ECE 510 Lecture 6
Confidence Limits

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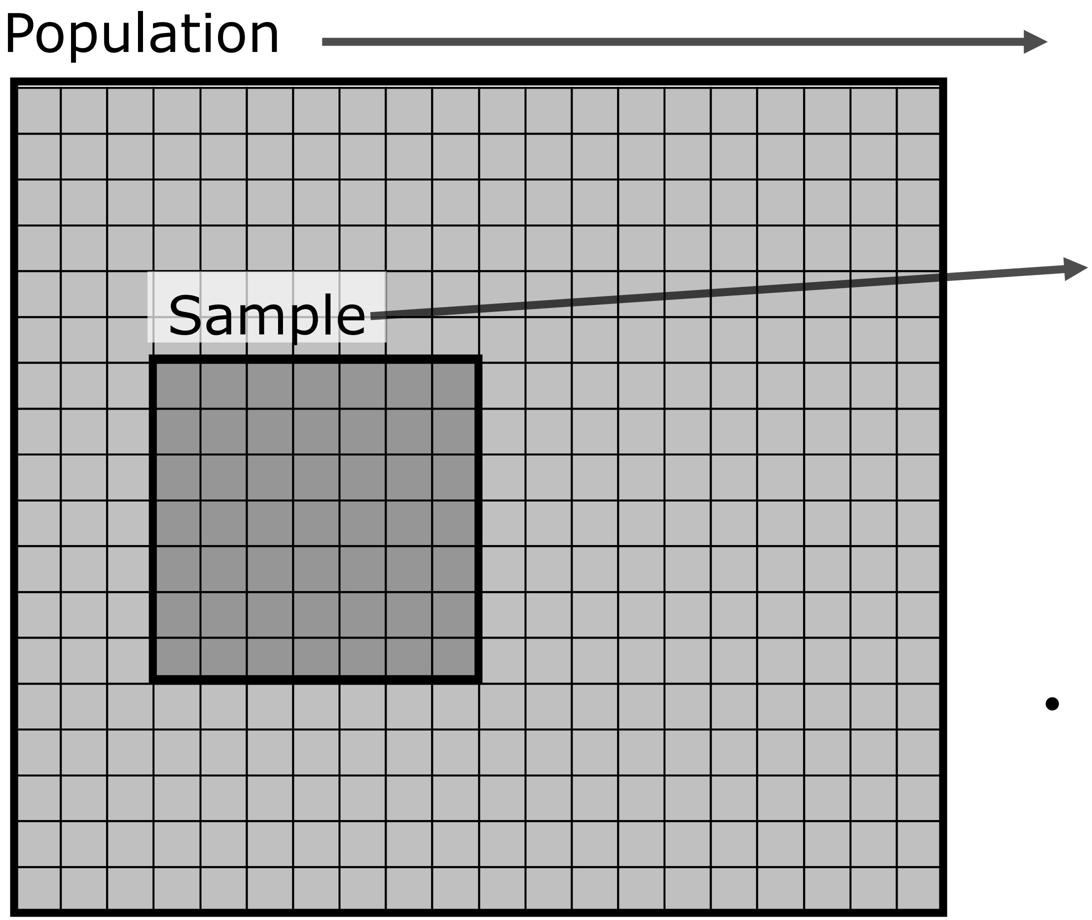
Concepts

Statistical Inference

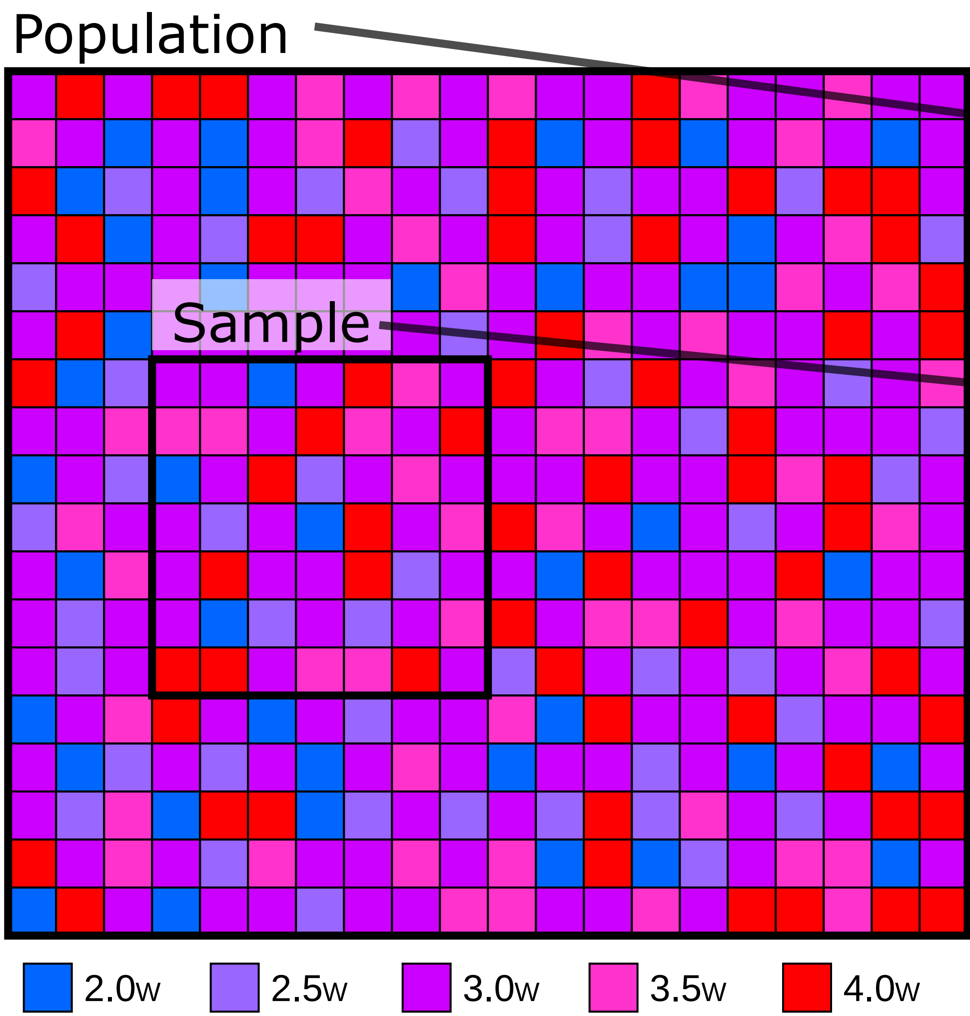


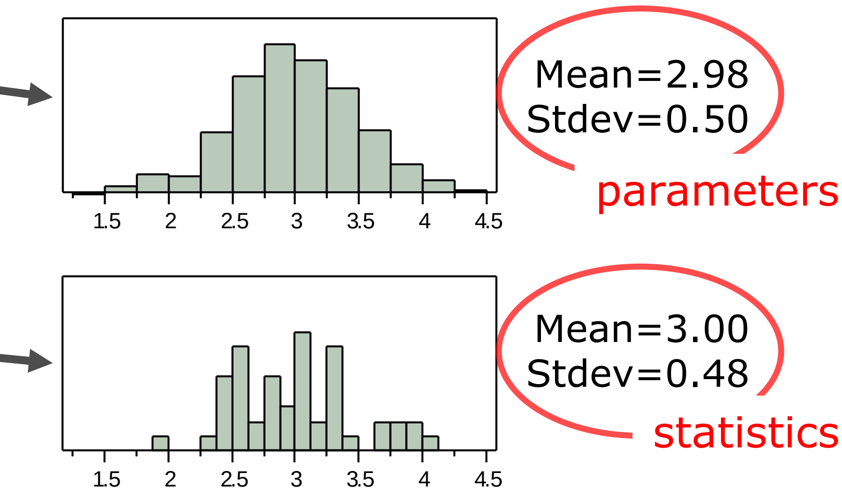
True ("population") value
= parameter

Sample value
= statistic

- Use a sample ***statistic*** to estimate a population ***parameter***

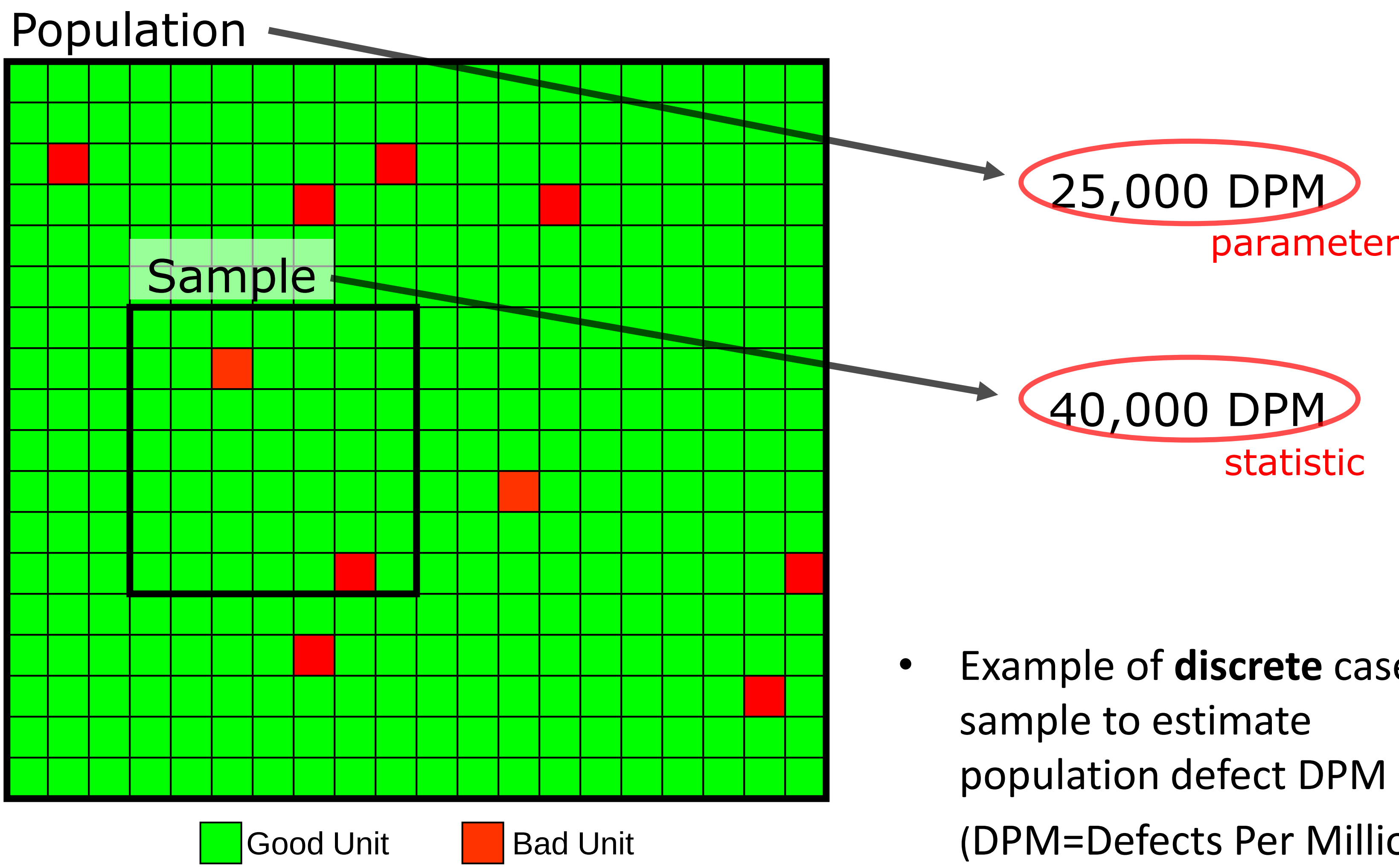
Statistical Inference (Continuous)





