ECE 510 Lecture 8
Acceleration, Maximum Likelihood

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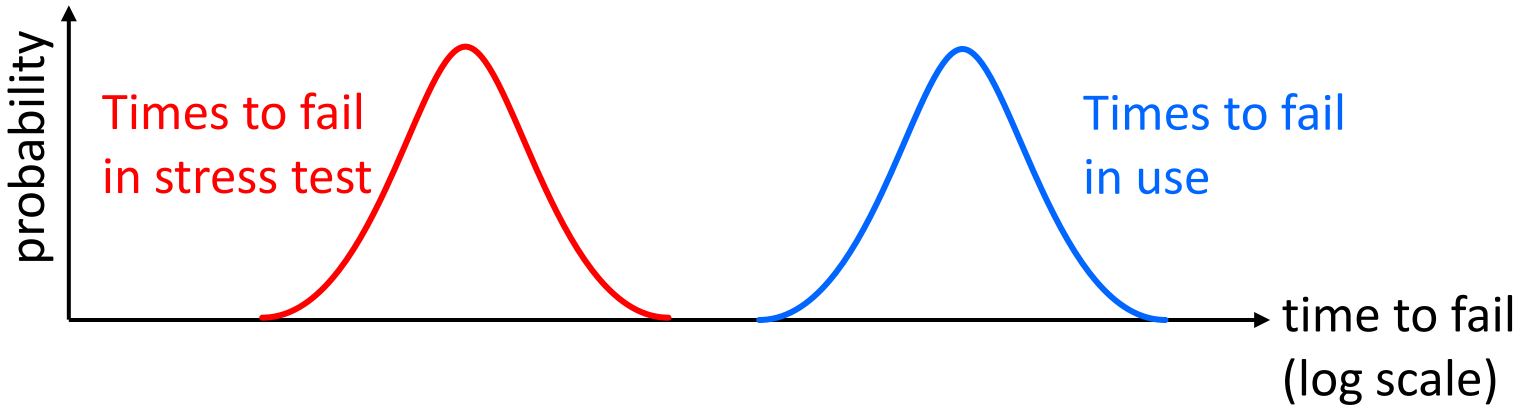
Acceleration Concept

Stress and Failure

- How long is our product going to last?
- We can't wait until it fails to see – that takes too long!
- We need to identify the stresses that cause it to fail
  - …and then apply them harder to make our parts fail in a reasonable amount of time
- Our stresses include
  - Voltage
  - Temperature
  - Current
  - Humidity
  - Mechanical stress
  - …and others

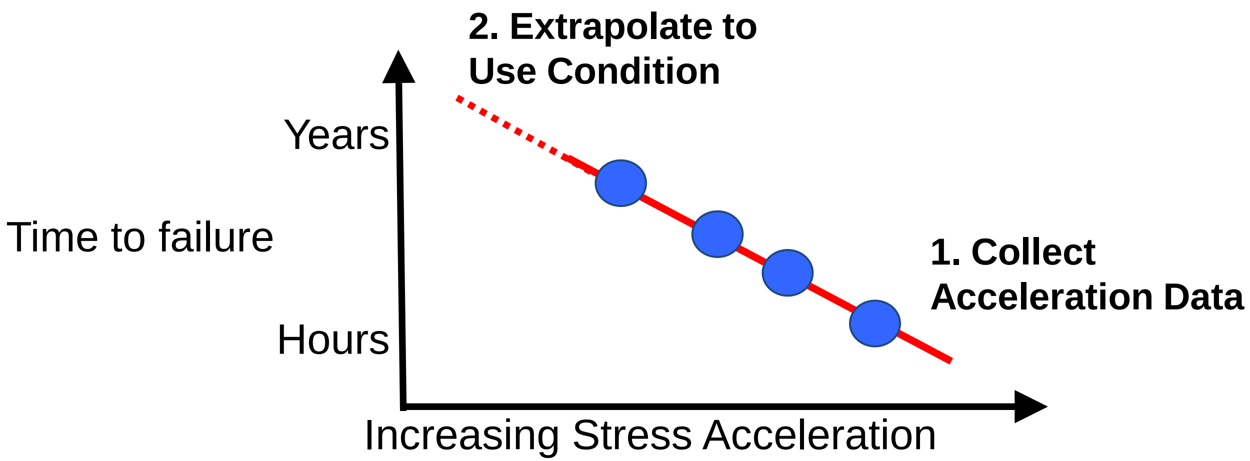
Reliability Models

Probability distributions of times-to-fail at two stress conditions



- Knowledge-based qual based on a *reliability model*
  - Model is built at one test condition
  - It can be scaled (“*accelerated*”) to other use conditions
- Models are built from data from reliability tests

Accelerated Test



- Accelerated test increases one or more conditions (e.g., T, V, etc.) to reduce times to failure

  Life Test (years) → Accelerated Test (hours)

- Intention is to accelerate a mechanism without inducing new mechanisms

Semiconductor Failure Mechanisms

Category	**Mechanism**	**Cause**	**Stress**
Constant	Electrical Overstress	ESD and Latchup	V, I
IM	Infant Mortality	Extrinsic Defects	V, T
Wearout	Hot Carrier	e- Impact ionization	V, I
Wearout	Neg. Bias-T Instability	Gate dielectric damage	V, T
Wearout	Electromigration	Atoms move by e- wind	I, T
Wearout	Time-Dep Diel. B'down	Gate dielectric leakage	V, T
Wearout	Stress Migration	Metal diffusion, voiding	T
Wearout	Interlayer Cracking	Interlayer stress	ΔT
Wearout	Solder Joint Cracking	Atoms move w/ stress	ΔT
Wearout	Corrosion	Electrochemical reaction	V, T, RH
Constant	Soft Error	n & α e-h pair creation	radiation

V = Voltage, I = Current, T = Temperature, ΔT = Temp cycle, RH = Relative Humidity

Reliability Tests

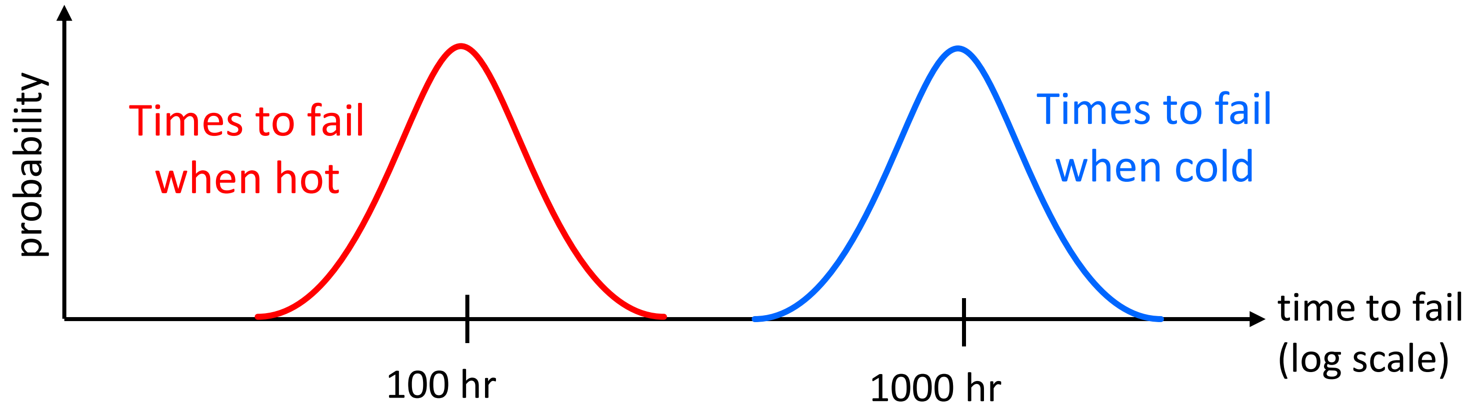
Name	Count	Time and Stress	Mechanisms
Infant Mortality Experiment	~10,000 units	48 hr at hi-V, hi-T	Latent reliability defects (IM)
Extended Life Test	~300 units	500 hr at hi-V, hi-T	Wearout (oxide, PBT, Fmax, Vccmin)
Test structure stress tests	100’s of devices	Hours at hi-V, hi-T	Oxide breakdown, PMOS bias-temp, electromigration, other silicon mechs
Bake	~300 units	100’s of hours at hi-T	TIM degradation, cracking and delaminating
Highly Accelerated Stress Test (HAST)	~300 units	50-150 hr at hi-T, hi-RH	Metal migration, adhesion fail
Temperature Cycling	~300 units	~1000 cycles -55C to 125C	Cracks anywhere, TIM degradation

Accelerated Testing Pitfalls

- Different mechanisms might accelerate differently
- No universal test:
  - Stress tests are idealizations of real life
  - Some mechanisms might get too much acceleration
  - Single stress does not stimulate all relevant behaviors
  - May not comprehend effects like materials creep
- The most accurate data is the most difficult or unrealistic to acquire:
  - Long test times are required at low acceleration conditions

