

ECE 510 Lecture 7

Goodness of Fit, Maximum Likelihood

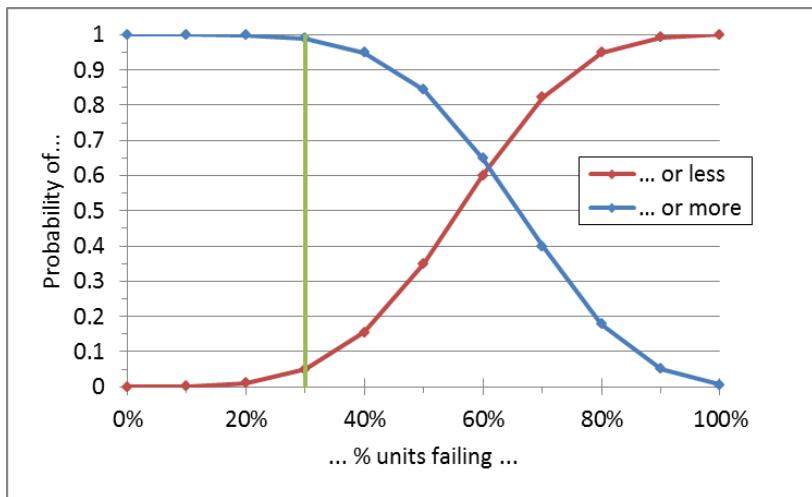
Scott Johnson

Glenn Shirley

Confidence Limits

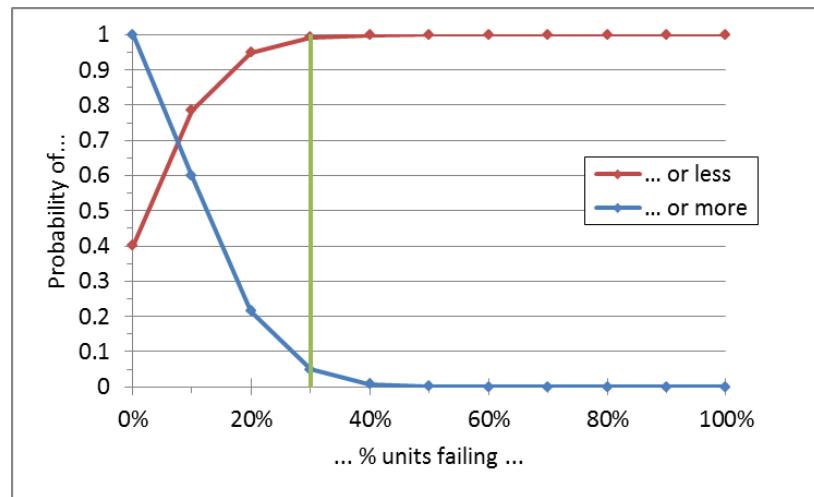
Binomial Confidence Limits (Solution 6.2)