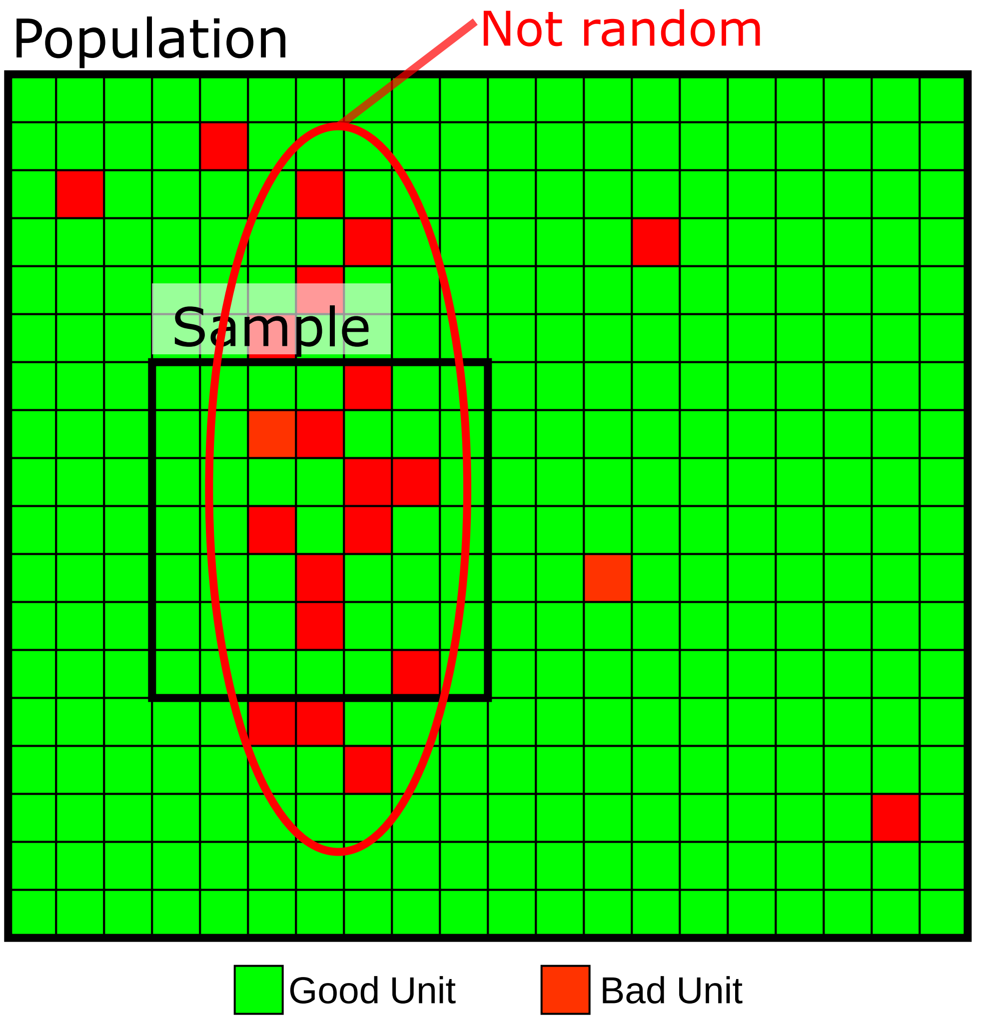
- Example of **continuous** case: Use sample to estimate population mean and standard deviation

Statistical Inference (Discrete)



- Example of **discrete** case: Use sample to estimate population defect DPM (DPM=Defects Per Million)

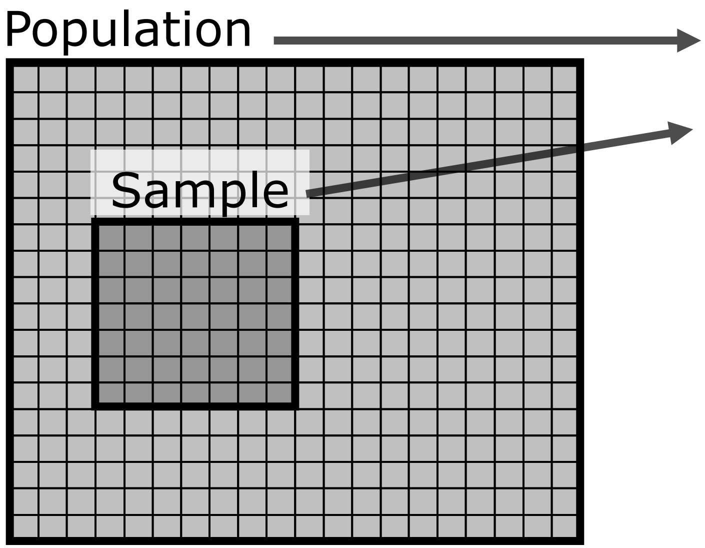
Note: Samples Must Be Random!



Population = 55,000 DPM
Sample = 204,000 DPM

- Samples must be representative of the entire population!
- Best to select samples truly randomly
  - Not the first lot available or other partly-random methods
- No statistical analysis can correct for non-random samples

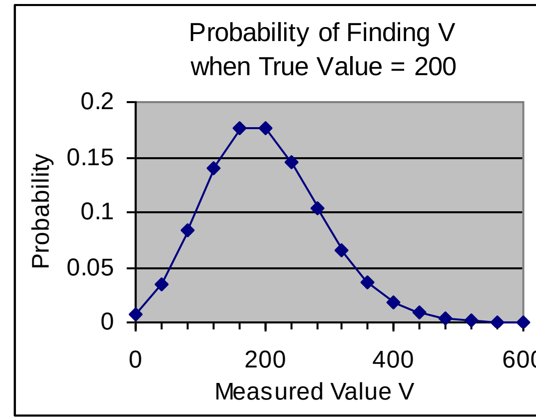
Distributions of Statistics



True (“population”) value = parameter

Sample value = statistic

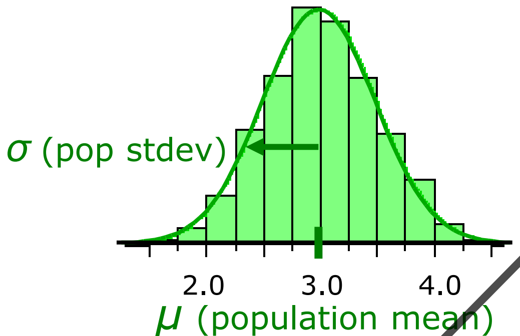
Distribution of statistic:



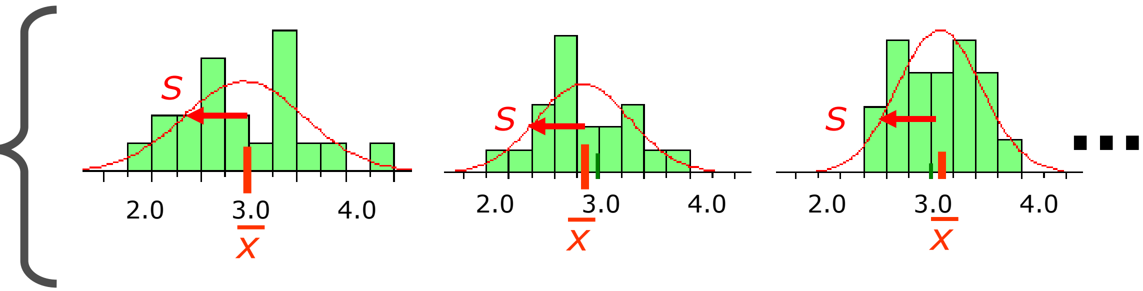
- Measured statistic is not enough
- Need to add either
  - Confidence interval or limits
  - Answer to a statistically-well-posed question (“hypothesis test”)
- Calculated from distributions of statistics
  - If we looked at many samples from many identical populations, what values of the statistics might we get?