Acceleration Calculation

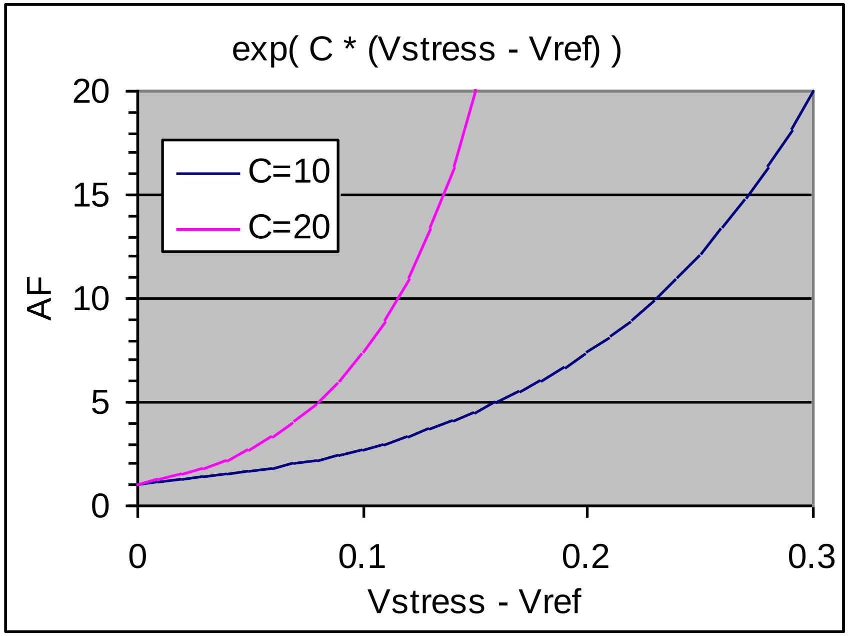
Acceleration Factor



$$AF = \frac{t_{cold}}{t_{hot}} = \frac{1000\text{hr}}{100\text{hr}} = 10$$

- An acceleration factor describes how much a particular stress accelerates degradation or failure

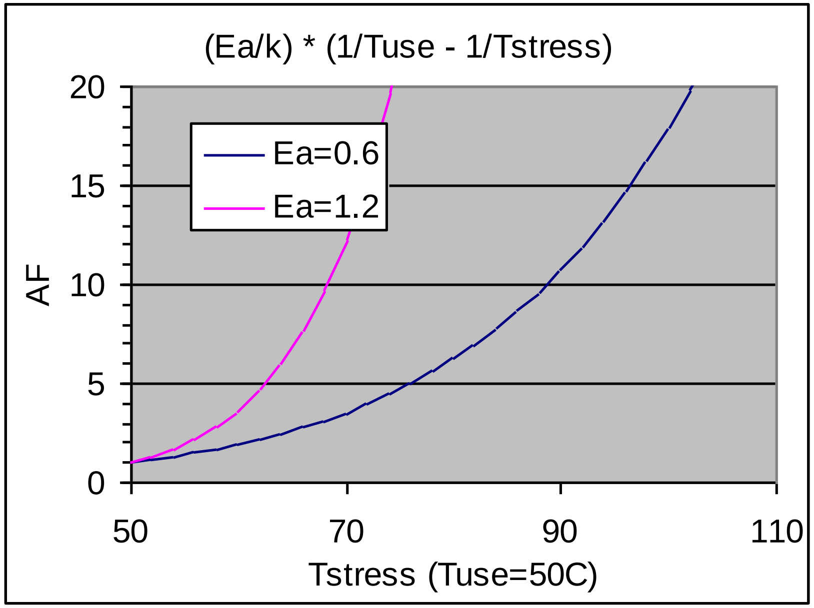
Voltage Acceleration Model



$$AF = \exp\left\{C \times \left(V_{stress} - V_{use}\right)\right\}$$

- Acceleration models are determined empirically
- Voltage acceleration is usually exponential, like this example

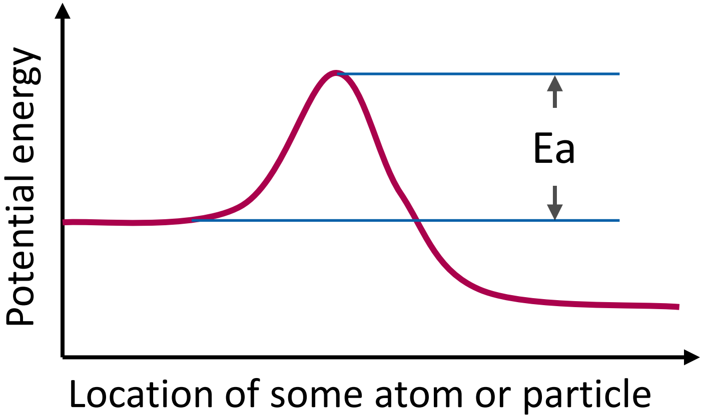
Temperature Acceleration Model



Activation energy

$$AF = \exp\left\{\left(\frac{E_a}{k}\right) \times \overbrace{\left(\frac{1}{T_{use}} - \frac{1}{T_{stress}}\right)}^{\text{Must be Kelvin}}\right\}$$

Boltzmann constant
$k=8.62\text{x}10^{-5}$ eV/K



- Temperature acceleration is usually like a chemical reaction
  - Arrhenius model with an energy barrier

Exercise 8.1

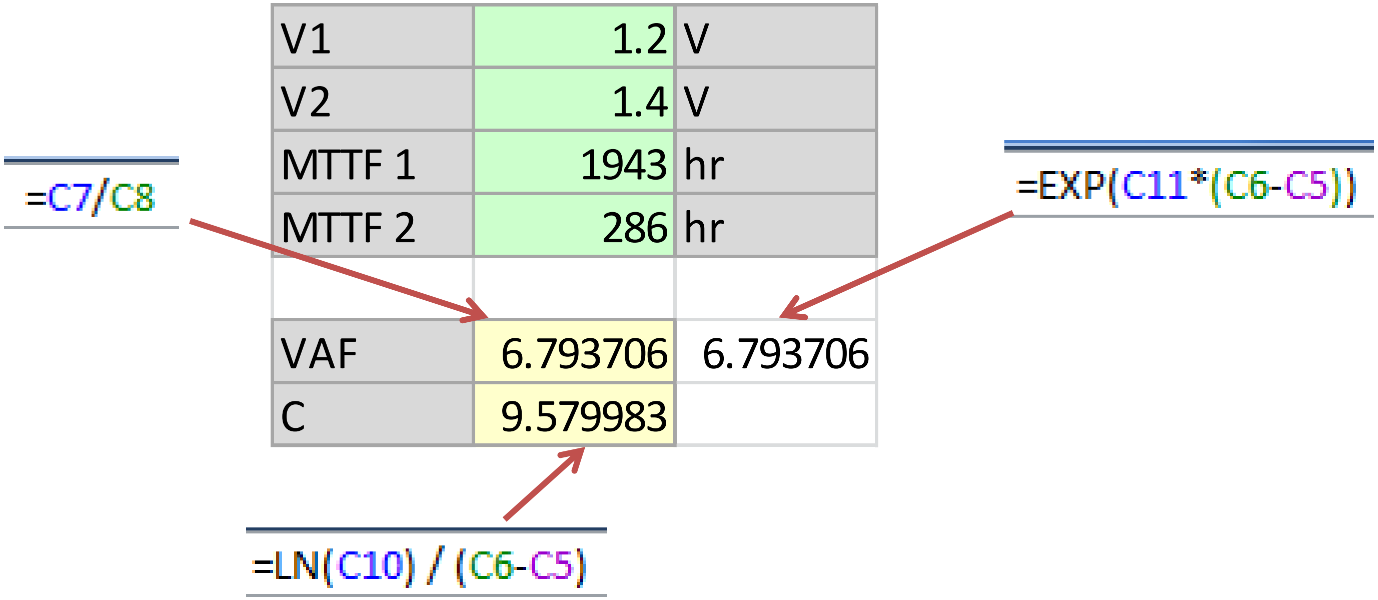
If two samples of devices give these MTTFs:

- 1943 hours at 1.2V
- 286 hours at 1.4 V

find the

- Voltage Acceleration Factor (VAF)
- Constant C in the an exponential voltage acceleration model

Solution 8.1



V1	1.2	V
V2	1.4	V
MTTF 1	1943	hr
MTTF 2	286	hr
VAF	6.793706	6.793706
C	9.579983	

=C7/C8

=EXP(C11*(C6-C5))

=LN(C10) / (C6-C5)

