ECE 510 Lecture 3
Functions

Reliability Functions, T&T 2.1-6, 9
Distributions, T&T 3.1-4, 4.1-4, 5.1-3

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Reliability Functions

Reliability Functions

- Functions of time
  - CDF(x) → F(t)
- Survival function S(t) = 1-F(t)
- PDF(x) → f(t)

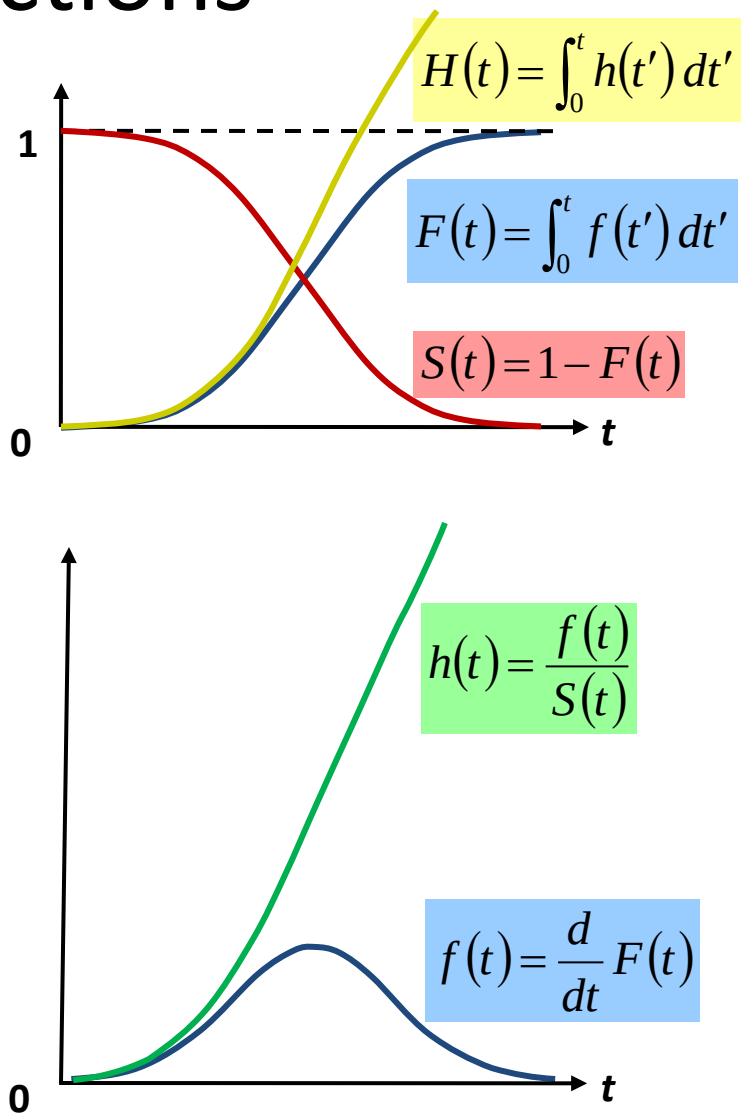
$$f(t) = \frac{\text{fraction of ORIGINAL population that fails in } dt}{dt}$$
$$= \frac{dF(t)}{dt} = -\frac{dS(t)}{dt}$$

- Hazard function h(t)

$$h(t) = \frac{\text{fraction of CURRENT population that fails in } dt}{dt}$$
$$= \frac{f(t)}{S(t)} = -\frac{dS(t)}{dt}\frac{1}{S(t)} = -\frac{d\ln S(t)}{dt}$$

- Cum hazard function H(t)

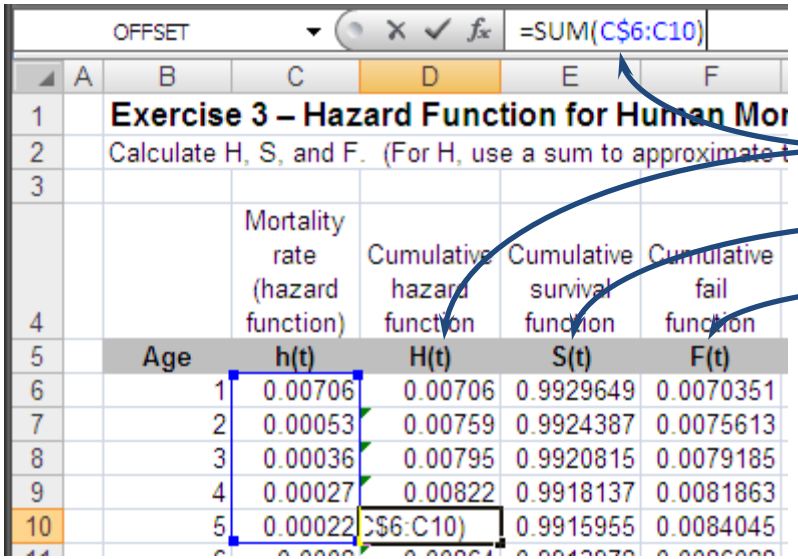
$$H(t) = \int_0^t h(t)\,dt$$
$$S(t) = \exp[-H(t)]$$
$$F(t) = 1 - \exp[-H(t)]$$