UCL: Prob of 30% units failing *or less* is < 0.05



$$\text{UCL} = 60.7\%$$

LCL: Prob of 30% units failing *or more* is < 0.05



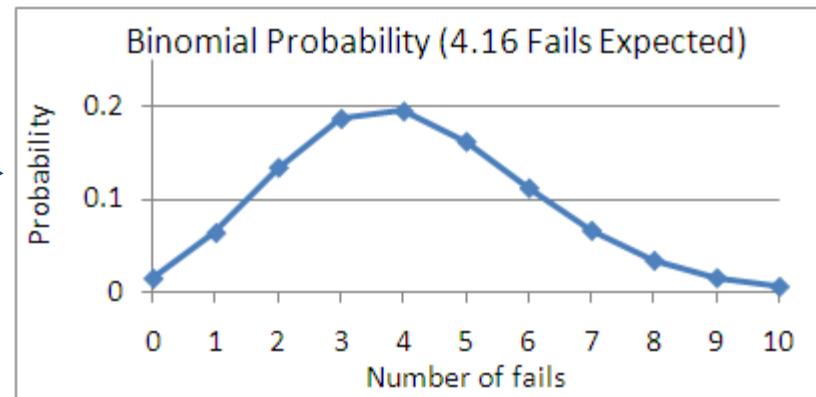
$$\text{LCL} = 8.7\%$$

Synthesizing Binomial Data

1000 units

0.416% prob of fail

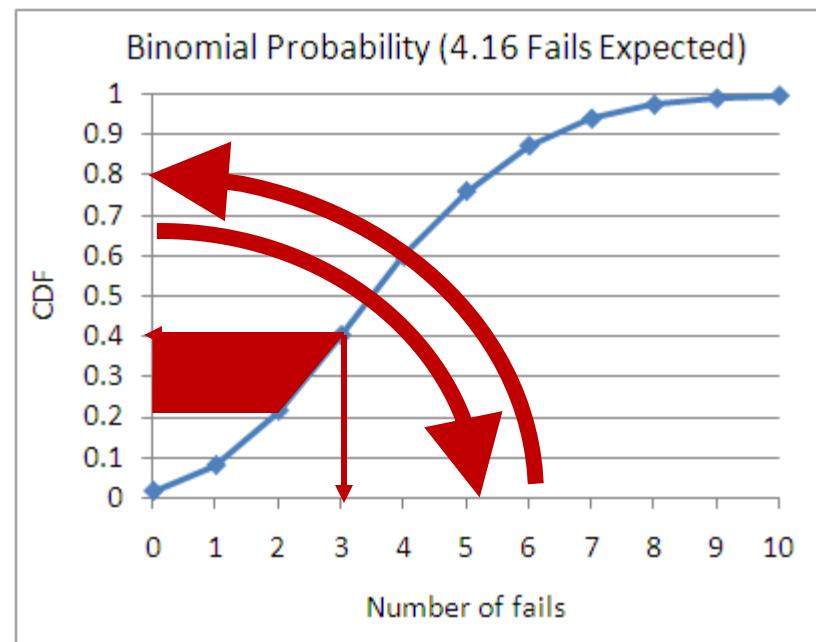
4.16 units expected



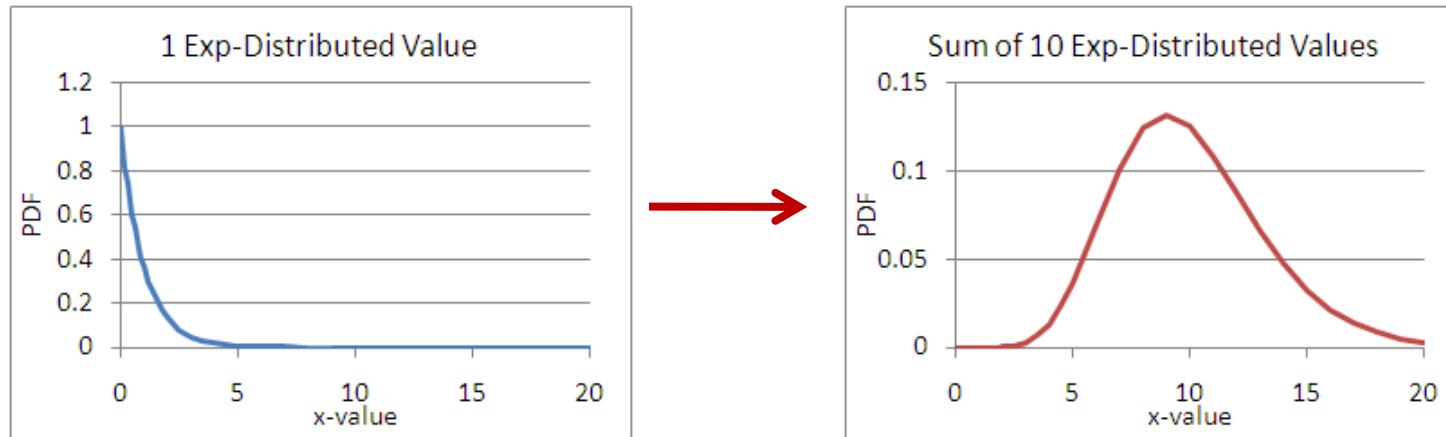
BINOMDIST(fails, samples, prob, TRUE)

CRITBINOM(samples, prob, CDF)

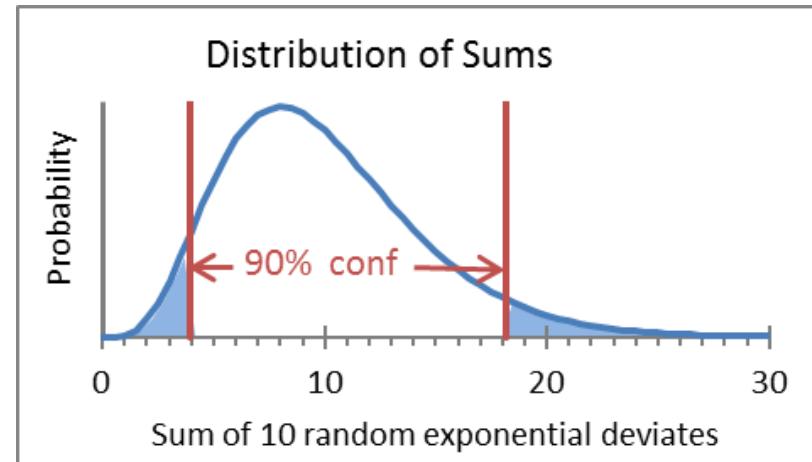
RAND()