Exercise 8.2

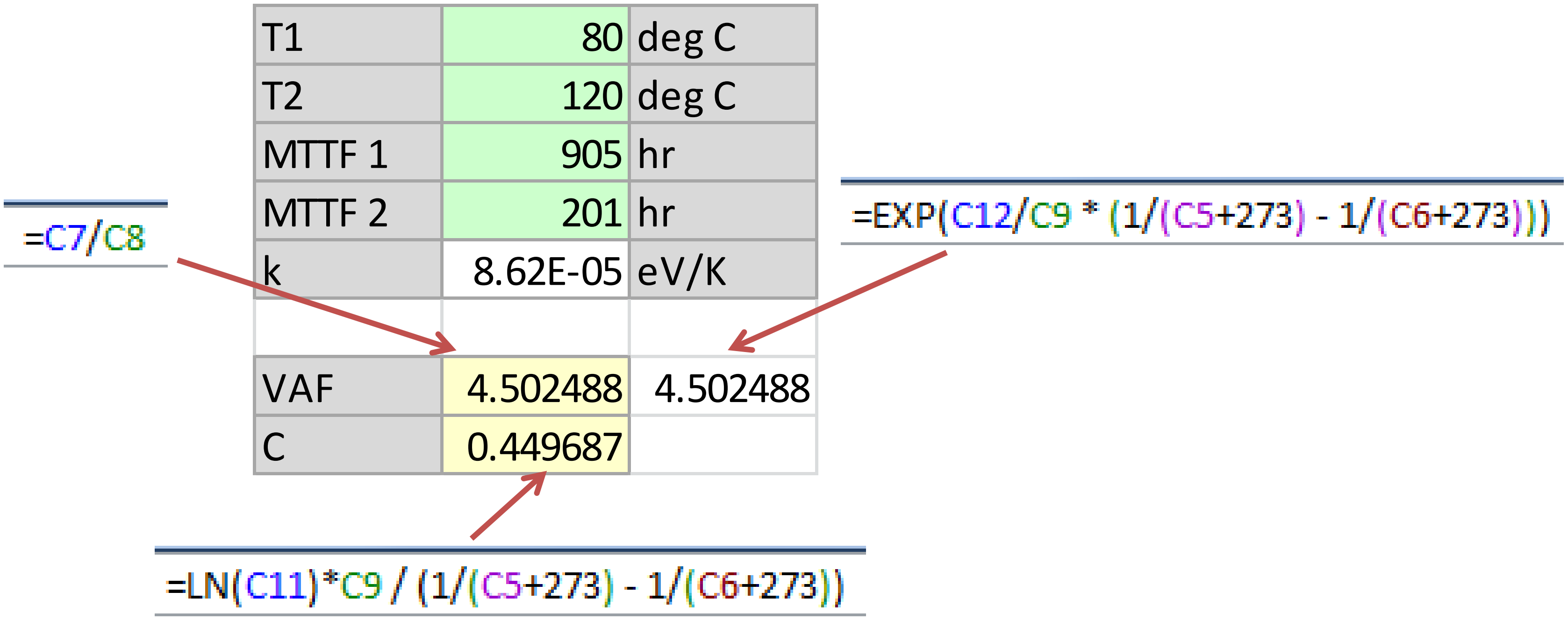
If two samples of devices give these MTTFs:

- 905 hours at 80 deg C
- 201 hours at 120 deg C

find the

- Temperature Acceleration Factor (TAF)
- Activation energy Ea in the an Arrhenius temperature acceleration model

Solution 8.2



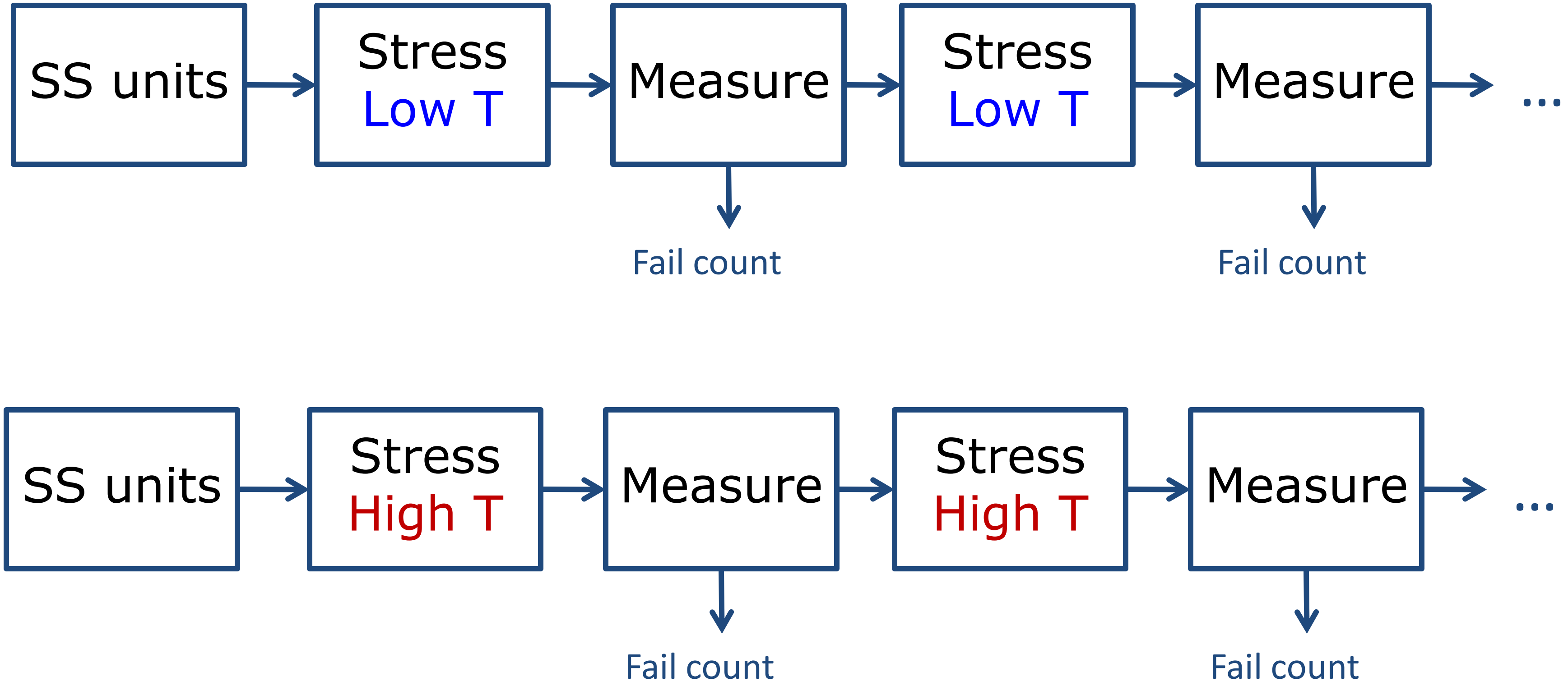
T1	80	deg C
T2	120	deg C
MTTF 1	905	hr
MTTF 2	201	hr
k	8.62E-05	eV/K
VAF	4.502488	4.502488
C	0.449687	

=C7/C8

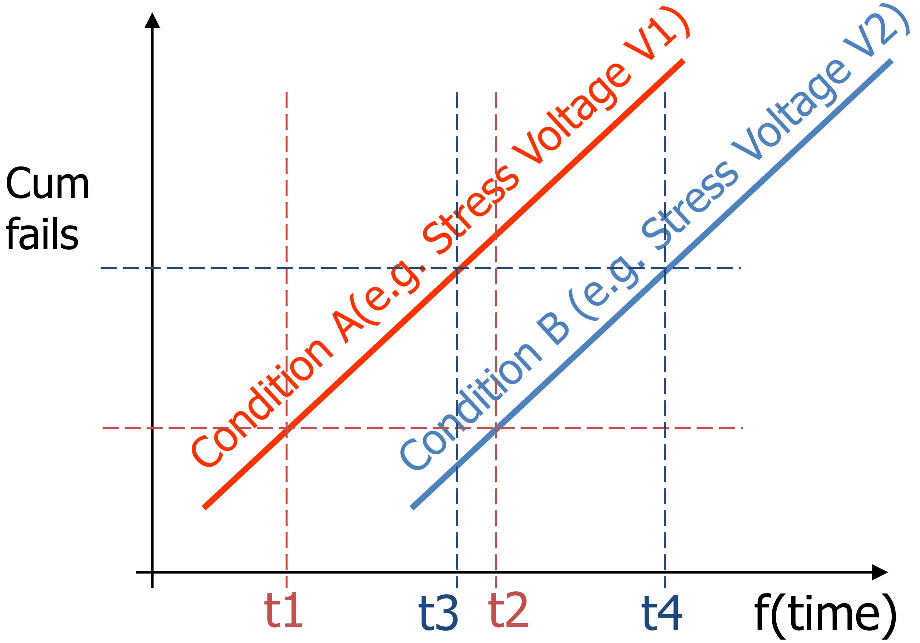
=EXP(C12/C9 * (1/(C5+273) - 1/(C6+273)))

=LN(C11)*C9 / (1/(C5+273) - 1/(C6+273))

