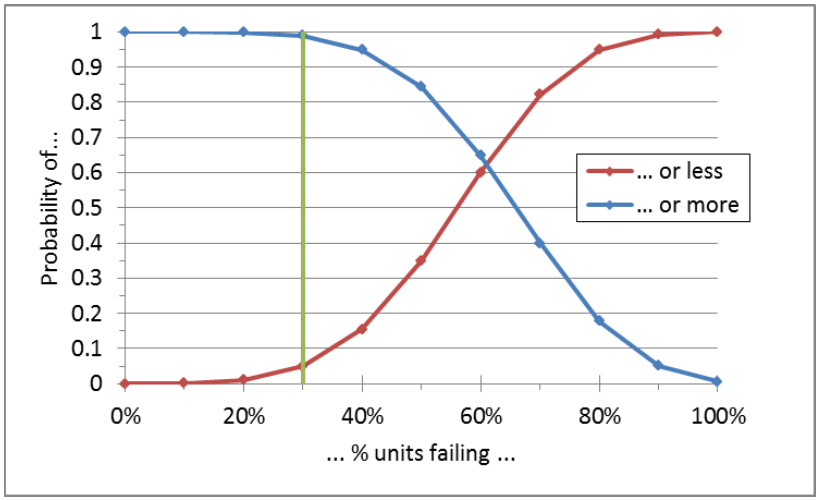
ECE 510 Lecture 7
Goodness of Fit, Maximum Likelihood

Scott Johnson
Glenn Shirley

Confidence Limits

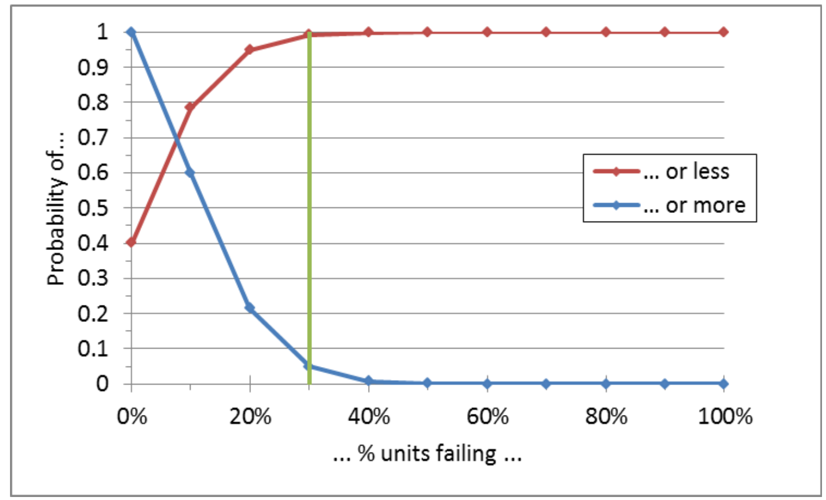
Binomial Confidence Limits (Solution 6.2)

UCL: Prob of 30% units failing *or less* is < 0.05



UCL = 60.7%

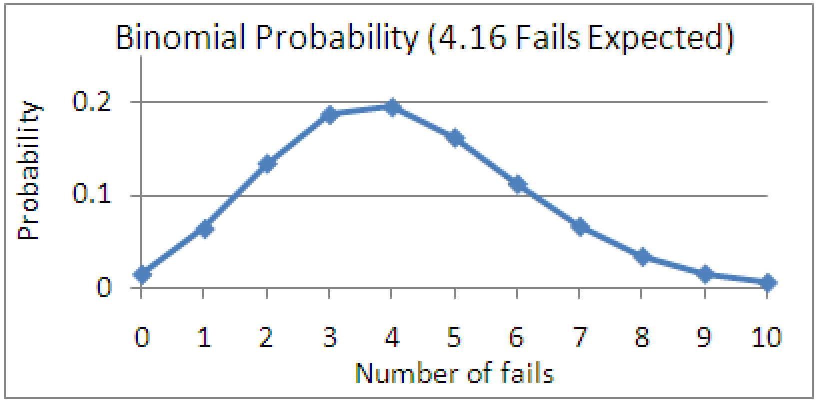
LCL: Prob of 30% units failing *or more* is < 0.05