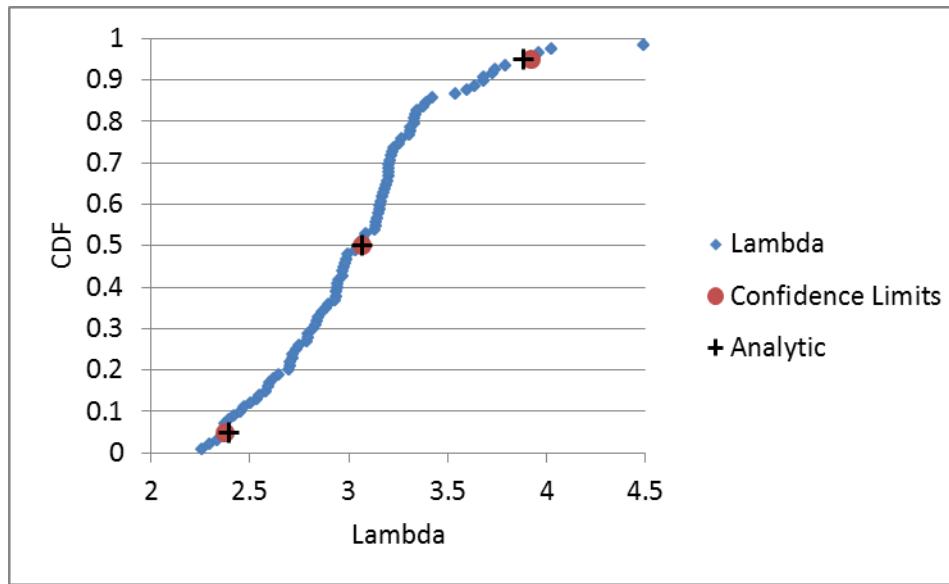
Why Chi-Square for Exponential CL?



- For $f(t) = \lambda e^{-\lambda t}$, best estimate for $1/\lambda$ is $\frac{1}{N} \sum t_i$ where t_i are the data
- So, what is the *distribution* of $\sum t_i$ where t_i are distributed exponentially?
- Answer: a gamma or a chi-square distribution
- Confidence intervals taken from that



Solution 6.3



Confidence Limits	CL values	MC	Analytic
Upper CL	0.95	3.920005	3.884124
Best estimate	0.5	3.068655	3.068655
Lower CL	0.05	2.376862	2.391386

=LARGE(\$C\$24:\$C\$123, 100*(1-C12))

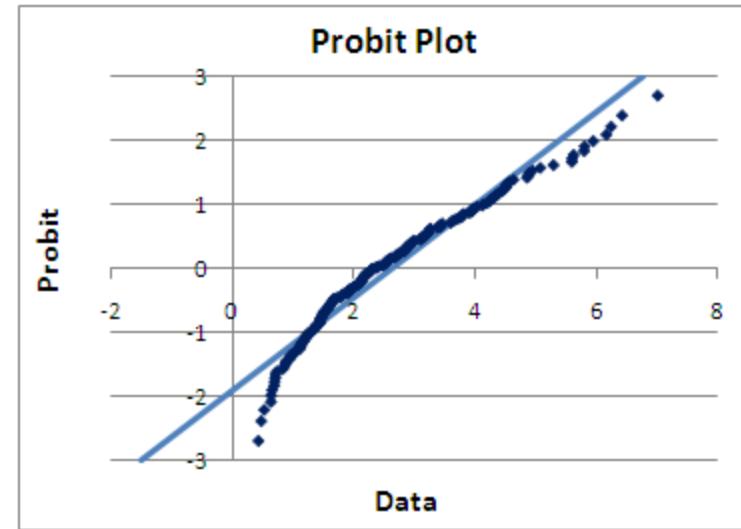
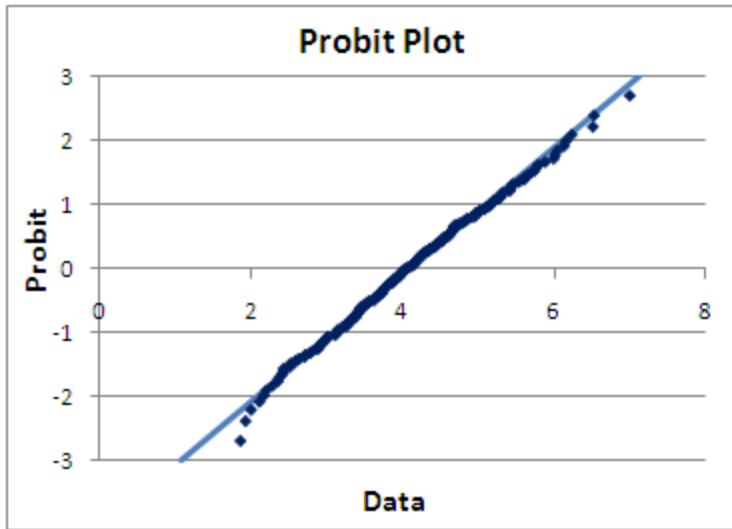
=E11 * CHIINV(95%, 2*(\$C\$5))/(2*\$C\$5)