Acceleration Experiment



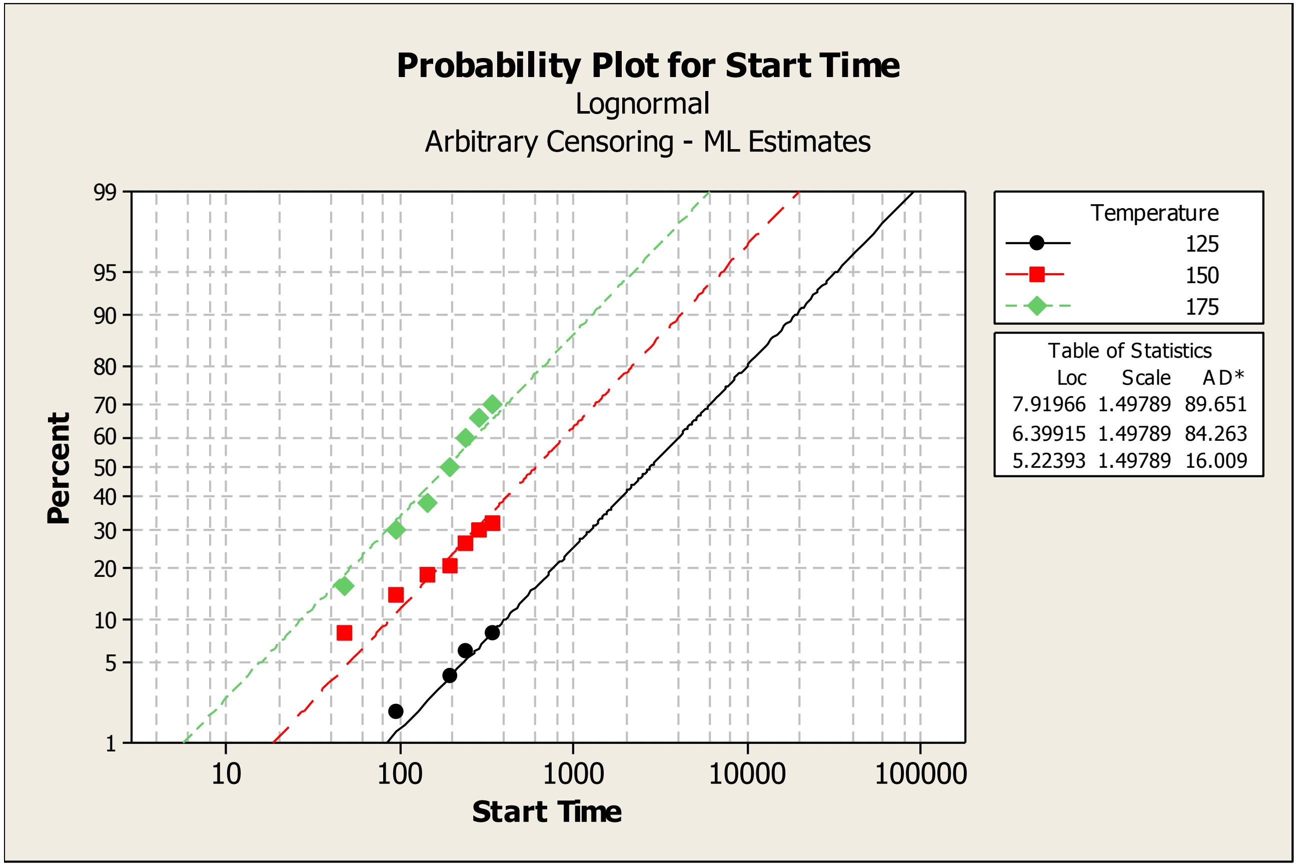
Acceleration Concept



$$AF = \frac{t2}{t1} = \frac{t4}{t3}$$

- Distributions at both conditions must match for acceleration concept to make sense

Acceleration Example



A temperature acceleration experiment showing the same distribution shape (slope) at each stress temp

Accelerated Stress Testing

- Special-purpose equipment accelerates various fail mechanisms

An LCBI burn-in system gives V and T stress to accelerate Si fail mechanisms

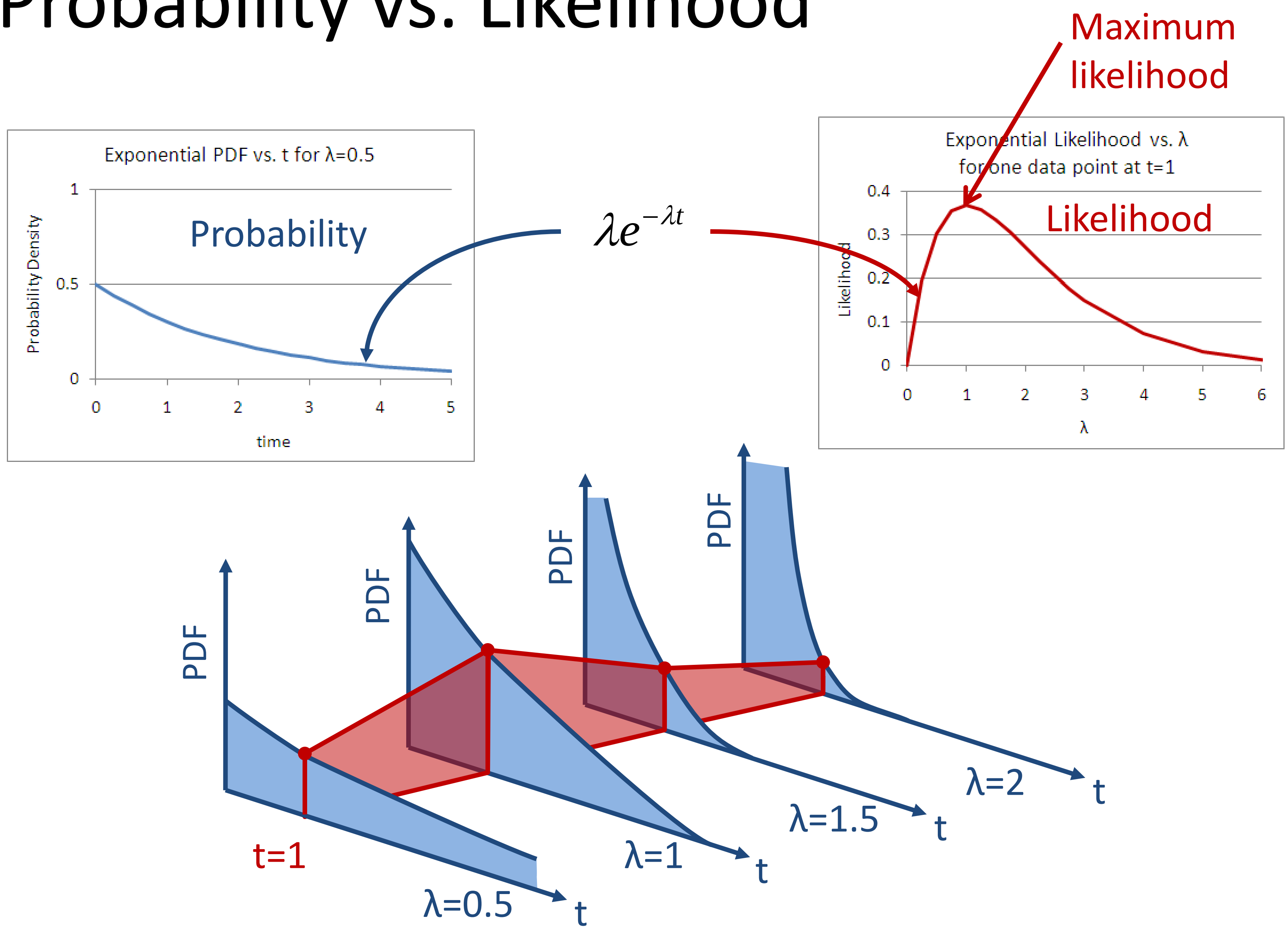
A HAST system gives pressure and humidity along with V and T to accelerate package fail mechanisms

Maximum Likelihood Method and the Exponential Distribution

MLE

- Maximum Likelihood Estimation (MLE) is a fitting technique that is good for any model
- Principle
  - We can't ask: What is the most likely model?
    - Because we don't have some well-defined space of possible models
  - We can ask: Given this model, how likely is this data set?
  - (This is a fairly Bayesian approach. We are usually frequentists.)

Probability vs. Likelihood



MLE

- Likelihood for each point
  - For exact values (exact times to fail), use the PDF
  - For ranges (failed between two readout times), use CDF delta
  - Multiply all together (or add logs)
- Use
  - Choose a model functional form with adjustable parameters
  - Adjust the parameters to maximize the likelihood

MLE for Exponential Data

- For a complete set of times to fail, likelihood is the PDF:

$$PDF_i = \lambda\, e^{-\lambda t_i}$$

- Take log of PDF:

$$\ln PDF_i = \ln \lambda - \lambda t_i$$

- Add up likelihood for each data point:

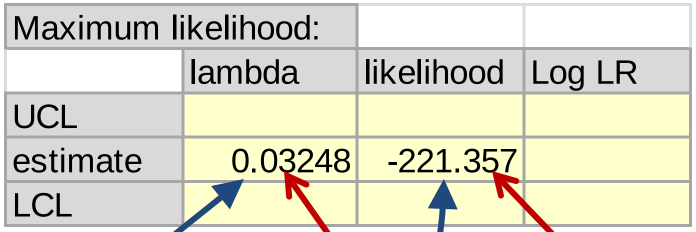
$$\text{Device hours} = \sum_i t_i$$

$$L = \sum_i \ln PDF_i = \sum_i \left(\ln \lambda - \lambda t_i \right) = N \ln \lambda - \lambda \sum_i t_i$$

