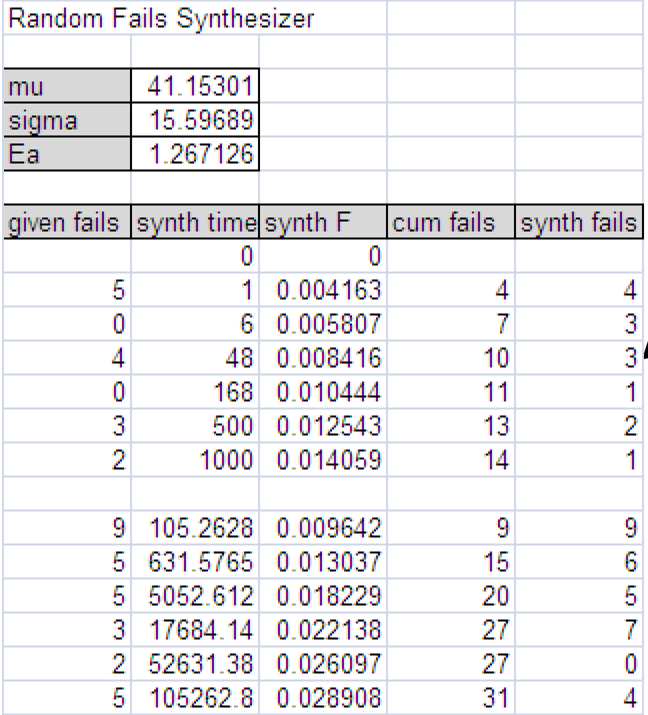
- Very high p-value at a corner indicates correlation



Exercise 9.3c

- Make a Monte Carlo simulation of the Ex 9.3 data. Use the procedure for generating random readout data sets, and make a "data synthesizer". Cut and paste each data set into the MLE calculator and use Solver to optimize. (No need to calculate confidence limits or goodness-of-fit.) Use the best estimates from the original data set to generate F for the synthesizer. Record mu, sigma, Ea, and the FOM for each trial. Do at least 5 trials, but more is better.

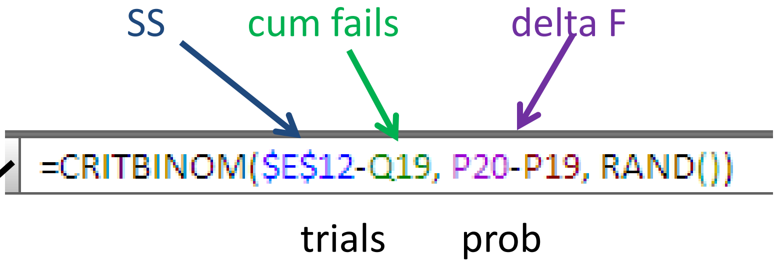
Solution 9.3c



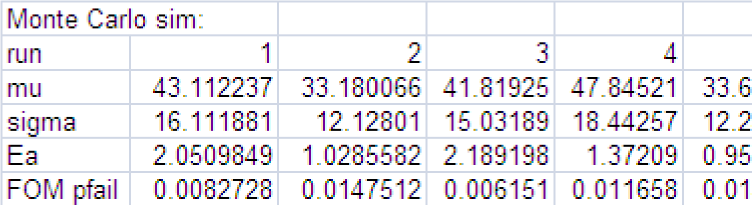
Random Fails Synthesizer

mu	41.15301
sigma	15.59689
Ea	1.267126

given fails	synth time	synth F	cum fails	synth fails
	0	0		
5	1	0.004163	4	4
0	6	0.005807	7	3
4	48	0.008416	10	3
0	168	0.010444	11	1
3	500	0.012543	13	2
2	1000	0.014059	14	1
9	105.2628	0.009642	9	9
5	631.5765	0.013037	15	6
5	5052.612	0.018229	20	5
3	17684.14	0.022138	27	7
2	52631.38	0.026097	27	0
5	105262.8	0.028908	31	4



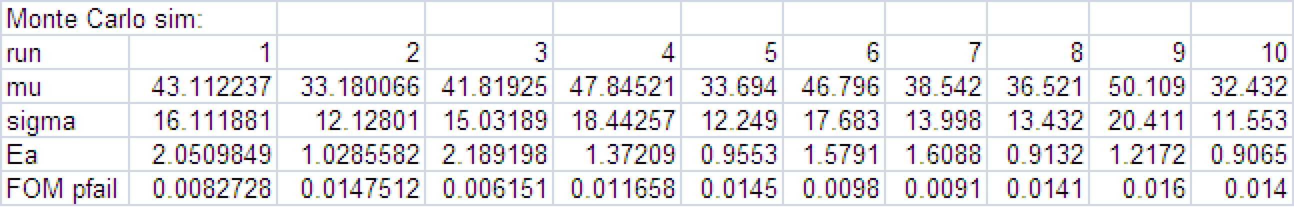
=CRITBINOM(E12-Q19, P20-P19, RAND())