Confidence Limits Summary

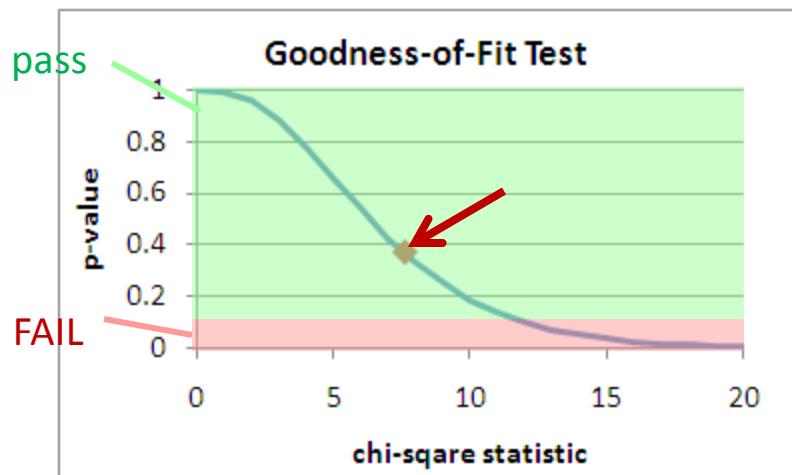
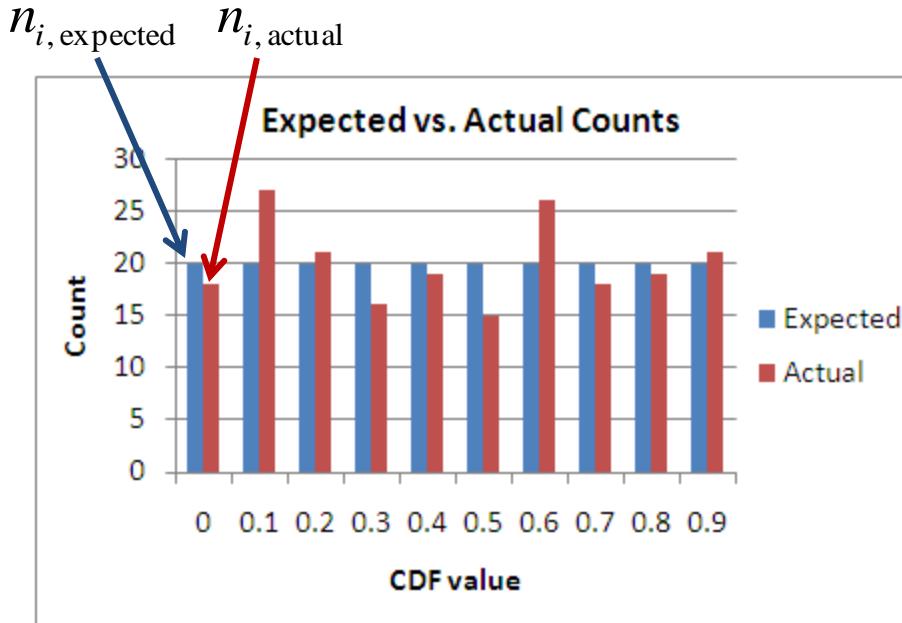
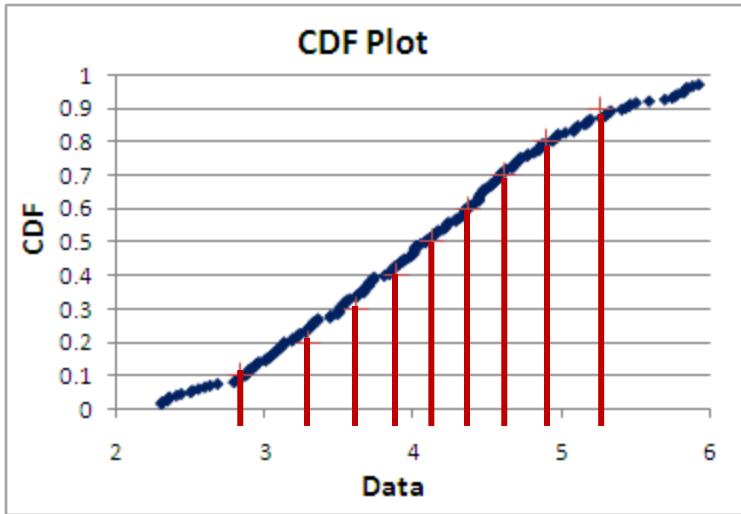
- Confidence limits (UCL and LCL) are values between which (2-sided) or above or below which (1-sided) the true population value falls with <confidence level> probability
 - Units are whatever units your data uses
- Confidence level is the probability that the true value lies between (or above or below) your confidence limit(s).
- “CL” can mean either confidence limit or confidence level
 - Use context to decide
- Confidence limits can be calculated
 - Analytically (best if available)
 - Monte Carlo (will work for any distribution)
 - Likelihood methods (coming soon)
- Monte Carlo confidence limits work regardless of how you calculate the best estimate

Goodness of Fit Tests

How Good Is Good Enough?