$$\text{Sample Size} = N$$

- Then choose λ to maximize L

Ex 8.3a – MLE for Exponential



Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

guess

```
=$C$3*LN(F8) - F8*$C$4
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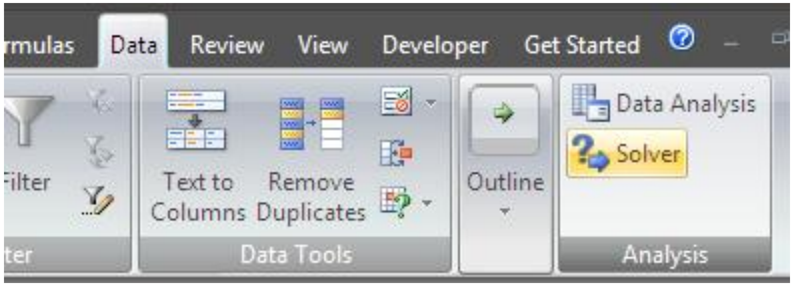
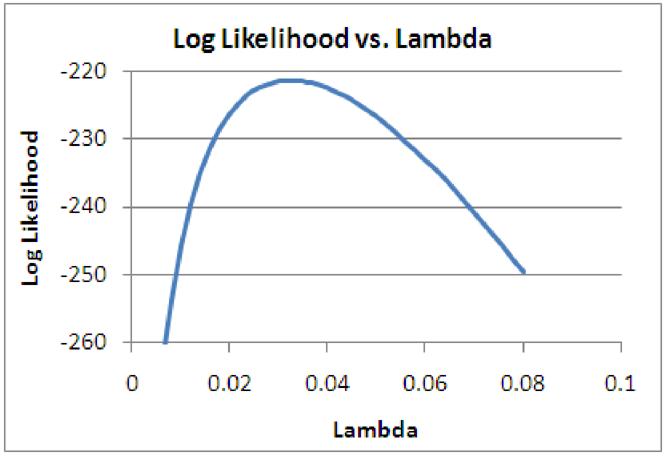
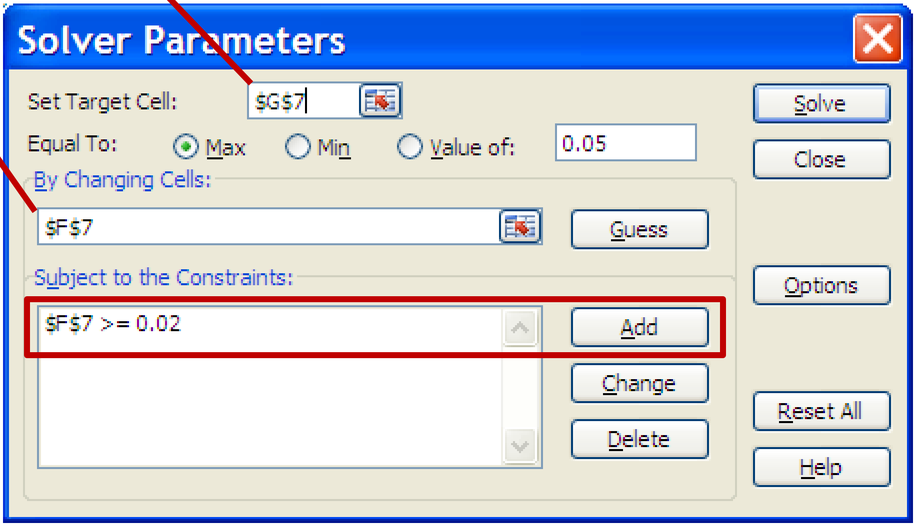






Solver Parameters

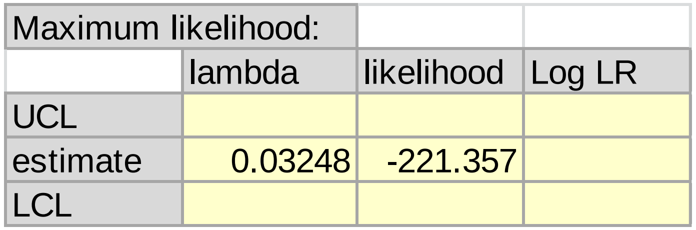
Set Target Cell: G7

Equal To: ● Max ○ Min ○ Value of: 0.05

By Changing Cells: F7

Subject to the Constraints: F7 >= 0.02

Solution 8.3a



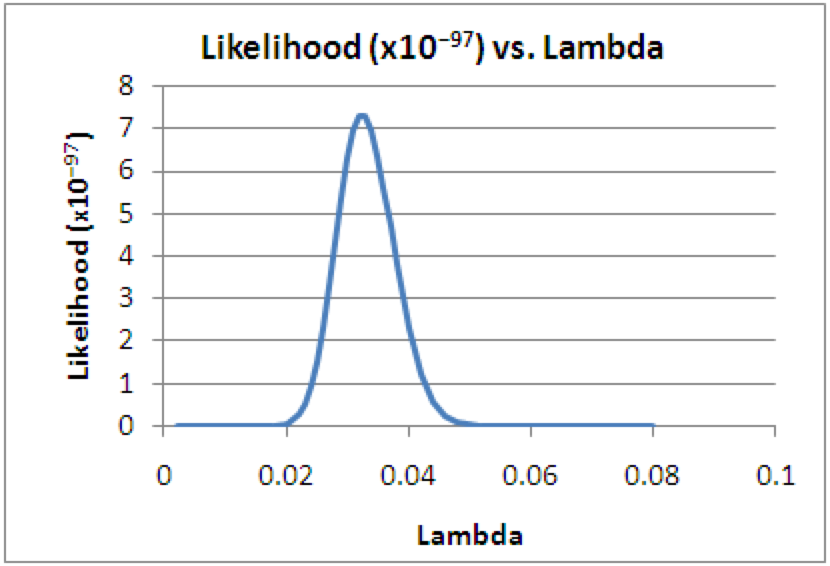
Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

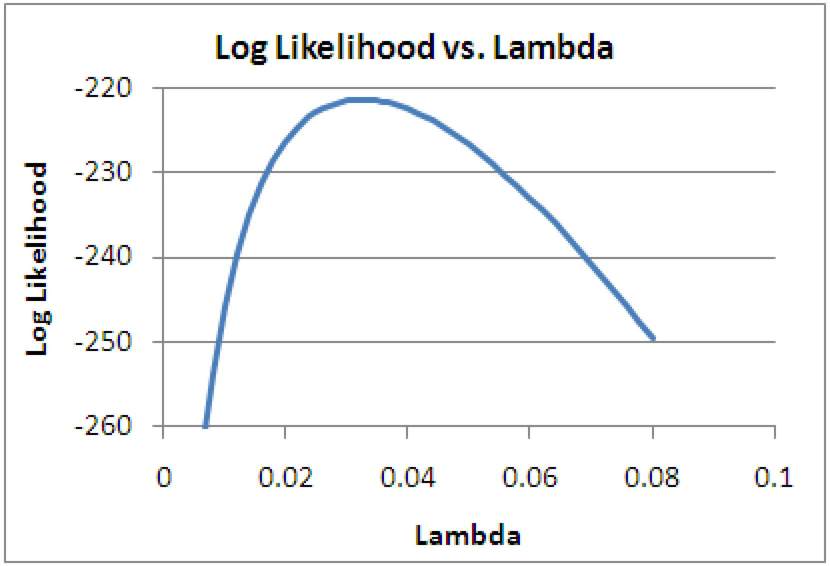
λ = 0.032 per hour = 3.2% per hour

MTTF = 1/λ = 30.8 hours

Graphs of Likelihood vs. Lambda

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			





Analytic λ

- For exponential, can maximize analytically:

$$L = N \ln \lambda - \lambda \sum_i t_i$$

$$\frac{dL}{d\lambda} = \frac{N}{\lambda} - \sum_i t_i = 0$$

$$\lambda = \frac{N}{\sum_i t_i} = \frac{\text{Number of fails}}{\text{Total device hours}}$$

Even works for type I censored data

Exercise 8.3b

- Calculate λ for the Ex 8.3 data set using the analytic expression and compare it to what you got from the MLE technique

Solution 8.3b