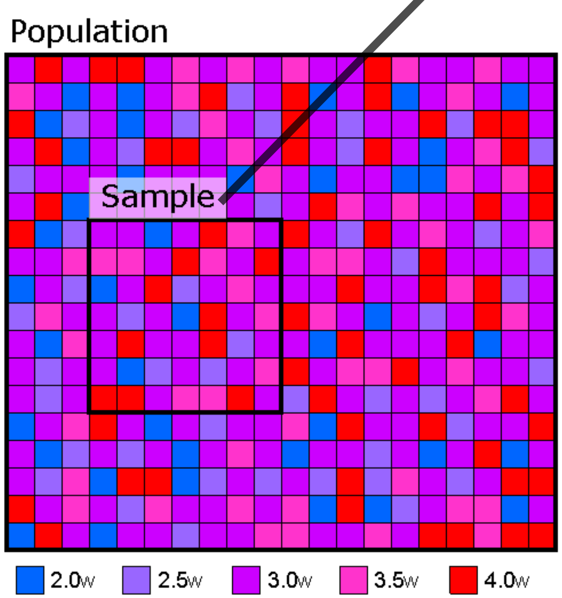
Distributions of Statistics (Continuous)

Population has one true distribution:

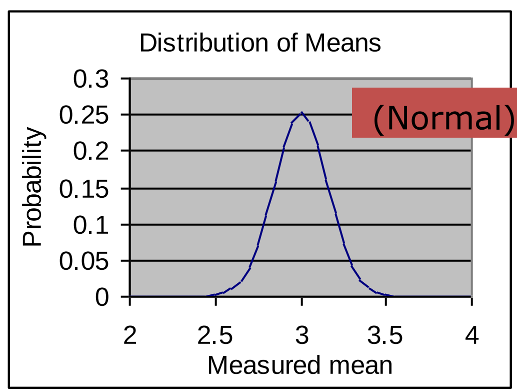


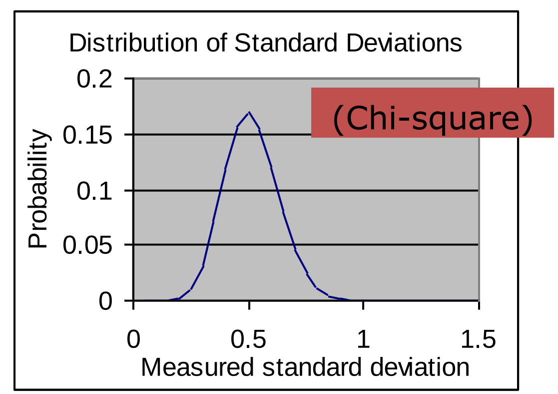
Different samples have different distributions:





Properties of sample distributions are *statistics*. We can calculate distributions of these statistics:





We get one value for each from our one sample.

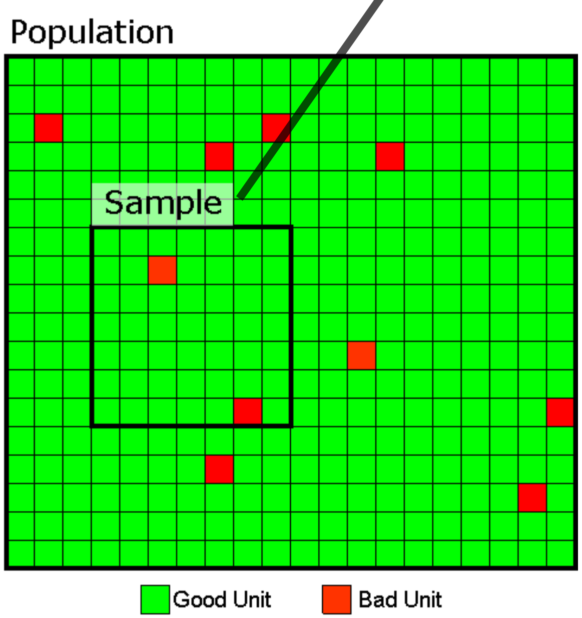
Distributions of Statistics (Discrete)

Population has one true DPM:

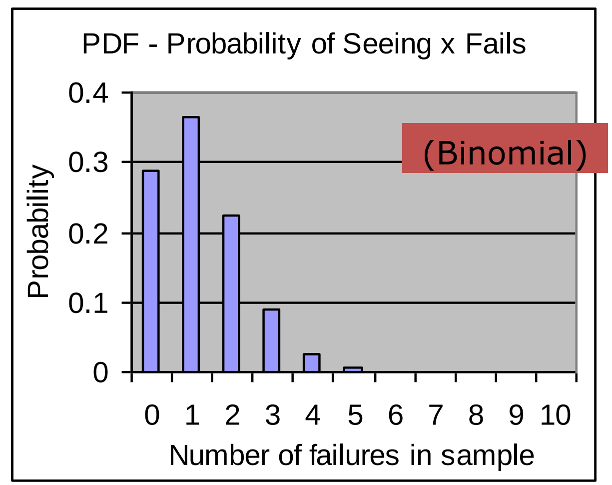
25,000 DPM

Different samples have different DPMs:

20,000 DPM (1 fail)	0 DPM (0 fail)
40,000 DPM (2 fail)	60,000 DPM (3 fail) ...
20,000 DPM (1 fail)	40,000 DPM (2 fail)



The measured sample DPM is a *statistic*.
We can calculate the distribution of this statistic:



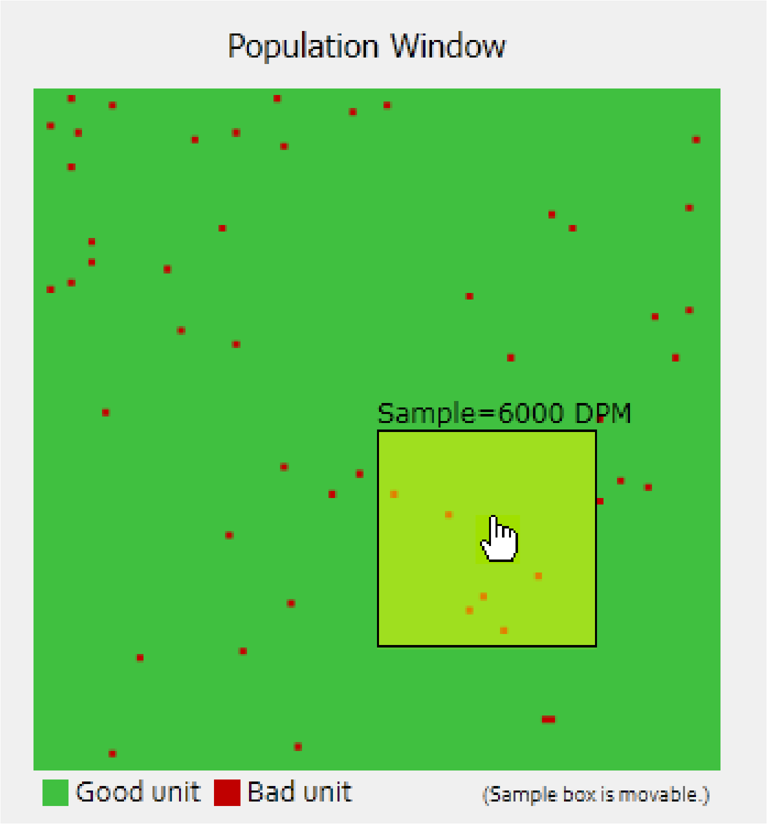
We get one value from our one sample.

DPM Simulation

Population Window

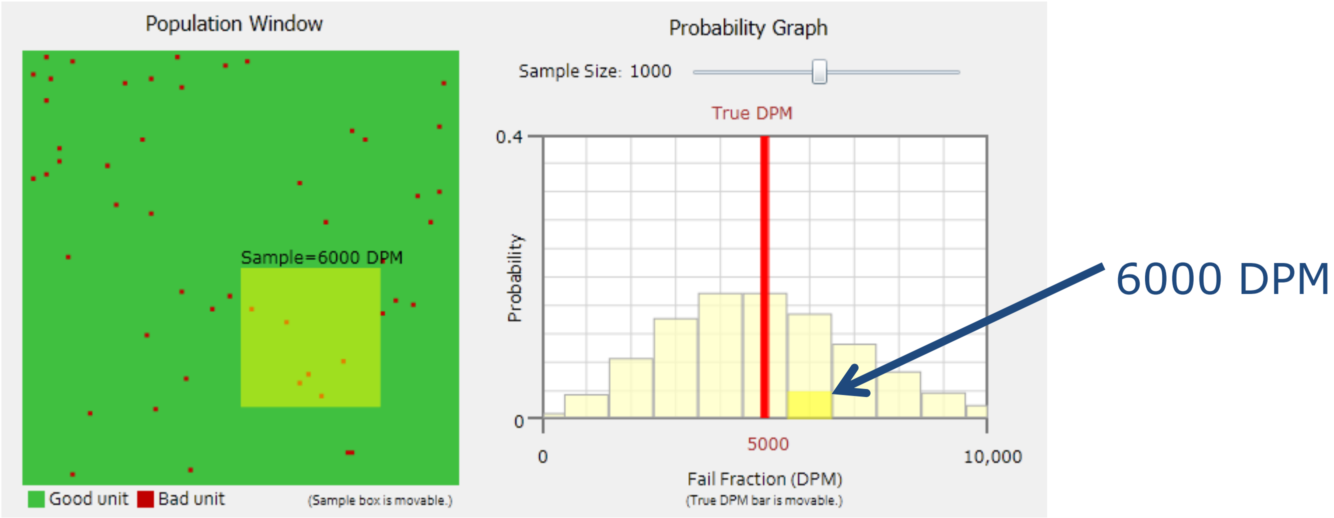
- Shows 10,000 units, most good, a few bad