Pearson's Chi-Squared Test



$$\chi_i^2 = \frac{(n_{i, \text{actual}} - n_{i, \text{expected}})^2}{n_{i, \text{expected}}}$$

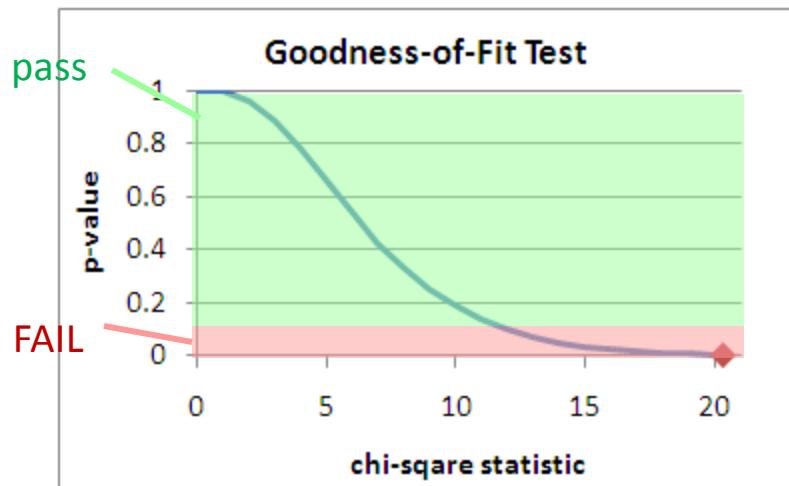
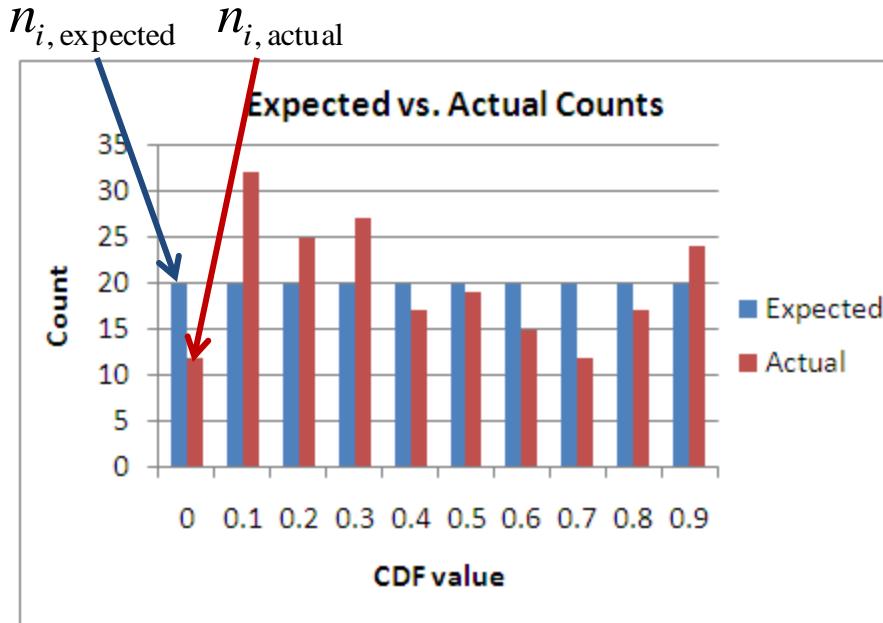
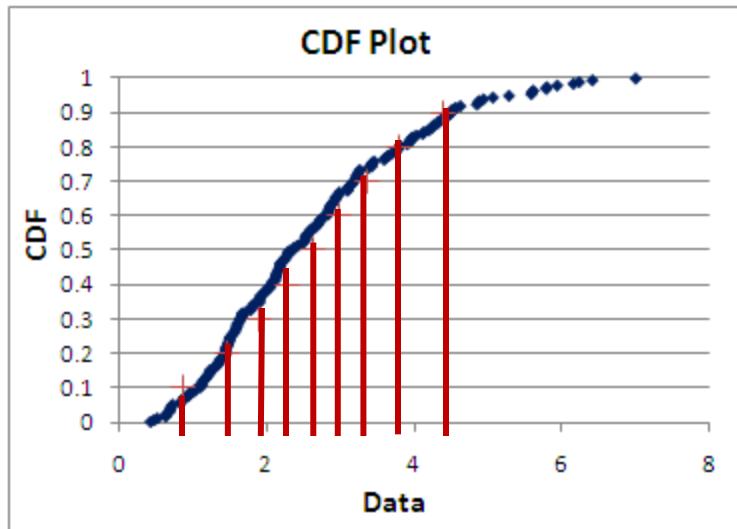
Chi-Sq:

$$\chi^2 = \sum_i \chi_i^2$$

DoF = bins – parameters – 1 = 7

p-value = CHIDIST(Chi-Sq, DoF)
= 0.378

Pearson's Chi-Squared Test



$$\chi_i^2 = \frac{(n_{i, \text{actual}} - n_{i, \text{expected}})^2}{n_{i, \text{expected}}}$$