- Same as MLE technique

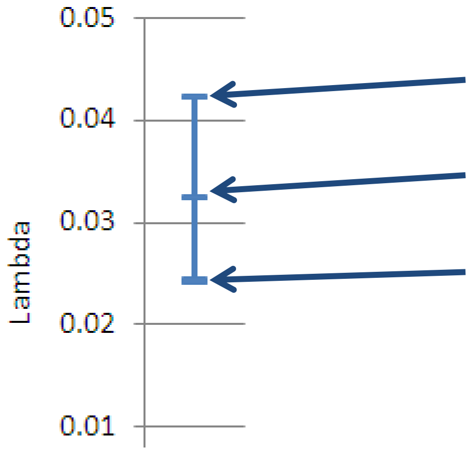
Analytic

fail count	50
device hours	1539.413
lambda (fails / dev hrs)	0.03248

MLE

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

Uncertainty Range of Lambda



Upper Confidence Limit (UCL)

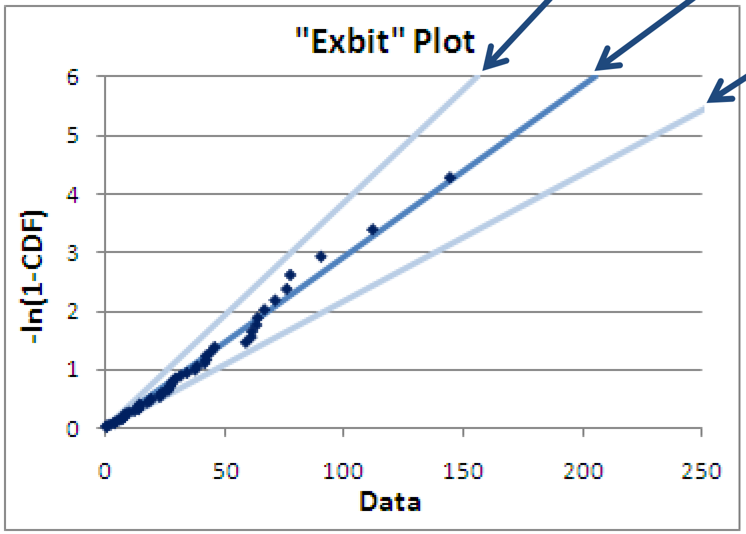
Best estimate

Lower Confidence Limit (LCL)

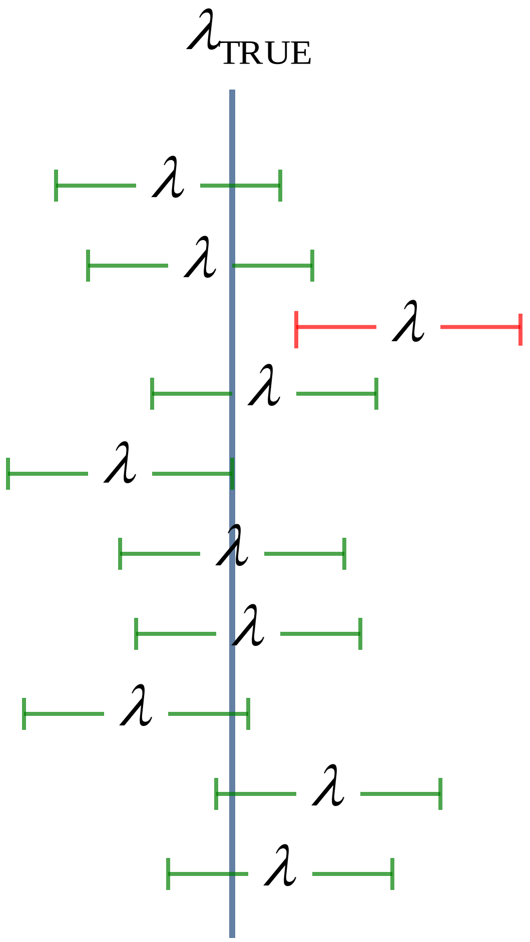
UCL

Best estimate

LCL

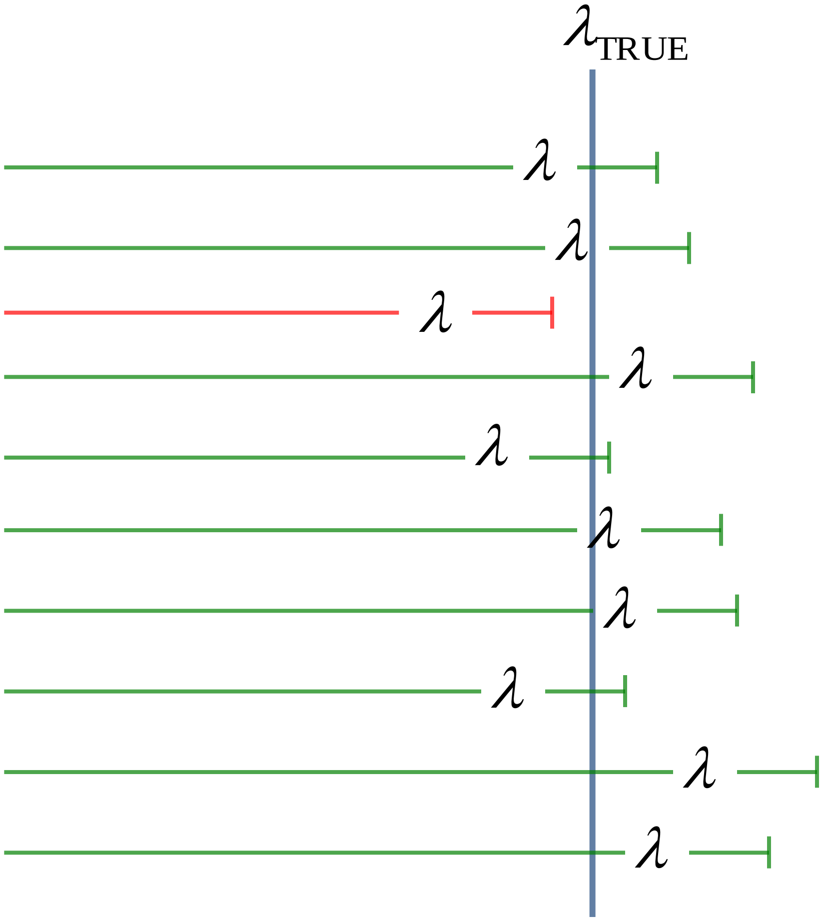


Confidence Interval (2-Sided)



- 90% of random sample λ's with this confidence interval include the true population λ

Confidence Interval (1-Sided)

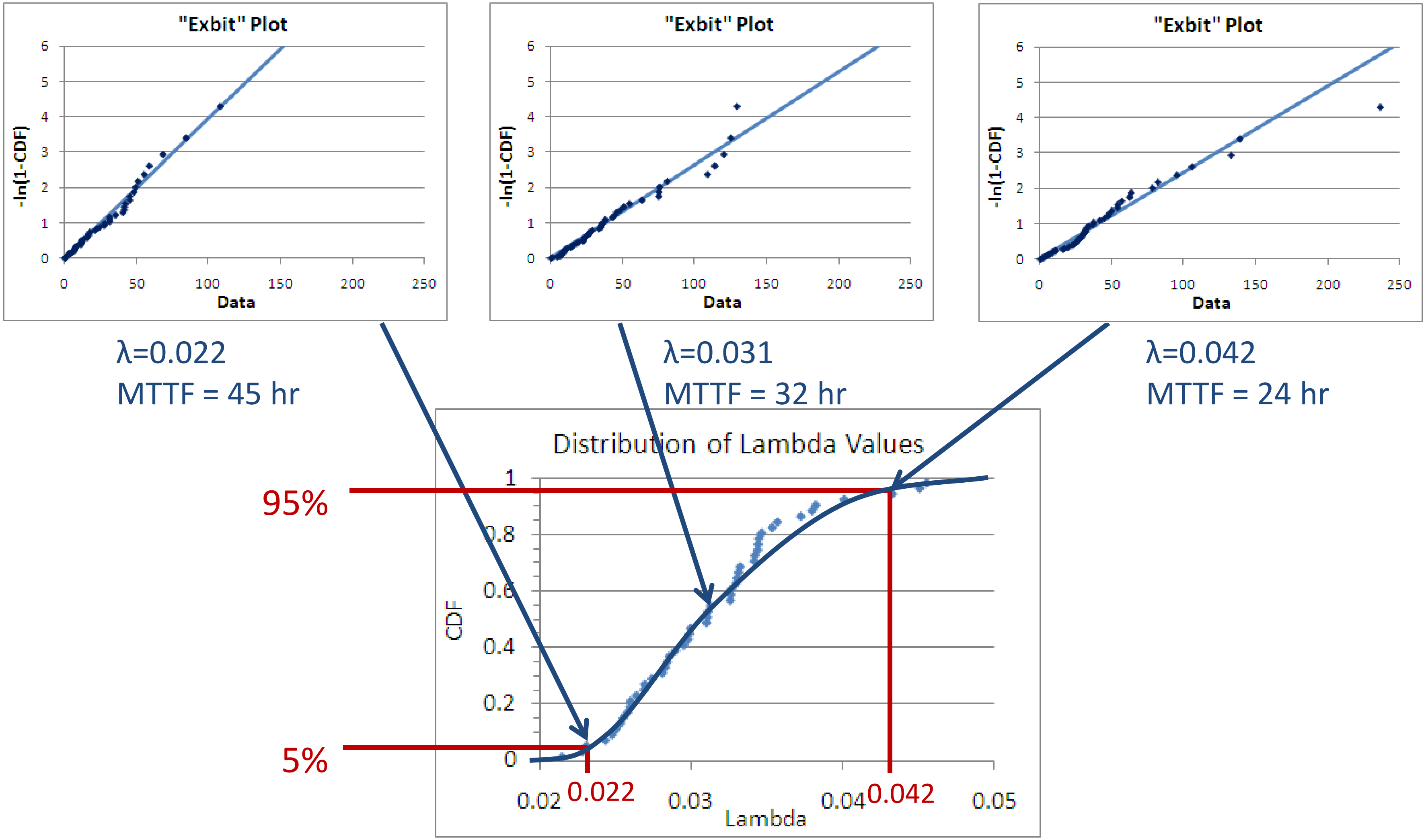


Uncertainties on Parameters

To calculate:

- Monte Carlo
- Likelihood ratio
- Analytic

Recall Monte Carlo Lambda Uncertainty



λ=0.022
MTTF = 45 hr

λ=0.031
MTTF = 32 hr

λ=0.042
MTTF = 24 hr

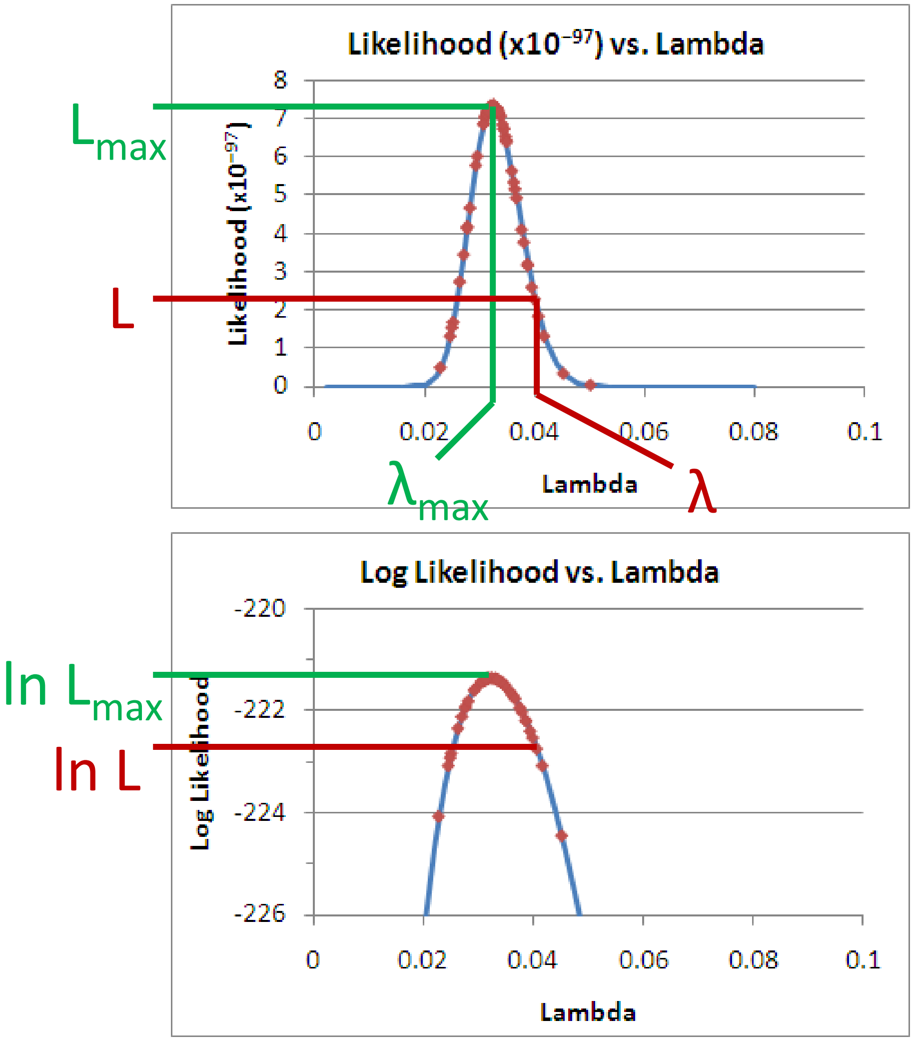
95%

5%

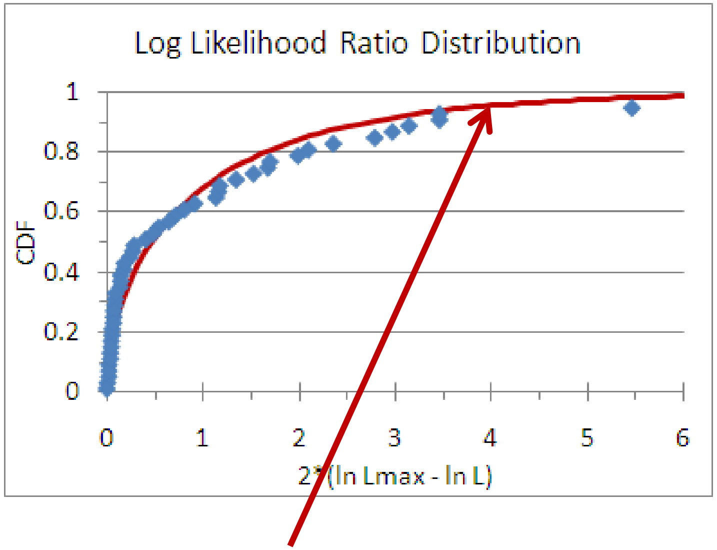
0.022

0.042

Likelihood Ratio Lambda Uncertainty



$$\ln\left(\frac{L_{\max}}{L}\right)^2 = 2\times\left(\ln L_{\max} - \ln L\right)$$

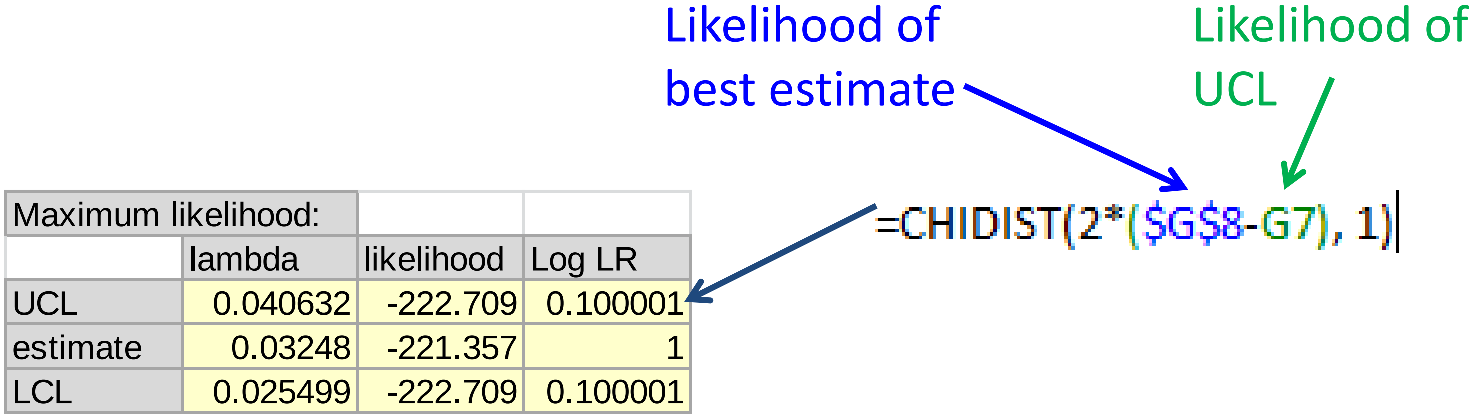


1–CHIDIST(Log LR, 1)

Number of parameters in model (=1 for exponential)

Exercise 8.3c

- Calculate UCL and LCL for lambda:
  - Calculate Log LR for each (below)