LCL = 8.7%

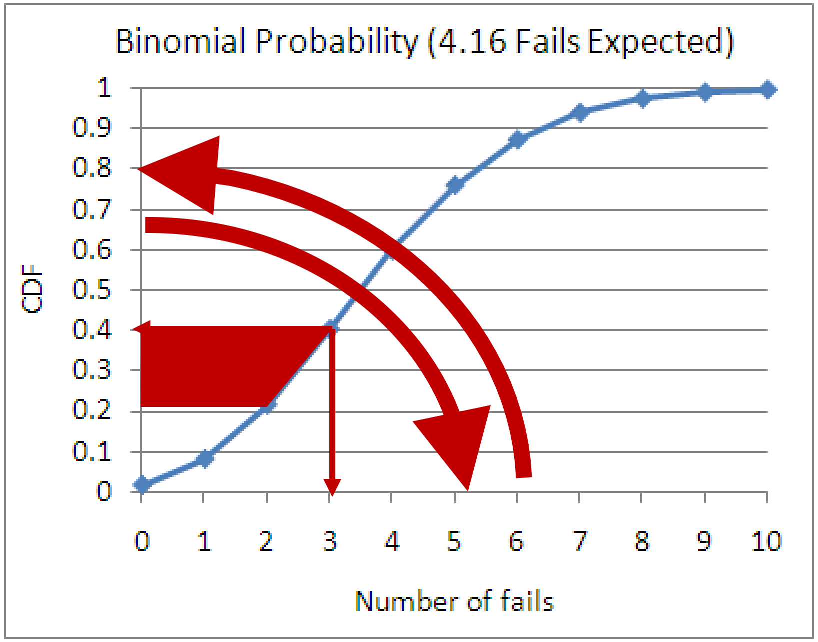
Synthesizing Binomial Data

1000 units
0.416% prob of fail
4.16 units expected

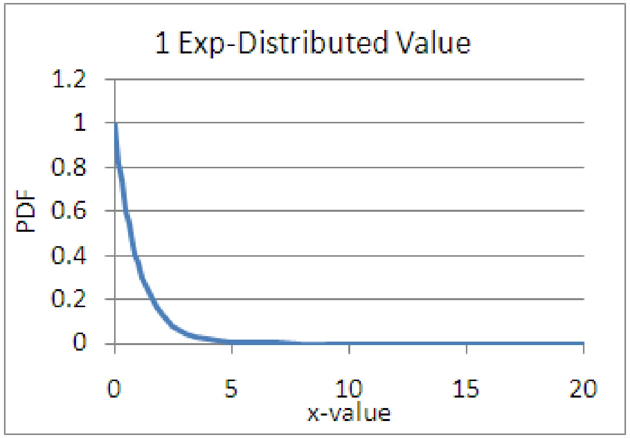


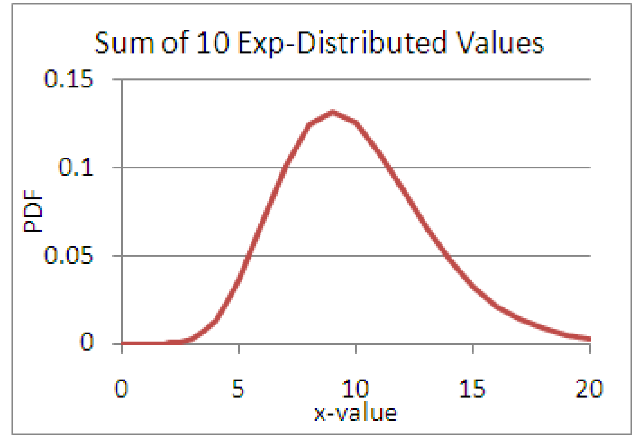
BINOMDIST(fails, samples, prob, TRUE**)**
CRITBINOM(samples, prob, CDF**)**

RAND()