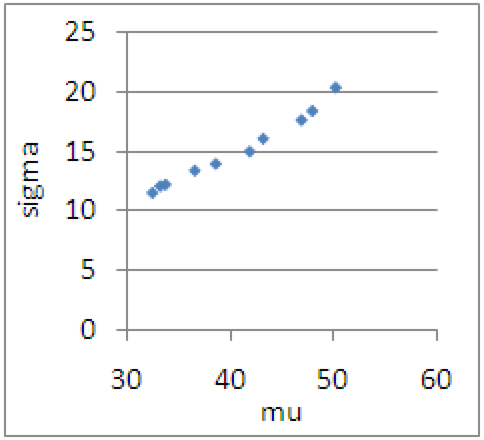
Monte Carlo sim:

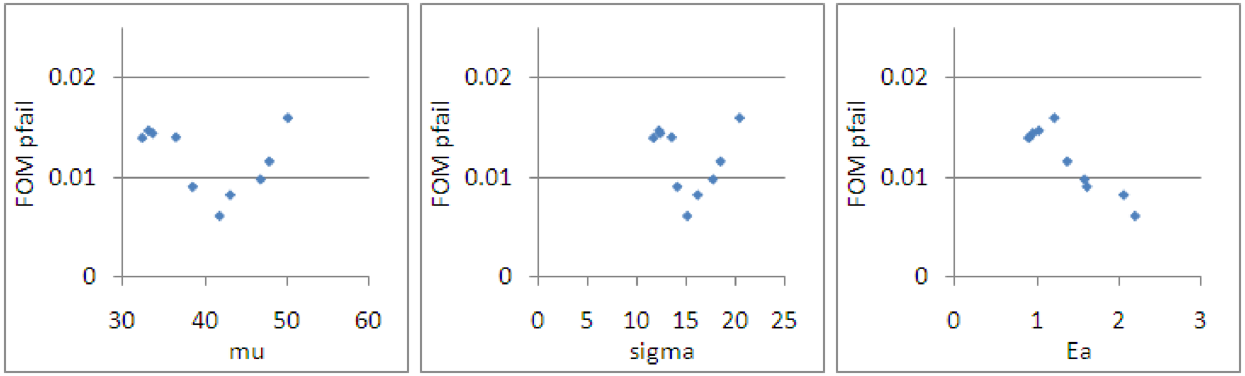
run	1	2	3	4	
mu	43.112237	33.180066	41.81925	47.84521	33.6
sigma	16.111881	12.12801	15.03189	18.44257	12.2
Ea	2.0509849	1.0285582	2.189198	1.37209	0.95
FOM pfail	0.0082728	0.0147512	0.006151	0.011658	0.01

Solution 9.3c

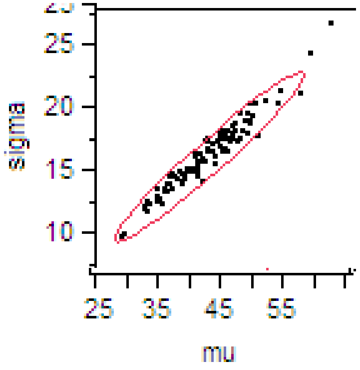


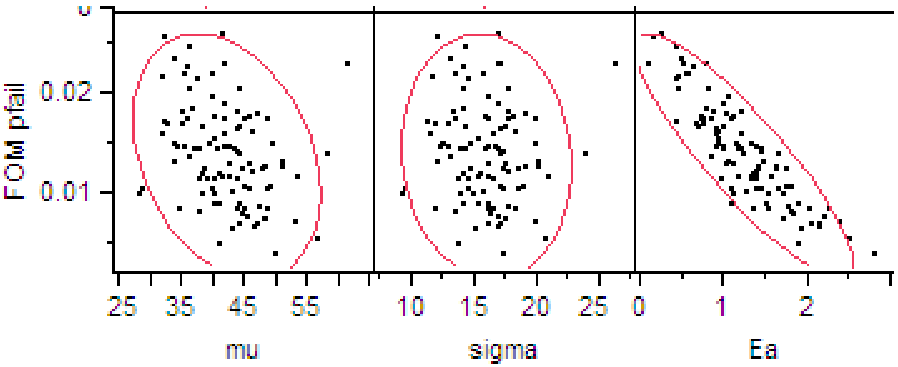
Monte Carlo sim:										
run	1	2	3	4	5	6	7	8	9	10
mu	43.112237	33.180066	41.81925	47.84521	33.694	46.796	38.542	36.521	50.109	32.432
sigma	16.111881	12.12801	15.03189	18.44257	12.249	17.683	13.998	13.432	20.411	11.553
Ea	2.0509849	1.0285582	2.189198	1.37209	0.9553	1.5791	1.6088	0.9132	1.2172	0.9065
FOM pfail	0.0082728	0.0147512	0.006151	0.011658	0.0145	0.0098	0.0091	0.0141	0.016	0.014





From the “Hard Fail DOE” tool:





The End