  - Choose lambda for each to set Log LR = 0.1
    - Do by hand first, then use Solver to fine-tune



Likelihood of best estimate

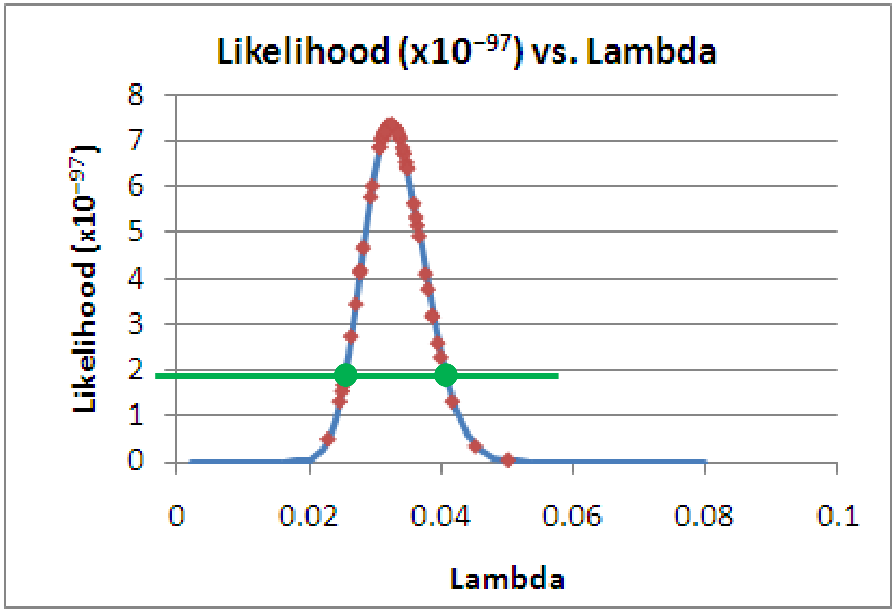
Likelihood of UCL

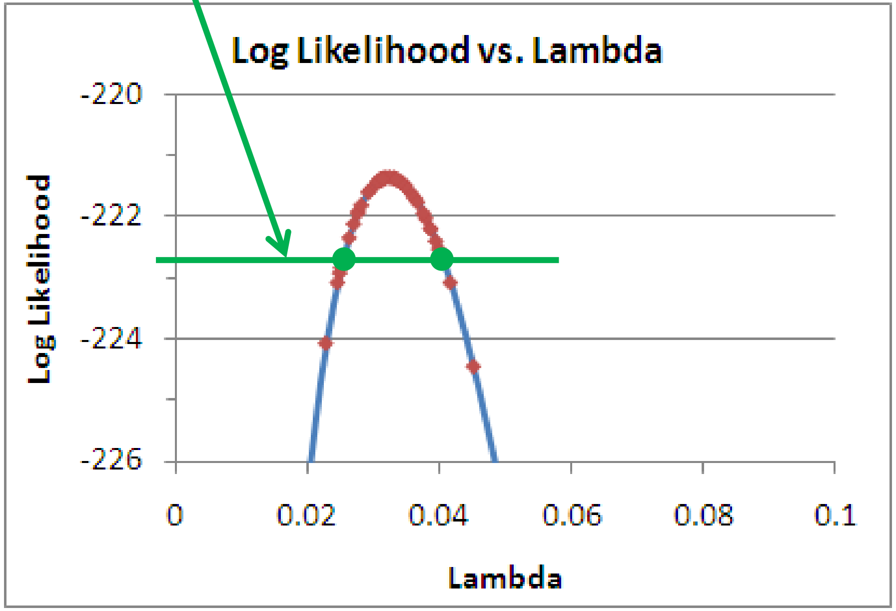
=CHIDIST(2*(G8-G7), 1)

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001

Solution 8.3c

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001





Analytic Lambda Uncertainty

for 90% CL

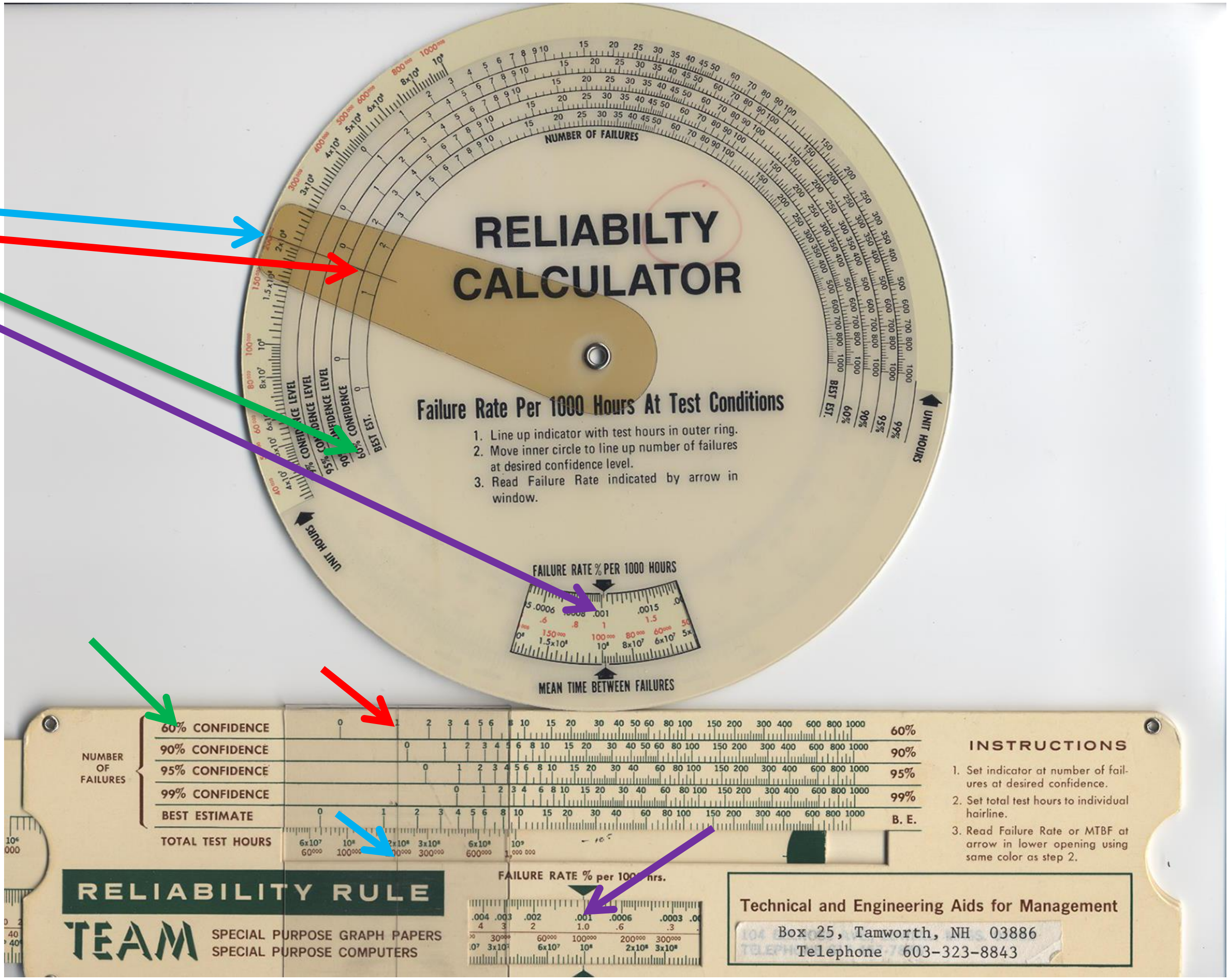
Note N+1 vs. N

$$\lambda_{UCL} = \lambda_{BE} \frac{\text{CHIINV}\left(5\%,\ 2(N+1)\right)}{2N}$$

$$\lambda_{LCL} = \lambda_{BE} \frac{\text{CHIINV}\left(95\%,\ 2N\right)}{2N}$$

Venerable Calculation

Reliability Calculator	
Device hours	200
Fails	1
Confidence	60%
Fail rate (%/hr)	1.01%



1. Line up indicator with test hours in outer ring.
2. Move inner circle to line up number of failures at desired confidence level.
3. Read Failure Rate indicated by arrow in window.

INSTRUCTIONS

1. Set indicator at number of failures at desired confidence.
2. Set total test hours to individual hairline.
3. Read Failure Rate or MTBF at arrow in lower opening using same color as step 2.

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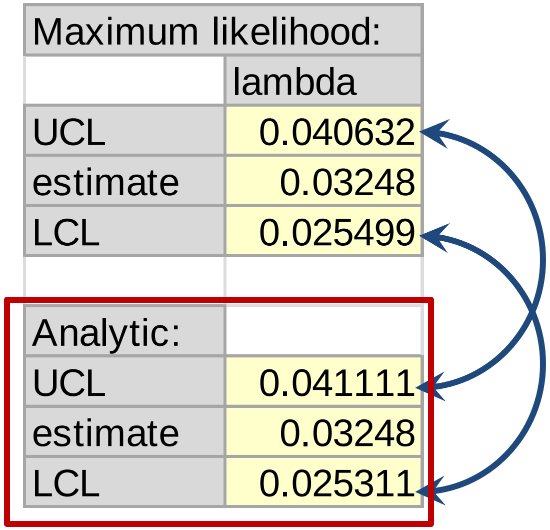
Exercise 8.3d

- Calculate lambda UCL and LCL analytically

Solution 8.3d