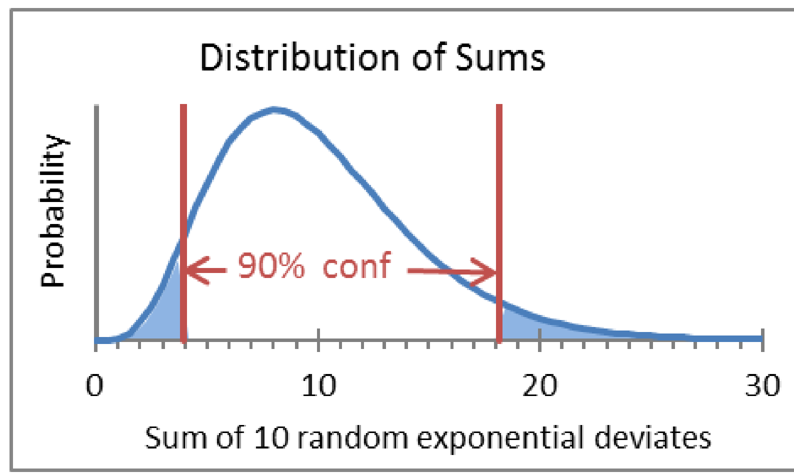
Why Chi-Square for Exponential CL?



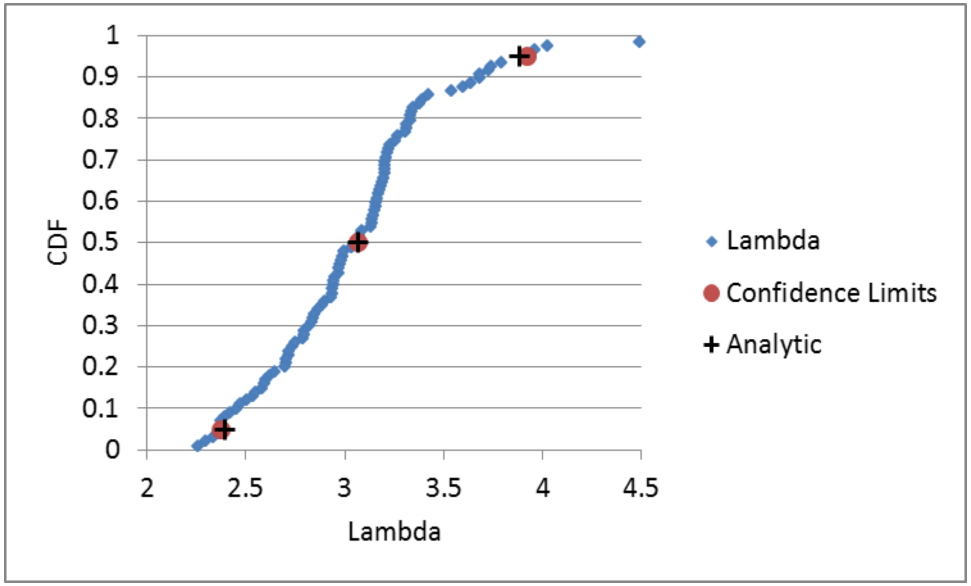


- For $f(t) = \lambda\, e^{-\lambda t}$, best estimate for $1/\lambda$ is $\frac{1}{\mathrm{N}}\sum t_i$ where t_i are the data
- So, what is the *distribution* of $\sum t_i$ where t_i are distributed exponentially?

- Answer: a gamma or a chi-square distribution
- Confidence intervals taken from that



Solution 6.3



Confidence Limits	CL values	MC	Analytic
Upper CL	0.95	3.920005	3.884124
Best estimate	0.5	3.068655	3.068655
Lower CL	0.05	2.376862	2.391386

=LARGE($C$24:$C$123, 100*(1-C12))

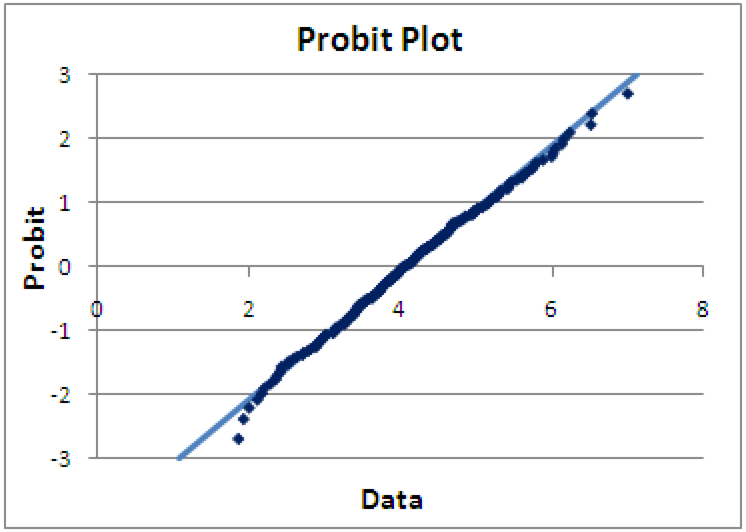
=E11 * CHIINV(95%, 2*($C$5))/(2*$C$5)