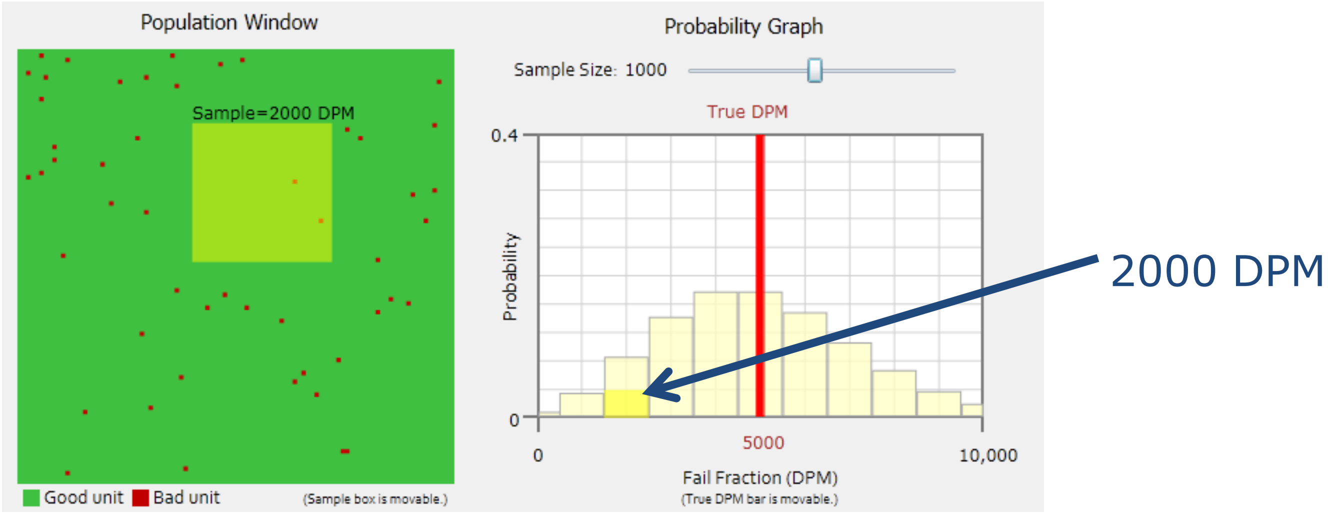
The Sample



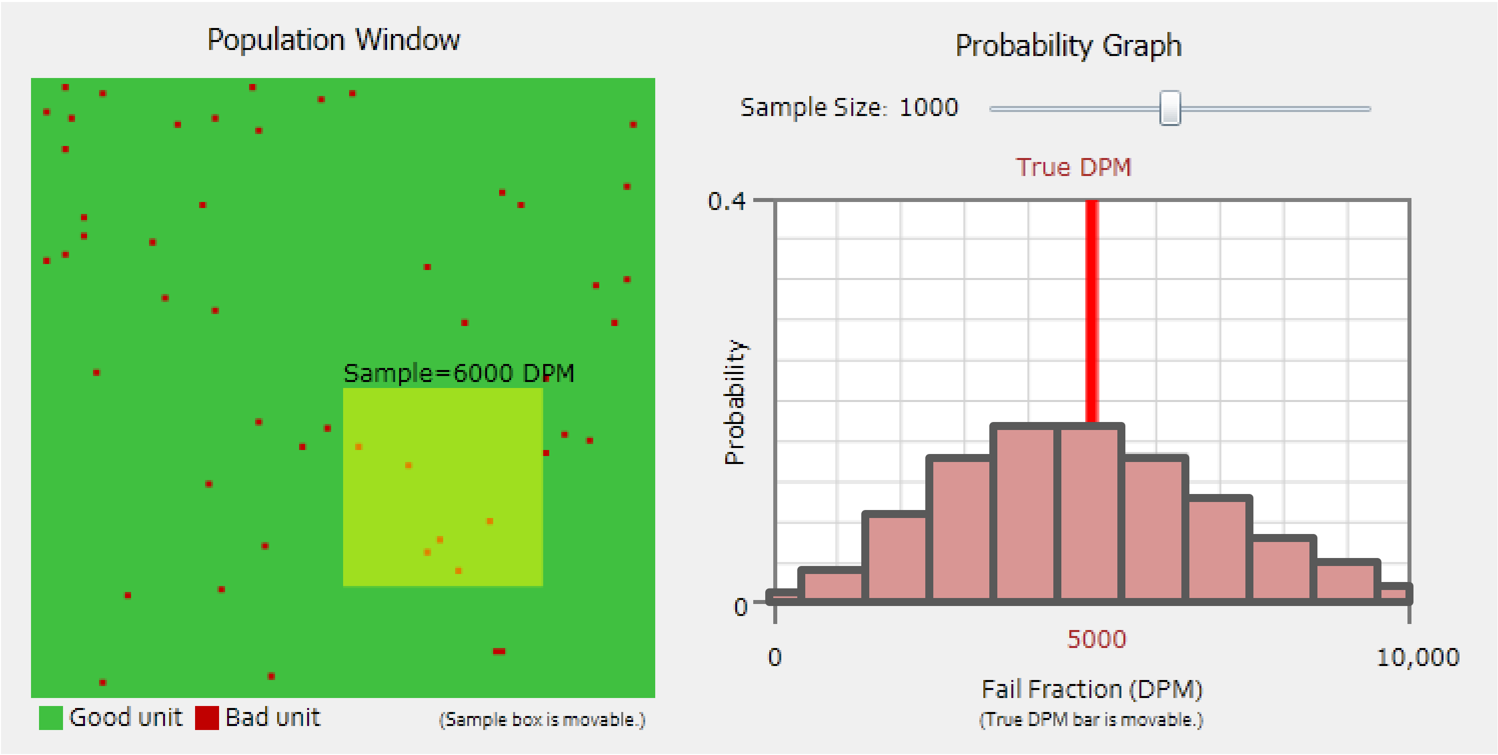
- You can move the sample box

DPM Indicator on DPM Histogram



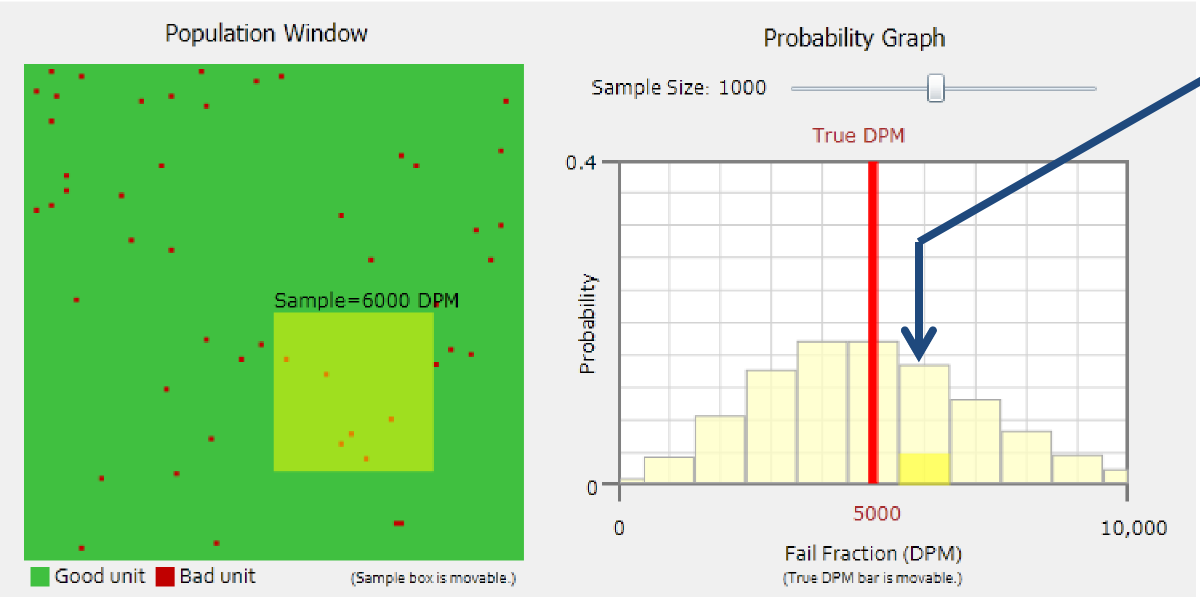


Binomial Histogram



- Gives probability of getting each measurement given the true DPM

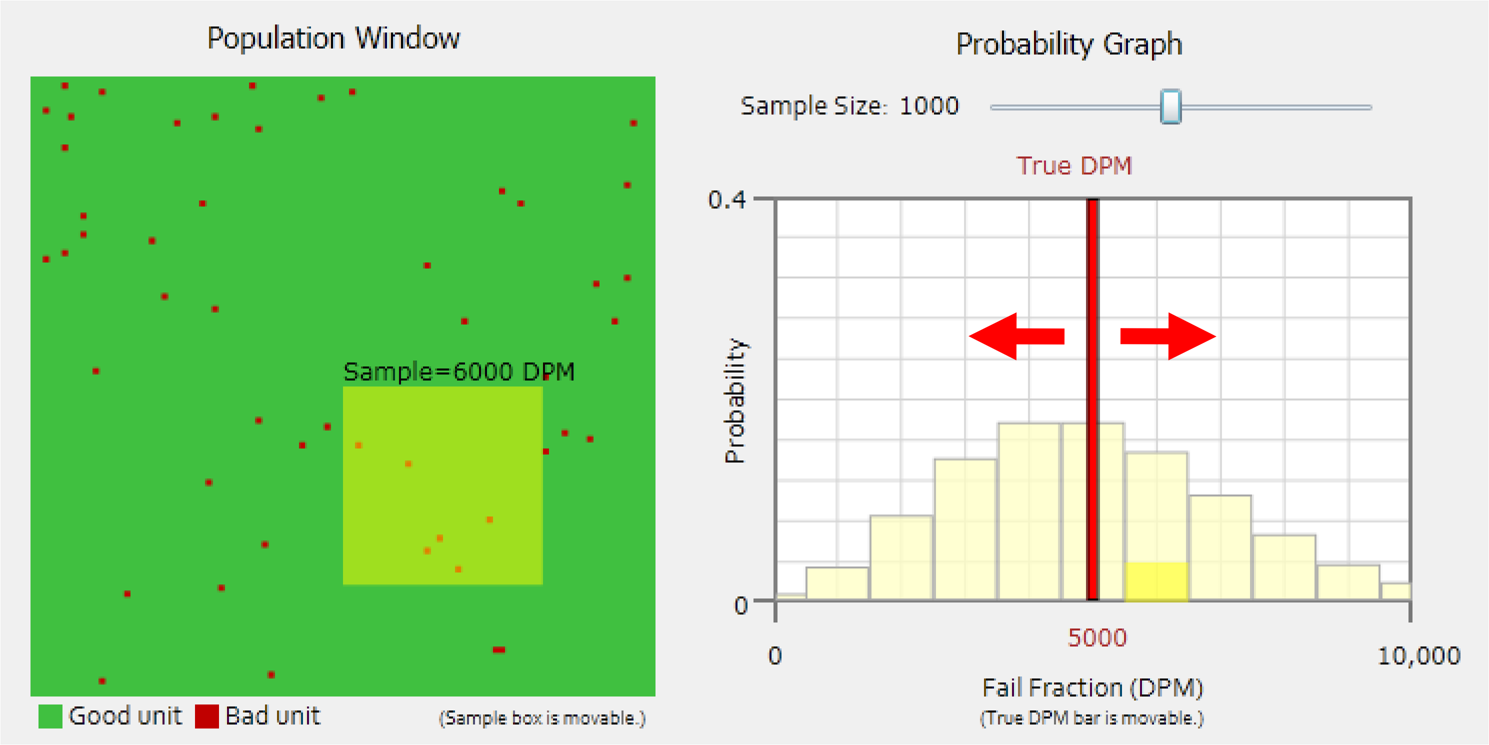
Binomial Distribution



=binomdist
(6, 1000, 0.005, false)
6 fails
1000 samples
5000 DPM
Not cumulative

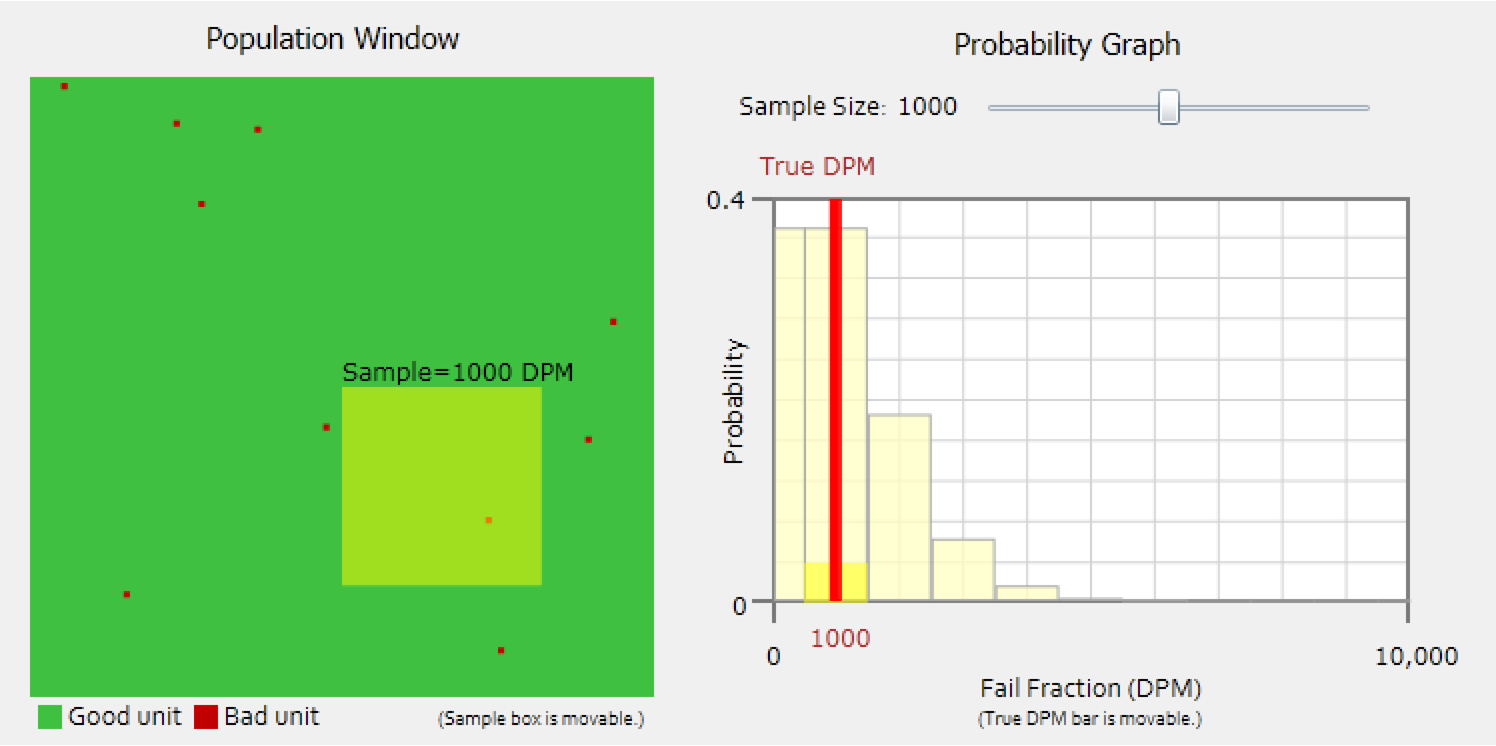