Chi-Sq:
 $\chi^2 = \sum_i \chi_i^2$

$$\text{DoF} = \text{bins} - \text{parameters} - 1 = 7$$

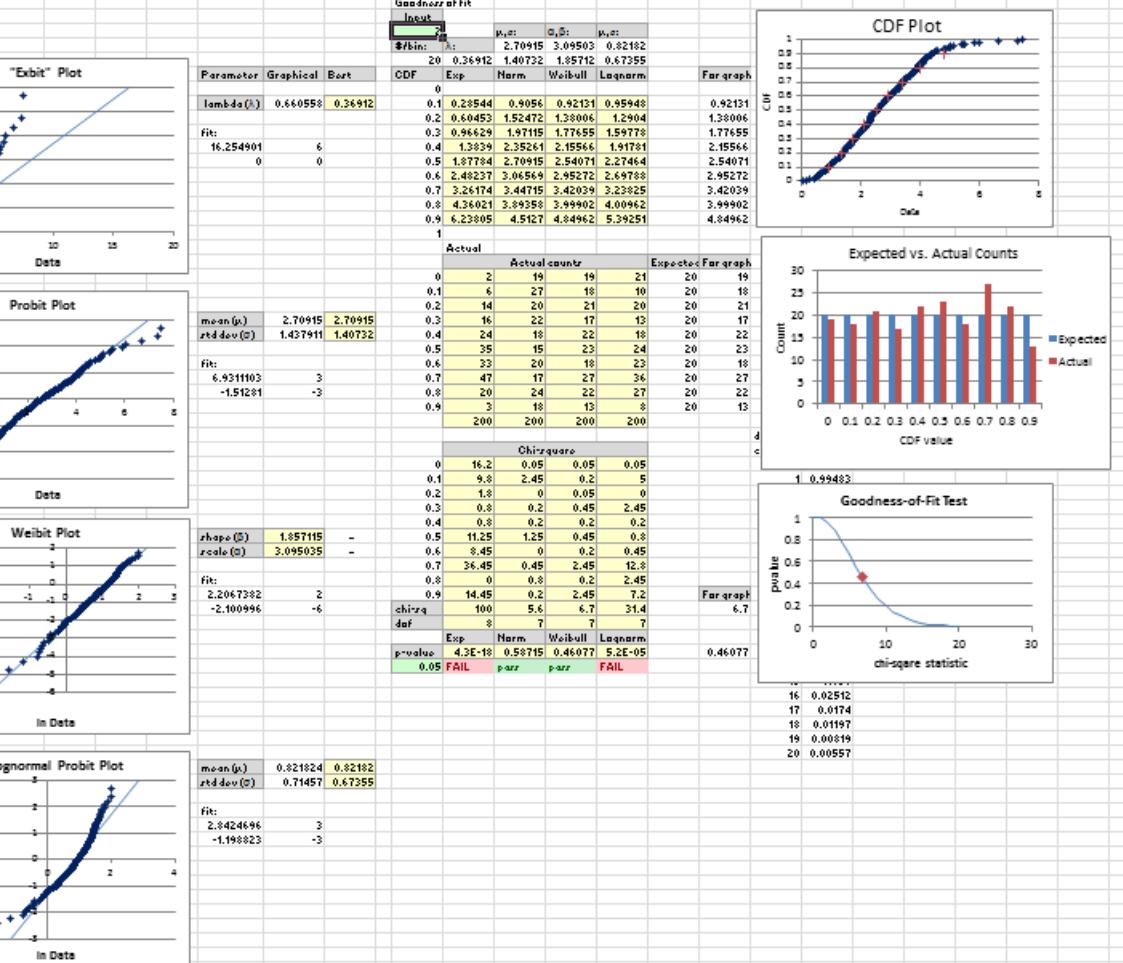
$$\begin{aligned} \text{p-value} &= \text{CHIDIST}(\text{Chi-Sq}, \text{DoF}) \\ &= 0.005 \end{aligned}$$

Exercise 7.1

- Add a chi-square goodness-of-fit test for each of the 4 fits in our synthesized data sheet

Add goodness-of-fit indicators for each of the 4 fits.