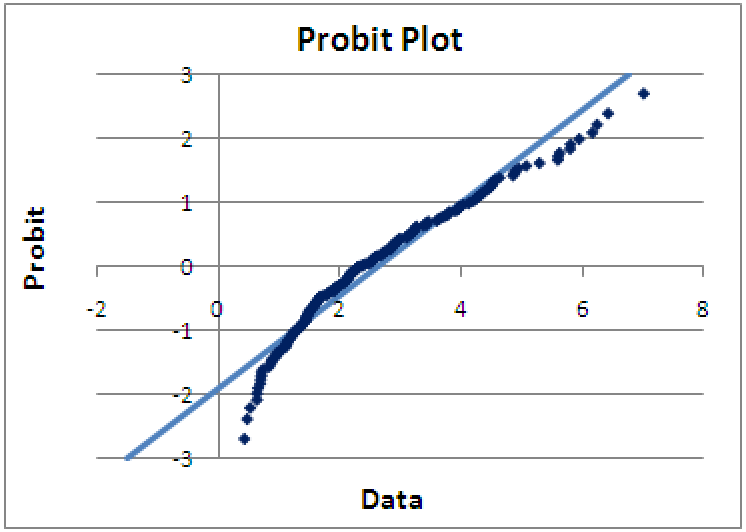
Confidence Limits Summary

- Confidence limits (UCL and LCL) are values between which (2-sided) or above or below which (1-sided) the true population value falls with <confidence level> probability
  - Units are whatever units your data uses
- Confidence level is the probability that the true value lies between (or above or below) your confidence limit(s).
- "CL" can mean either confidence limit or confidence level
  - Use context to decide
- Confidence limits can be calculated
  - Analytically (best if available)
  - Monte Carlo (will work for any distribution)
  - Likelihood methods (coming soon)
- Monte Carlo confidence limits work regardless of how you calculate the best estimate