$$\text{binomdist}(f, N, p, \text{false}) = \binom{N}{f} p^{f} (1-p)^{N-f}$$

True DPM



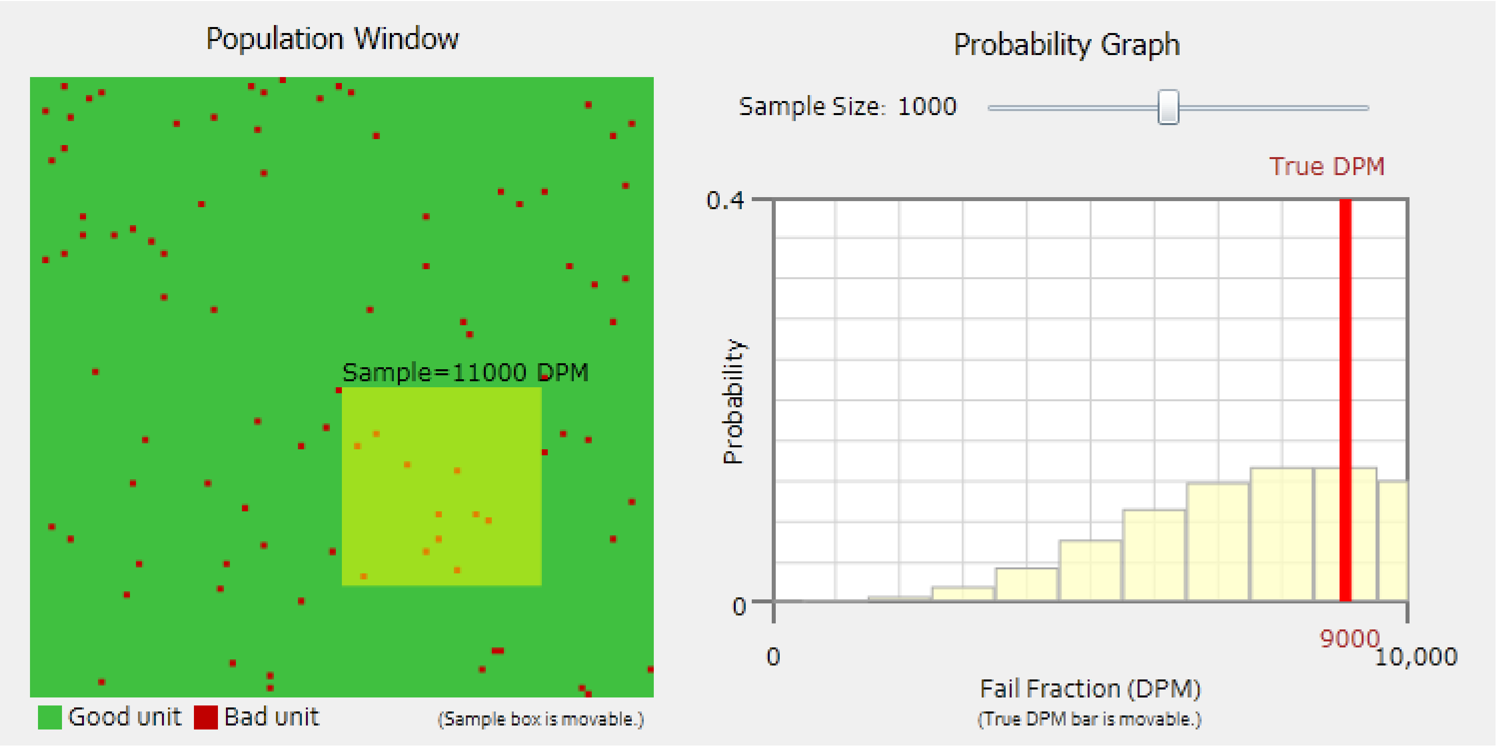
- True DPM is adjustable
  - Not in the real world, only the simulation!

Low DPM



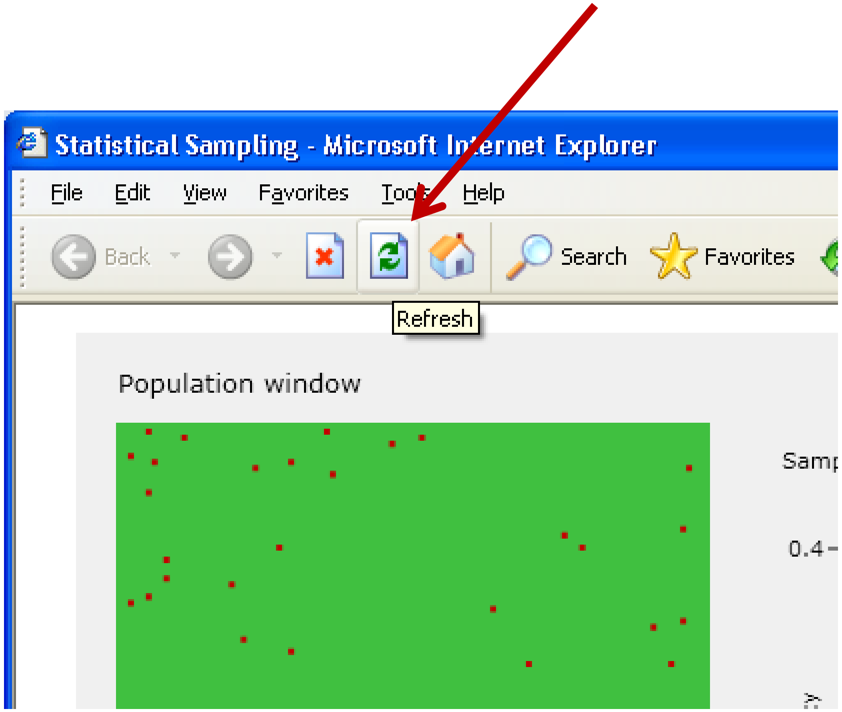
Population Window
Sample=1000 DPM
Good unit
Bad unit
(Sample box is movable.)
Probability Graph
Sample Size: 1000
True DPM
0.4
0
Probability
0
1000
10,000
Fail Fraction (DPM)
(True DPM bar is movable.)