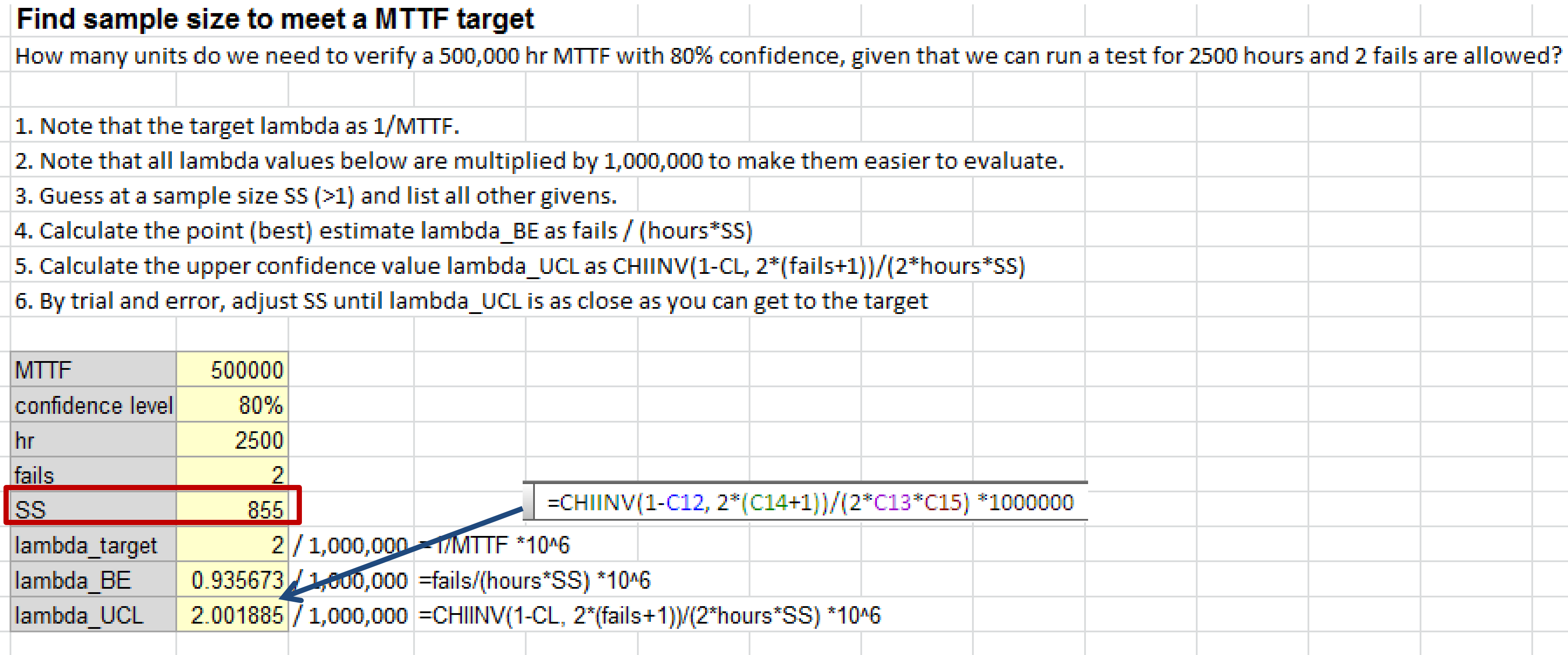
Maximum likelihood:	
	lambda
UCL	0.040632
estimate	0.03248
LCL	0.025499

Analytic:	
UCL	0.041111
estimate	0.03248
LCL	0.025311

Exercise 8.4

- This is Tobias & Trindade problem 3.1
- How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?
- Hint: you can do this by trial and error. Calculate the UCL on λ as a function of sample size SS and then adjust SS until the UCL equals the target λ.

Solution 8.4



Find sample size to meet a MTTF target

How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?

1. Note that the target lambda as 1/MTTF.
2. Note that all lambda values below are multiplied by 1,000,000 to make them easier to evaluate.
3. Guess at a sample size SS (>1) and list all other givens.
4. Calculate the point (best) estimate lambda_BE as fails / (hours*SS)
5. Calculate the upper confidence value lambda_UCL as CHIINV(1-CL, 2*(fails+1))/(2*hours*SS)
6. By trial and error, adjust SS until lambda_UCL is as close as you can get to the target

MTTF	500000		
confidence level	80%		
hr	2500		
fails	2		
SS	855		
lambda_target	2	/ 1,000,000	=1/MTTF *10^6
lambda_BE	0.935673	/ 1,000,000	=fails/(hours*SS) *10^6
lambda_UCL	2.001885	/ 1,000,000	=CHIINV(1-CL, 2*(fails+1))/(2*hours*SS) *10^6

=CHIINV(1-C12, 2*(C14+1))/(2*C13*C15) *1000000

The End