	mean(μ)	scale(σ)	mean(μ)
lambda(λ)	4	3	0.5
lambda(λ)	std dev(σ)	shape(S)	std dev(σ)
lambda(λ)	3	1	2
Exponential	Normal	Weibull	Lognormal
0.318701	4.207795	3.624966	1.672404
0.351963	0.73703	0.6342	1.3357
0.301261	3.004225	2.932833	0.170856
0.6804791	2.392992	0.59731	2.4368 0.0958
0.6976611	4.177201	3.126259	19.078085
0.5241005	3.149928	0.2253048	3.12659 0.63723
0.2125874	4.714864	2.71226	0.805152
0.3185871	3.419264	1.792177	0.527318
0.3477967	4.789384	1.716395	4.0770727
0.0922595	3.403283	0.489734	3.9275798
0.1192647	4.336124	3.331676	1.4509559
0.008063	0.459966	0.452853	0.3949592
0.4244995	1.812265	2.875728	2.754472
0.2661626	3.415767	2.048623	0.0078159
0.2411963	2.75811	2.625243	17.595402
0.8399583	0.766589	1.157399	0.7730567
0.2794041	4.110522	1.1504071	0.20226 0.34281
0.0074612	4.49711	1.5232357	1.522944 0.22804
0.1334514	3.62025	3.305001	9.447429
0.7357875	5.459598	2.725951	5.6126988
0.7357628	2.459637	2.899278	9.9073996
0.1446831	4.194395	4.422177	26.35742
0.5336618	2.009527	3.942803	3.2393936
0.3563451	0.465175	0.96123	1.8546729
0.0646963	3.357514	2.624774	1.3401621
0.7227827	4.486557	2.077581	1.3128827
0.3440088	2.204043	2.190298	0.6410772
0.5504468	3.250205	3.032373	-0.2983 0.48246
0.5072709	3.909429	4.81678	1.3078411
0.1845664	3.79107	0.477115	1.20372 0.15327
0.1562732	5.122024	1.96202	1.3169108
0.7871401	4.213805	6.701622	1.50179 0.22305
2.1646872	2.247382	3.116464	0.8630515
0.0219507	4.121593	1.00961	4.12159 0.32457
0.0690615	4.732166	4.1502	3.3473058
0.1016227	5.456174	3.259945	5.04232 0.94651
1.0542222	2.956487	5.405454	0.4842152
0.1860954	3.547242	3.299582	1.73242 0.27794
0.2664104	4.734773	1.992554	0.7936475
0.0032292	4.447589	4.617146	0.0371791
0.1266255	3.259162	1.040899	0.345051
0.0219507	4.121593	1.00961	4.12159 0.32457
0.0690615	4.732166	4.1502	3.3473058
0.1016227	5.456174	3.259945	5.04232 0.94651
1.0542222	2.956487	5.405454	0.4842152
0.1860954	3.547242	3.299582	1.73242 0.27794
0.2664104	4.734773	1.992554	0.7936475
0.0032292	4.447589	4.617146	0.0371791
0.1266255	3.259162	1.040899	0.345051
0.2604715	3.740324	1.879556	0.9463457
0.0509482	4.402913	0.89172	4.40291 0.47574
0.0706513	3.745383	3.124621	22.950725
0.1922243	4.241197	4.11916	0.0776164
0.1887931	4.344968	0.866831	0.144132
0.3794859	3.693642	4.167142	4.6071958
0.2898459	2.891765	3.431927	1.6207509
0.2119566	5.605272	4.336393	0.071108
0.4577802	4.201193	2.234923	1.061206
0.6815451	4.22241	2.392992	1.3249523
0.0801341	4.164594	0.949732	0.6959636
0.4941128	1.62924	2.359178	2.4712928
0.0561471	3.946183	2.772711	0.245146
0.0663324	4.076779	3.659458	0.4222487
0.5193401	5.605272	4.336393	0.0688742
0.4957288	4.692205	3.430263	0.224761
0.3506045	3.506308	0.659743	5.6252301
1.2011004	3.110226	3.509635	9.854519
0.7405229	1.937653	3.16732	0.5079244
0.6362625	2.056647	3.60526	0.6531912
0.3667376	3.226276	2.355957	1.112145
0.3048202	2.851045	2.395756	1.1117204



Other Goodness-of-Fit Tests

Exponential

Goodness-of-Fit Test

Kolmogorov's D

D	Prob>D
0.097489	> 0.1500

Note: H_0 = The data is from the Exponential distribution. Small p-values reject H_0 .

Weibull

Goodness-of-Fit Test

Cramer-von Mises W Test

W-Square	Prob>W^2
0.029696	> 0.2500

Note: H_0 = The data is from the Weibull distribution. Small p-values reject H_0 .