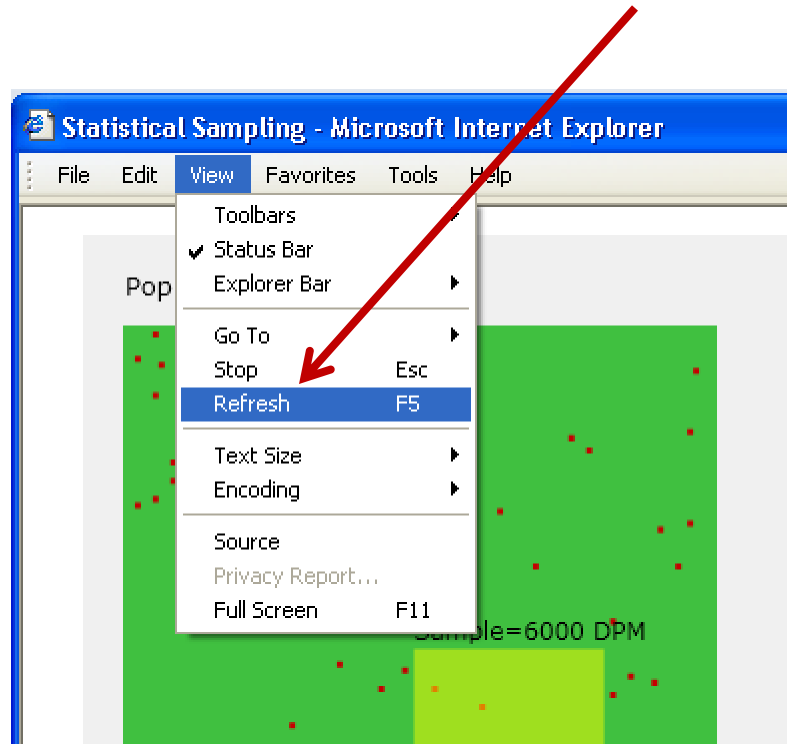
High DPM



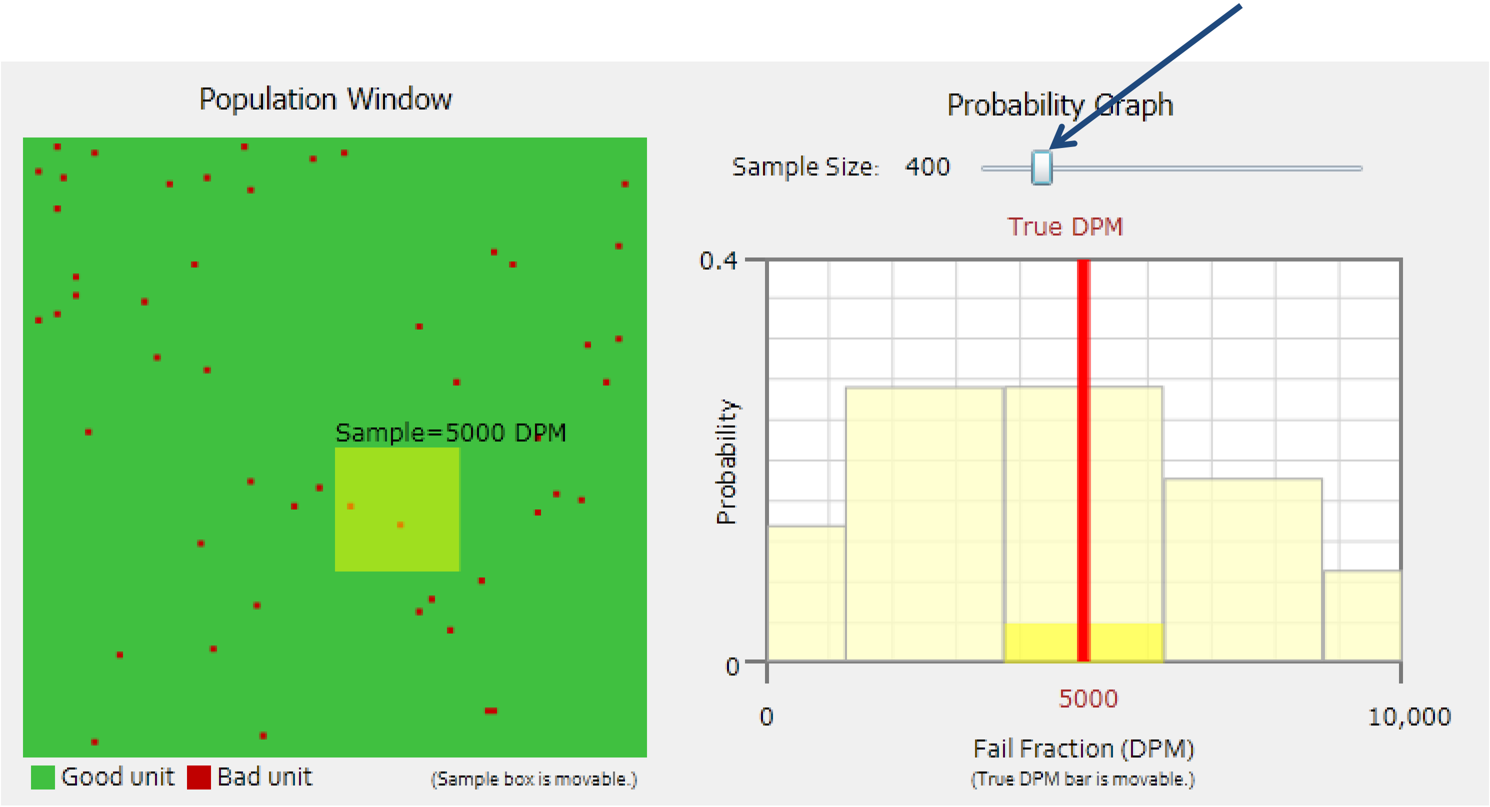
Population Window
Sample=11000 DPM
Good unit
Bad unit
(Sample box is movable.)
Probability Graph
Sample Size: 1000
True DPM
0.4
0
Probability
0
9000
10,000
Fail Fraction (DPM)
(True DPM bar is movable.)