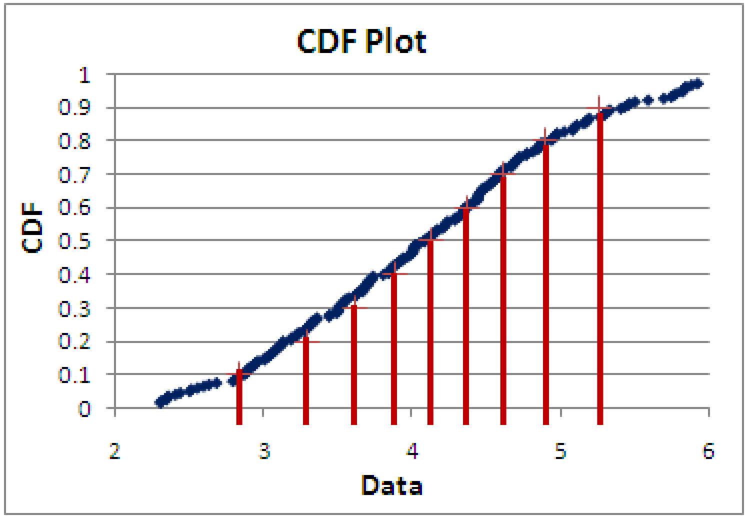
Goodness of Fit Tests

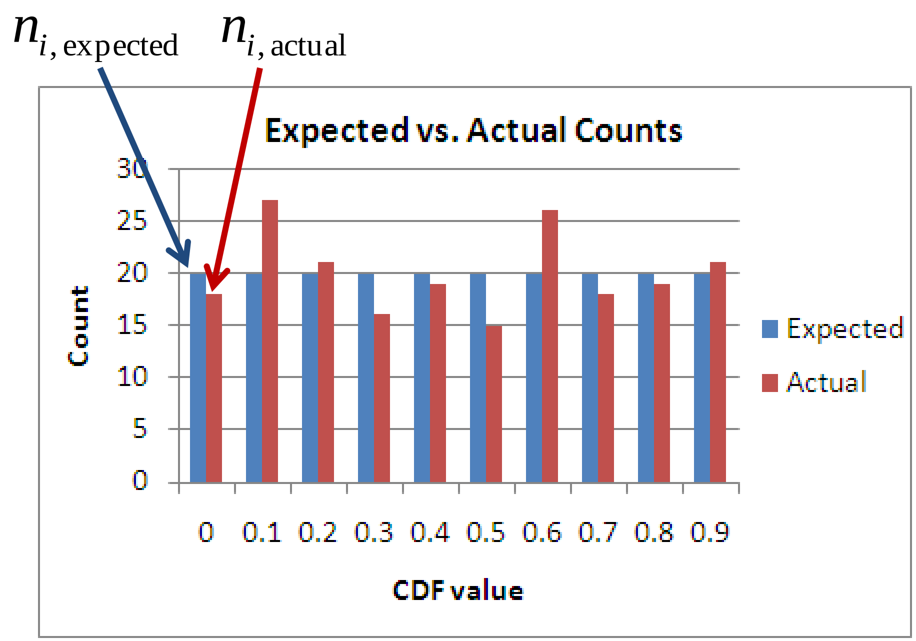
How Good Is Good Enough?

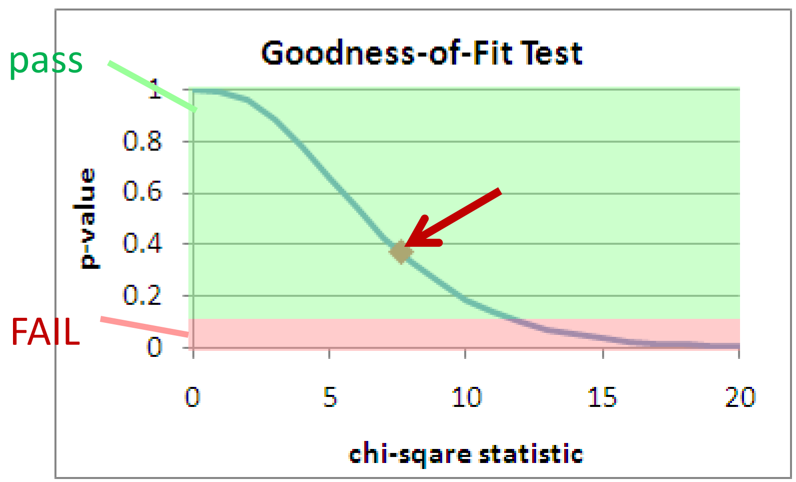




Pearson's Chi-Squared Test







$$\chi_i^2 = \frac{\left(n_{i,\text{actual}} - n_{i,\text{expected}}\right)^2}{n_{i,\text{expected}}}$$

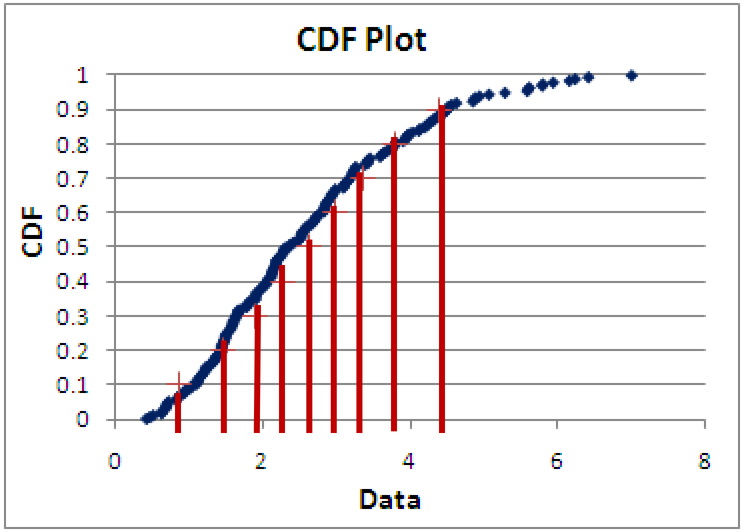
Chi-Sq:

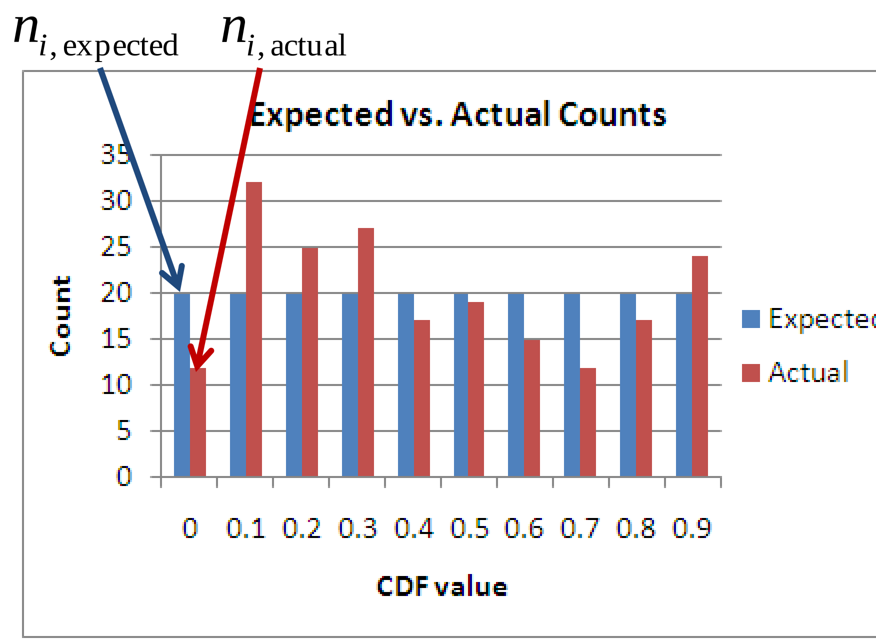
$$\chi^2 = \sum_i \chi_i^2$$

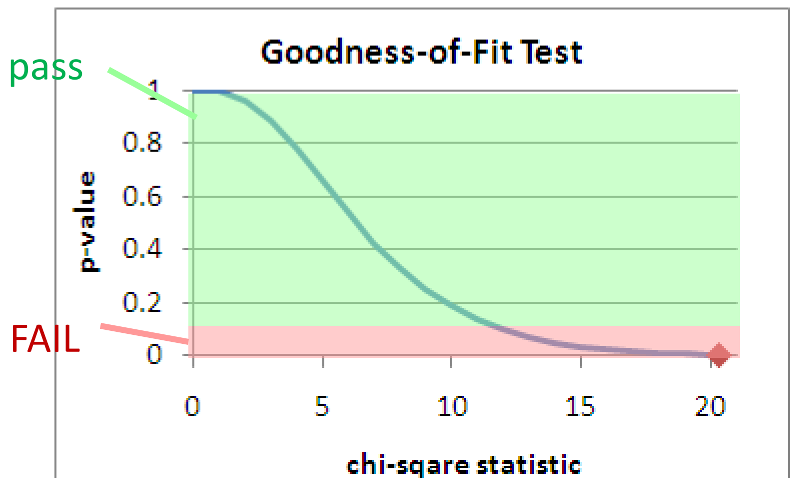
DoF = bins – parameters – 1 = 7

p-value = CHIDIST(Chi-Sq, DoF)
=0.378

Pearson's Chi-Squared Test







$$\chi_i^2 = \frac{\left(n_{i,\text{actual}} - n_{i,\text{expected}}\right)^2}{n_{i,\text{expected}}}$$

Chi-Sq:

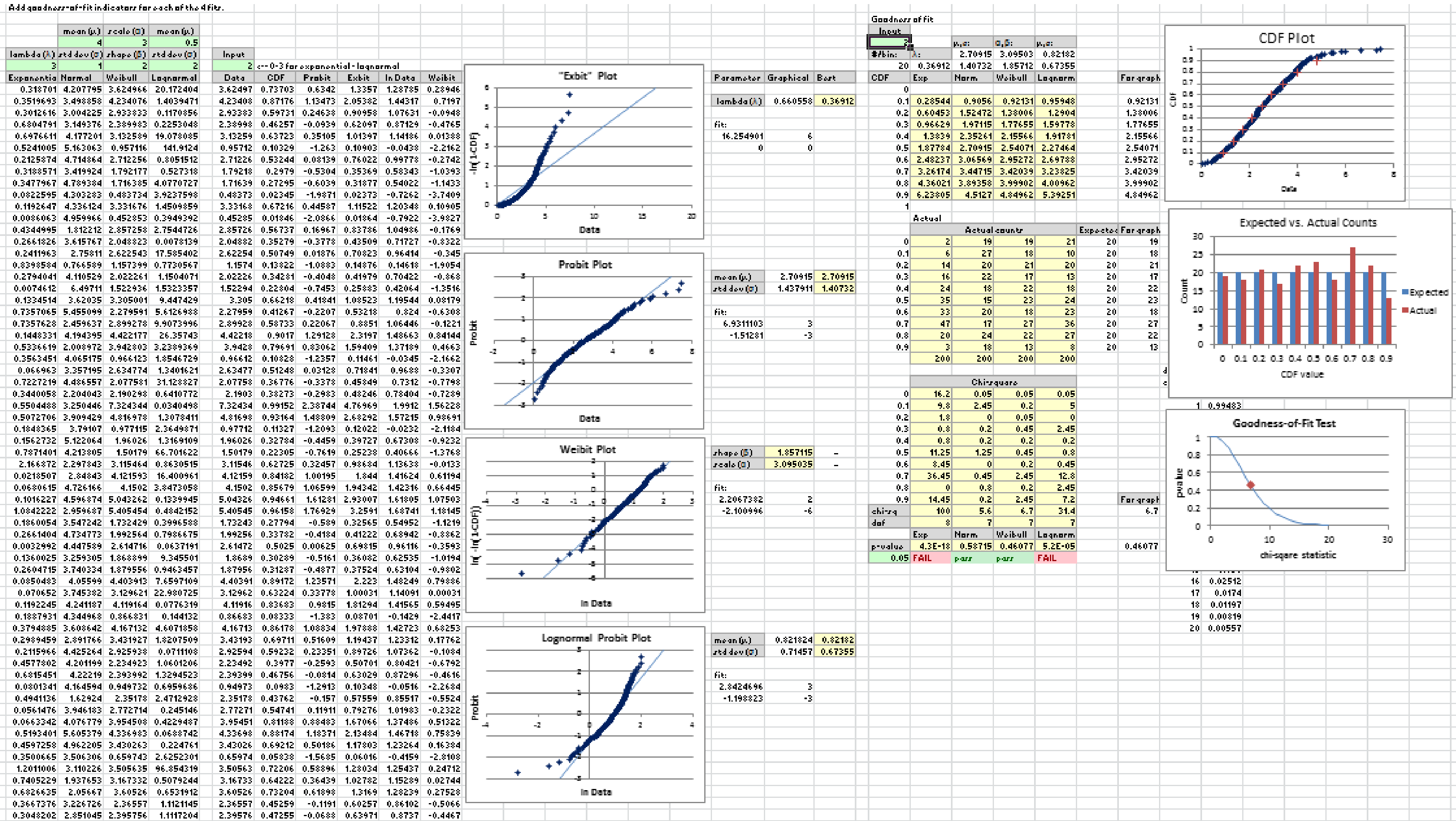
$$\chi^2 = \sum_i \chi_i^2$$

DoF = bins – parameters – 1 = 7

p-value = CHIDIST(Chi-Sq, DoF)
=0.005

Exercise 7.1

- Add a chi-square goodness-of-fit test for each of the 4 fits in our synthesized data sheet



Other Goodness-of-Fit Tests

Exponential

Goodness-of-Fit Test

Kolmogorov's D

D		Prob>D
0.097489	>	0.1500

Note: Ho = The data is from the Exponential distribution. Small p-values reject Ho.

Weibull

Goodness-of-Fit Test

Cramer-von Mises W Test

W-Square		Prob>W^2
0.029696	>	0.2500

Note: Ho = The data is from the Weibull distribution. Small p-values reject Ho.