Normal

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.947533	0.0270*

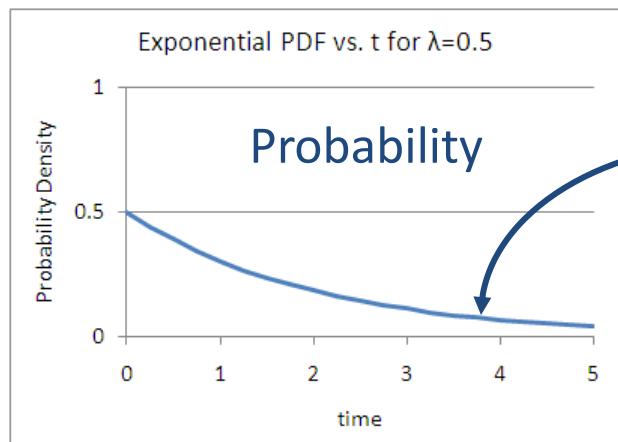
Note: H_0 = The data is from the Normal distribution. Small p-values reject H_0 .

Maximum Likelihood Method and the Exponential Distribution

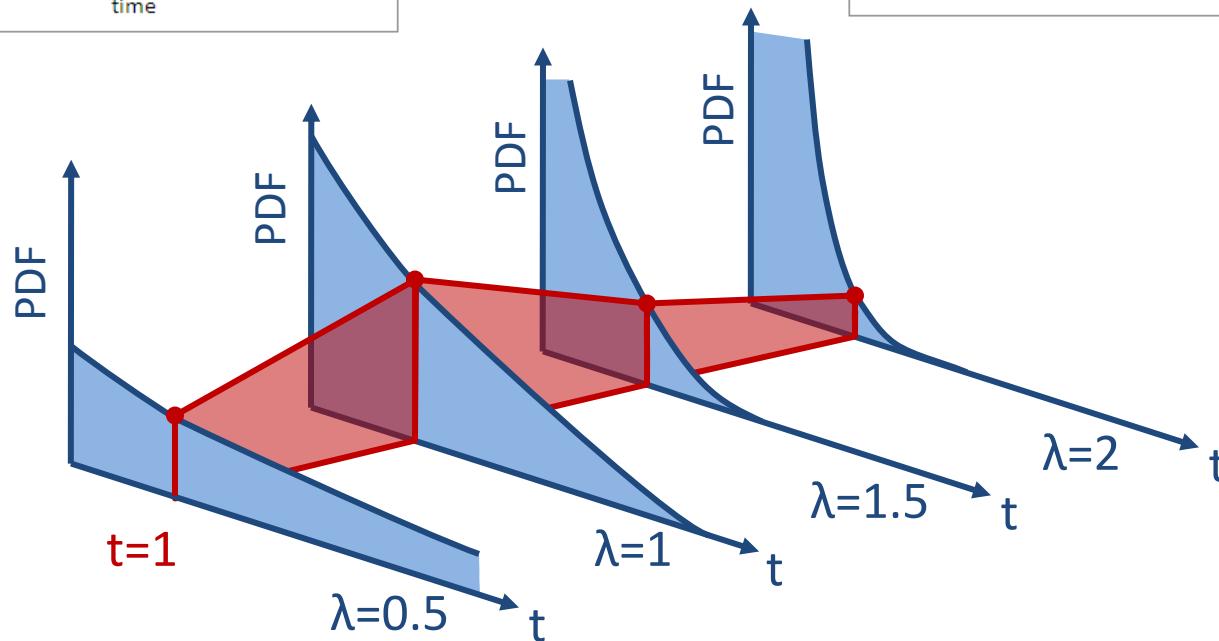
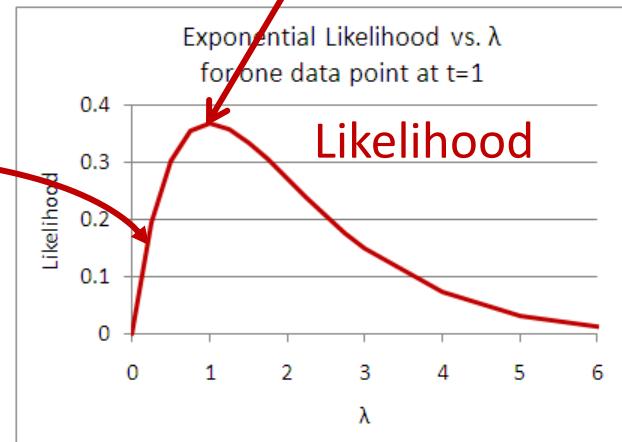
MLE

- Maximum Likelihood Estimation (MLE) is a fitting technique that is good for any model
- Principle
 - We can't ask: What is the most likely model?
 - Because we don't have some well-defined space of possible models
 - We can ask: Given this model, how likely is this data set?
 - (This is a fairly Bayesian approach. We are usually frequentists.)

Probability vs. Likelihood



$$\lambda e^{-\lambda t}$$



MLE

- Likelihood for each point
 - For exact values (exact times to fail), use the PDF
 - For ranges (failed between two readout times), use CDF delta
 - Multiply all together (or add logs)
- Use
 - Choose a model functional form with adjustable parameters
 - Adjust the parameters to maximize the likelihood

The End