Please put True DPM back to 5,000

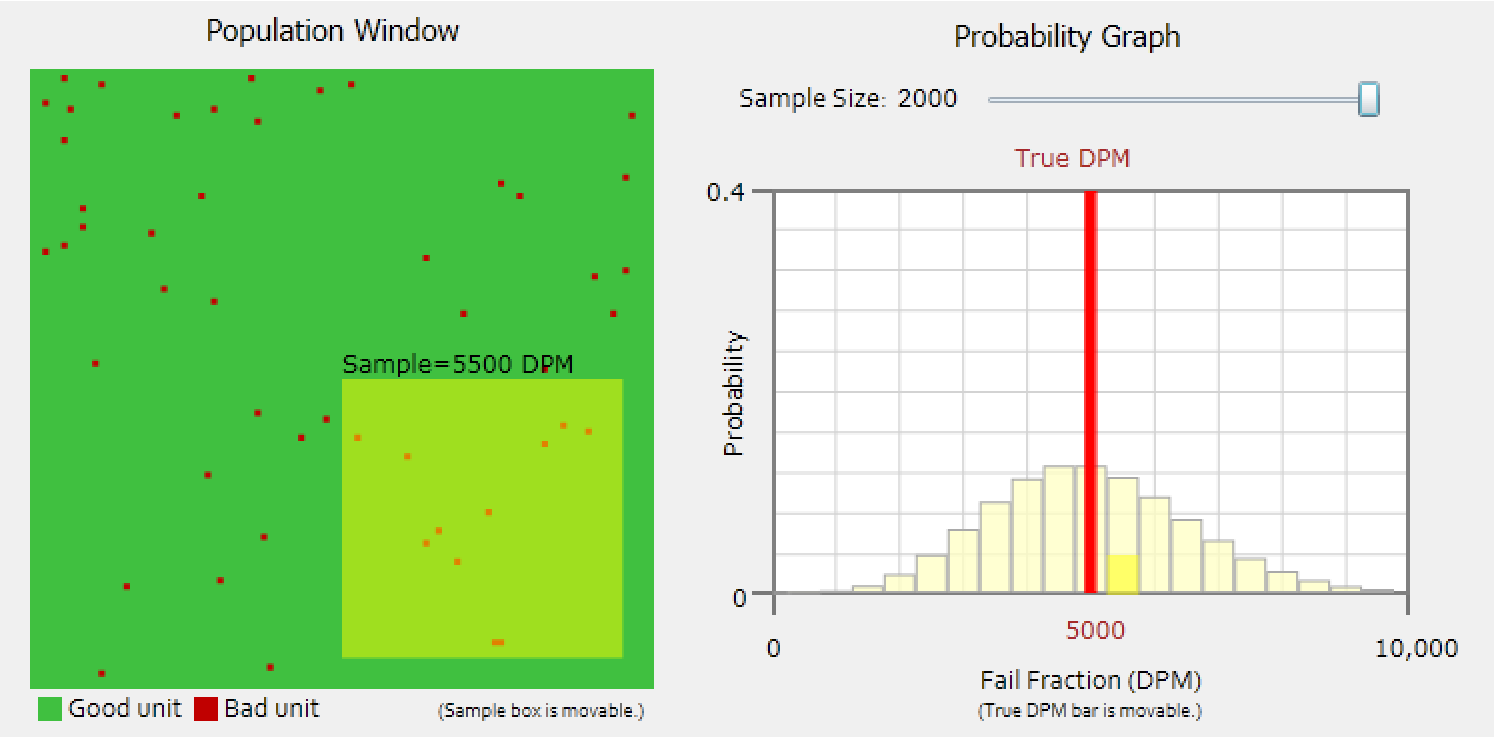




Small Sample Size

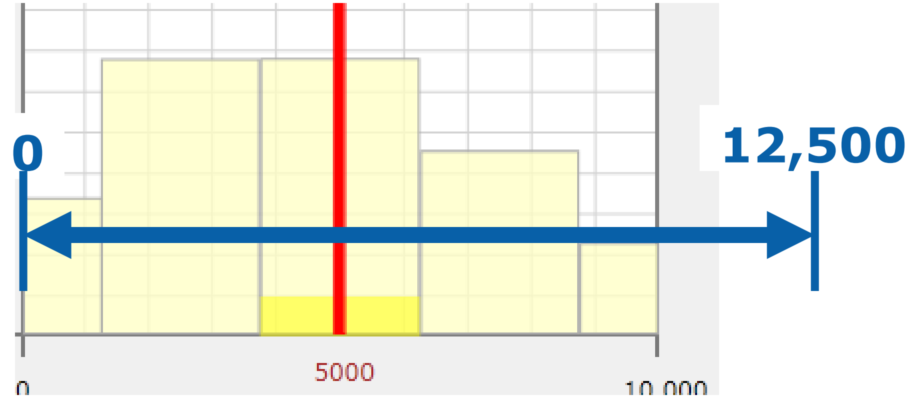


Large Sample Size

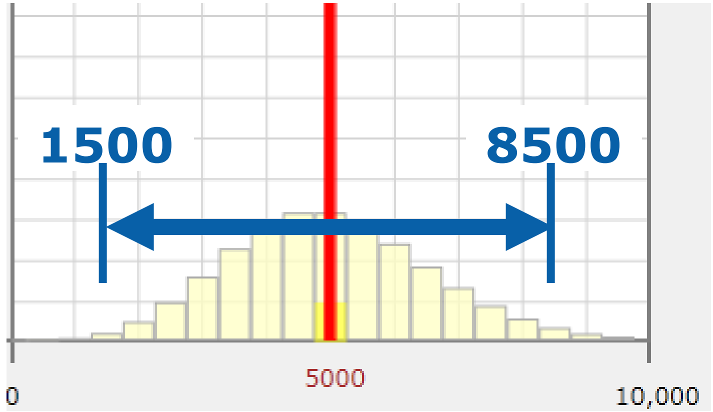


Statistical Measurement Uncertainty

Small sample (400)
= wide range



Large sample (2000)
= narrow range



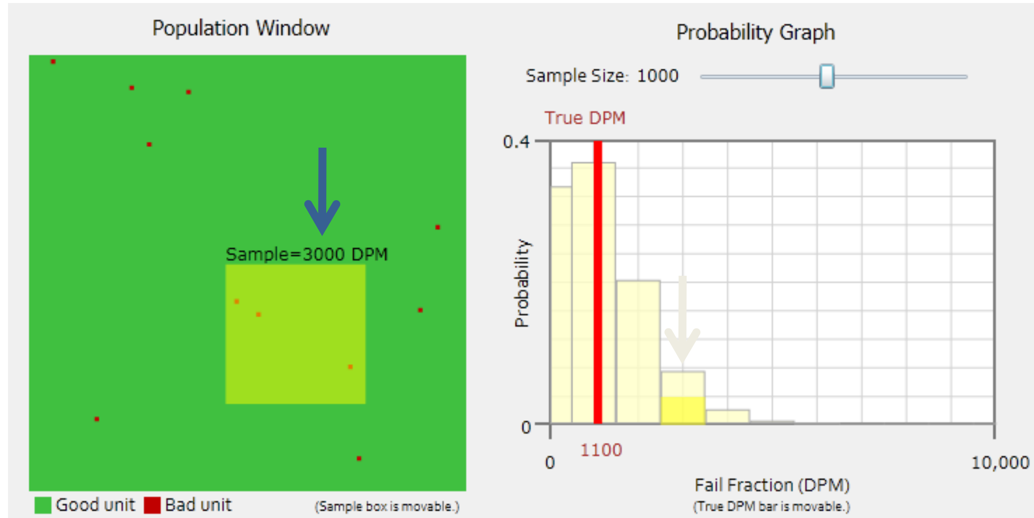
Exercise 6.1

- (A) Set sample size = 1000
- (B) Set True DPM = 1100 DPM and look for a sample with 3 fails – what DPM does that represent?
- (C) Set True DPM = 6700 DPM and look for a sample with 3 fails – what DPM does that represent?

Why We Need Confidence Limits

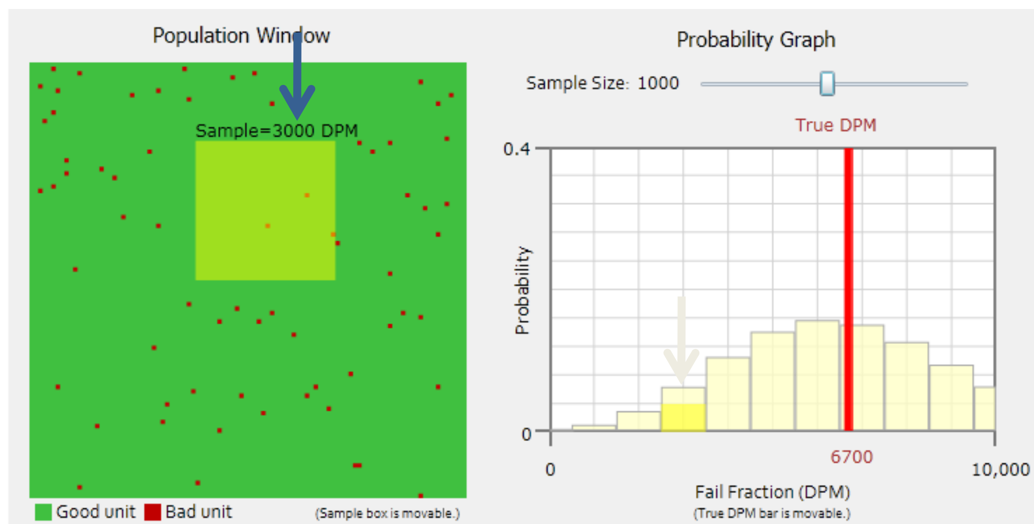
Did you get…

a *bad* sample from
a *good* population?



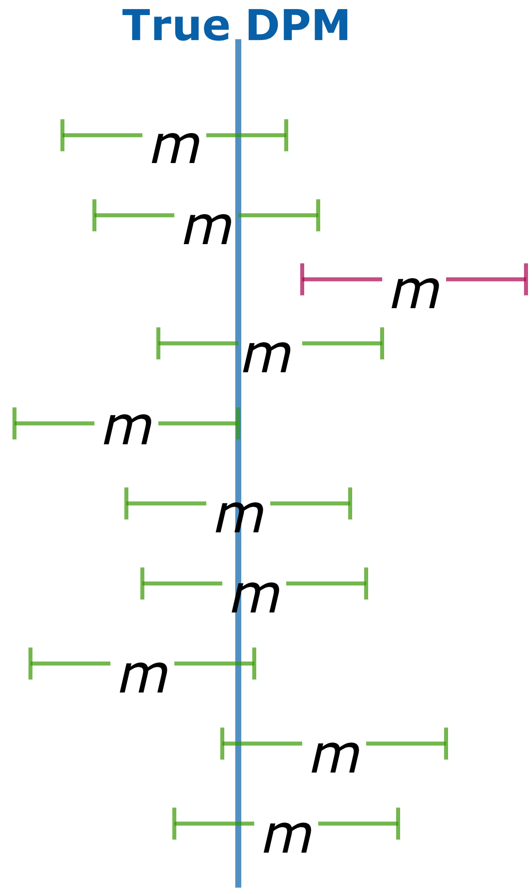
…or…

a *good* sample from
a *bad* population?