Normal

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.947533	0.0270*

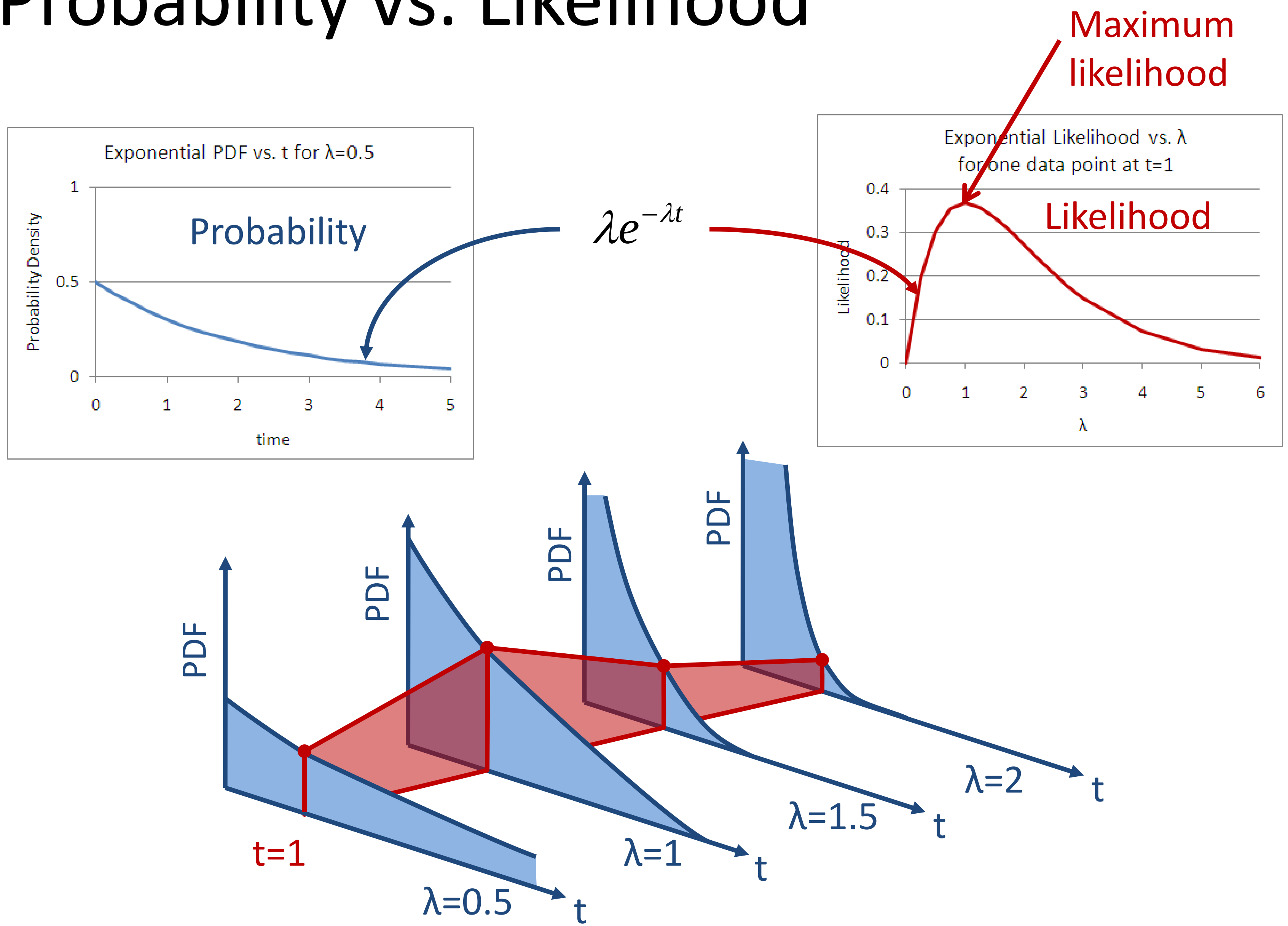
Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

Maximum Likelihood Method
and the
Exponential Distribution

MLE

- Maximum Likelihood Estimation (MLE) is a fitting technique that is good for any model
- Principle
  - We can't ask: What is the most likely model?
    - Because we don't have some well-defined space of possible models
  - We can ask: Given this model, how likely is this data set?
  - (This is a fairly Bayesian approach. We are usually frequentists.)

Probability vs. Likelihood



MLE

- Likelihood for each point
  - For exact values (exact times to fail), use the PDF
  - For ranges (failed between two readout times), use CDF delta
  - Multiply all together (or add logs)
- Use
  - Choose a model functional form with adjustable parameters
  - Adjust the parameters to maximize the likelihood

The End