Confidence Limits

Confidence Interval Meaning



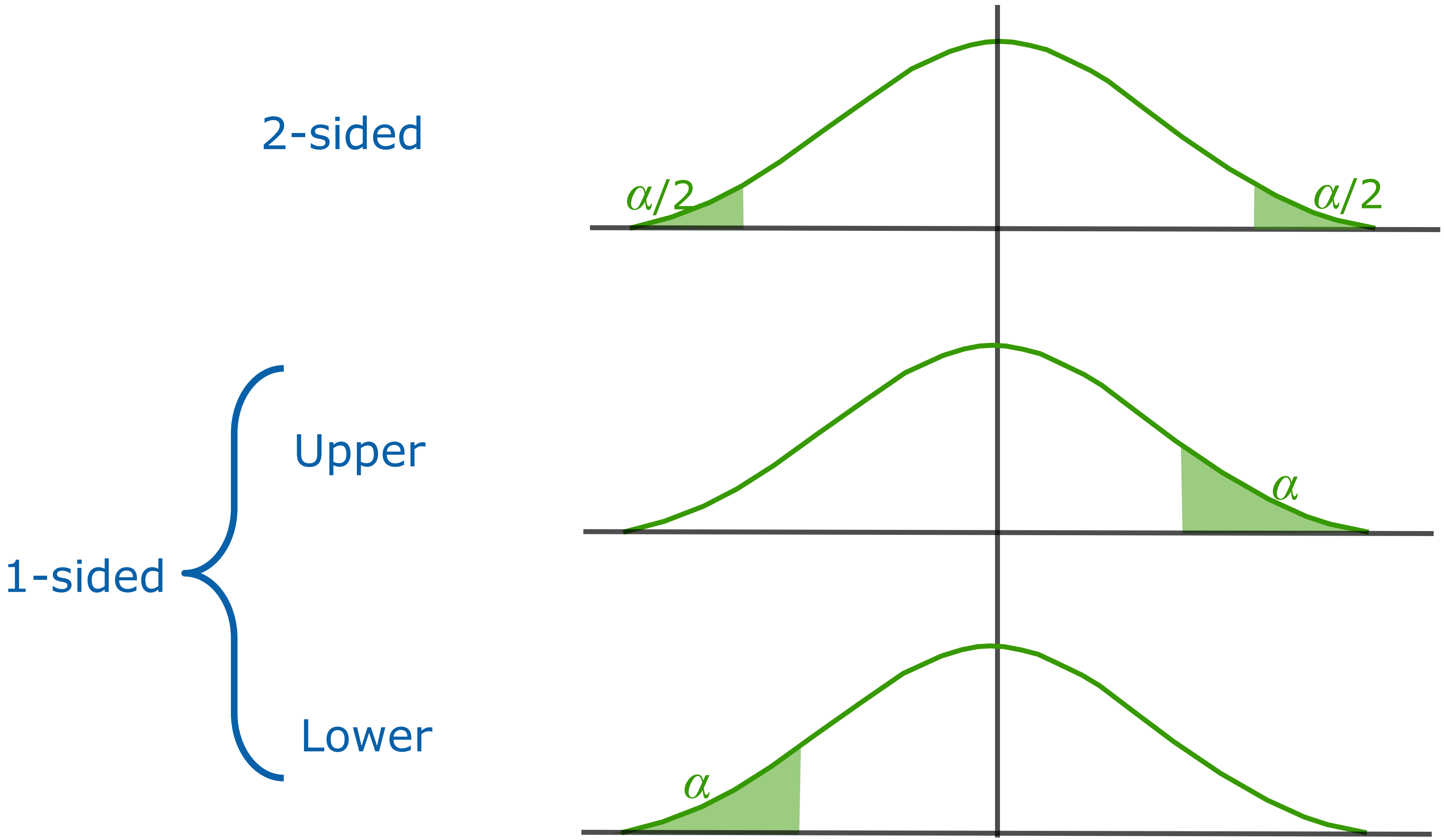
Confidence level = 90% = 0.9

Risk of being wrong
= 1 − confidence level

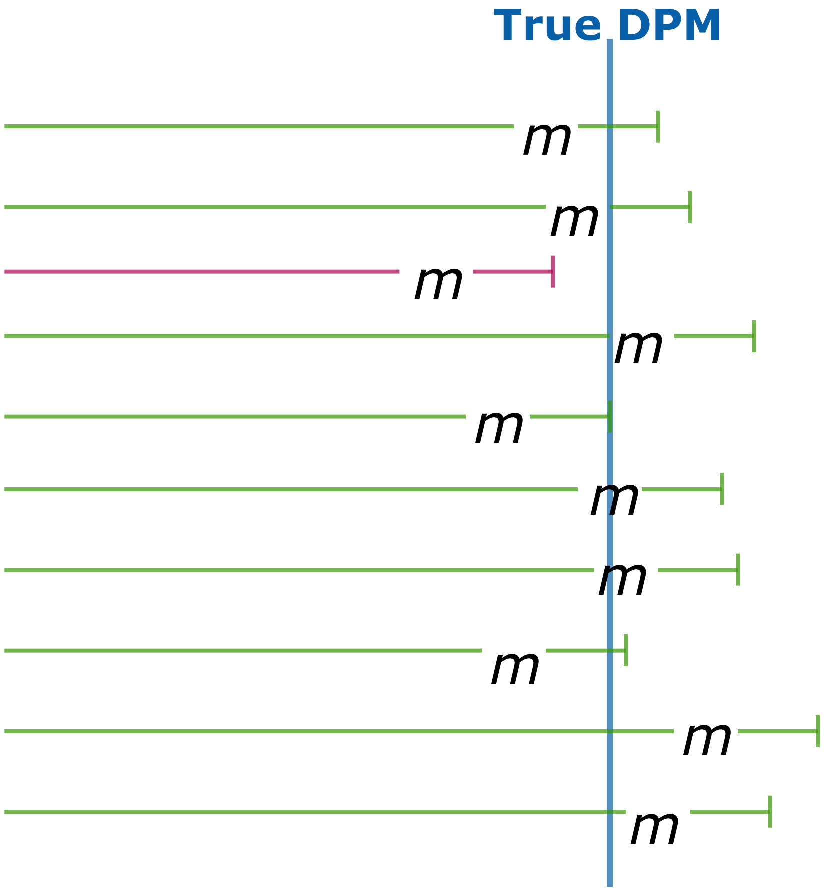
= α = 10% = 0.1

- 90% of random sample means with this confidence interval include the true population mean

1-Sided vs. 2-Sided



1-Sided UCL Meaning



- 90% of random sample means with this confidence interval include the true population mean

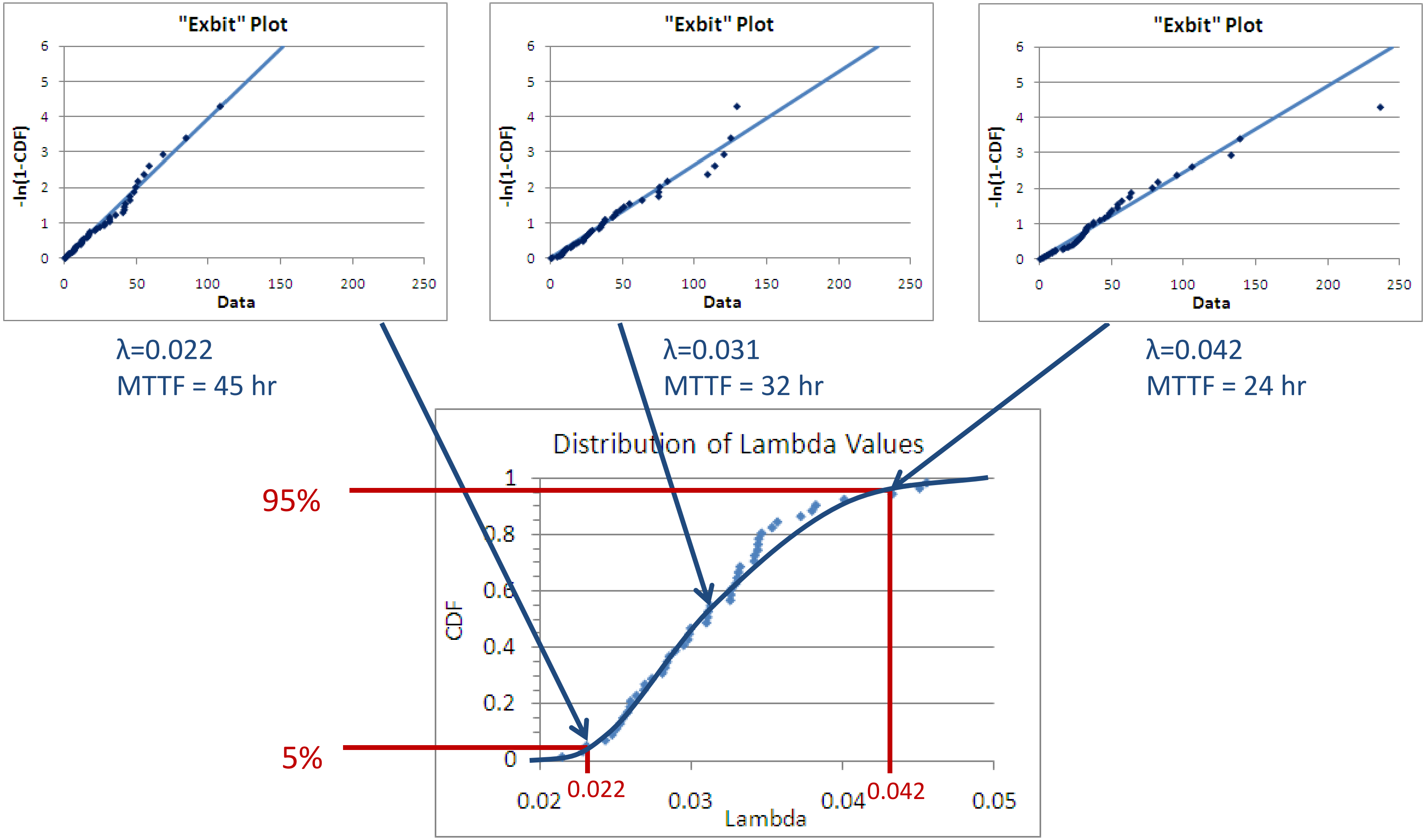
Calculating Confidence Limits

Exercise 6.2

Monte Carlo determination of binomial CL:

- In each row, simulate 10 pass/fail samples and count the number of fails
- Make a histogram of the count of runs that got each fail%
- Add the binomial prediction for each fail%
- Plot both as a bar chart
- Calculate cumulative values for your MC and calculated distributions
- Plot those with a line plot
- Use the cum plots to find the UCL and LCL for 3 fails / 10 units
- Compare to the analytic expressions (T&T section 11.3):
- LCL = BETAINV(5%, fails, samples—fails+1)
- UCL = BETAINV(95%, fails+1, samples—fails)

Monte Carlo Exponential CL

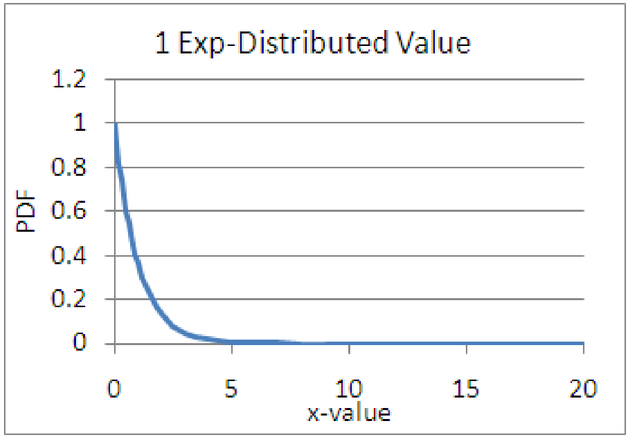


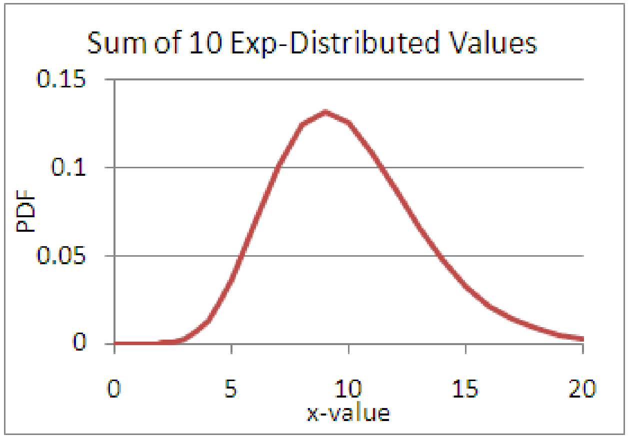
Exercise 6.3

Monte Carlo determination of exponential CL:

- In each row, simulate 50 exponentially distributed samples
- Determine the best lambda (exponential parameter) for each row
- Make a CDF plot of the lambda values
- Find the UCL and LCL for n=50 samples that found a lambda of 3
- Compare to the analytic expressions (T&T table 3.5):
- LCL = CHIINV(5%, 2*n) / (2*n)
- UCL = CHIINV(95%, 2*n+1) / (2*n)

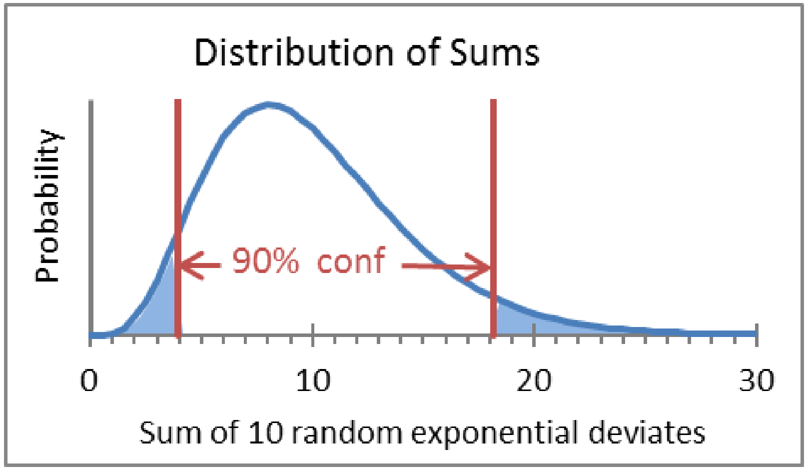
Analytic Exponential CL





- For $f(t) = \lambda\, e^{-\lambda t}$, best estimate for $1/\lambda$ is $\frac{1}{\mathrm{N}}\sum t_i$ where t_i are the data
- So, what is the *distribution* of $\sum t_i$ where t_i are distributed exponentially?

- Answer: a gamma or a chi-square distribution
- Confidence intervals taken from that



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