Exercise 3.1a

- Calculate H(t), S(t), and F(t) for the given human mortality data, and plot h(t), S(t), and F(t). The data is given as h(t) for each age, that is, the probability of a living person dying at the given age. Use a sum to approximate the integral for H(t).

Exercise 3.1a Solution, Part 1

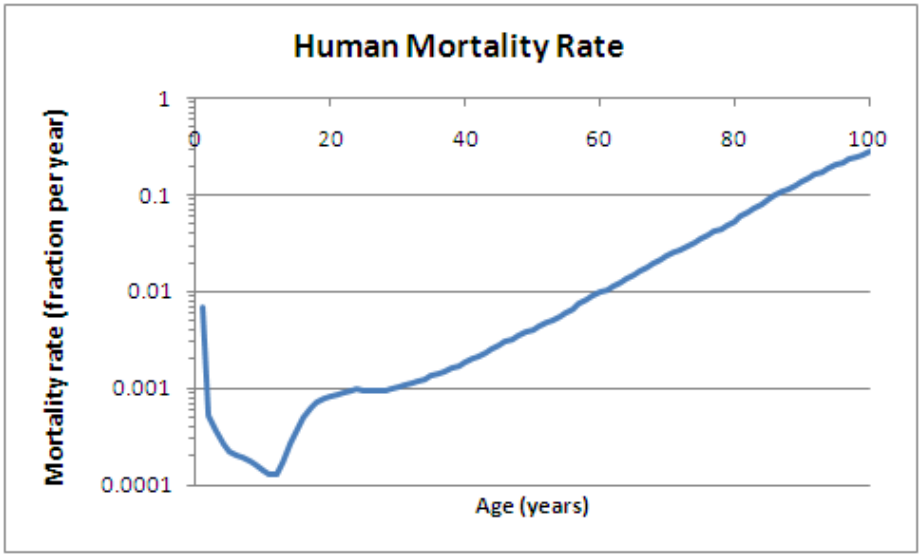


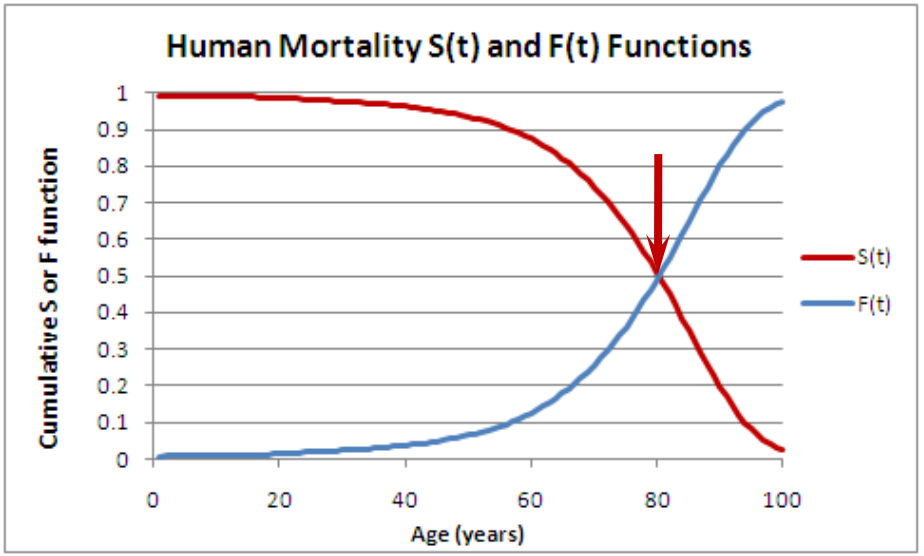
$$H(t) = \int_0^t h(t)\,dt$$

$$S(t) = \exp[-H(t)]$$

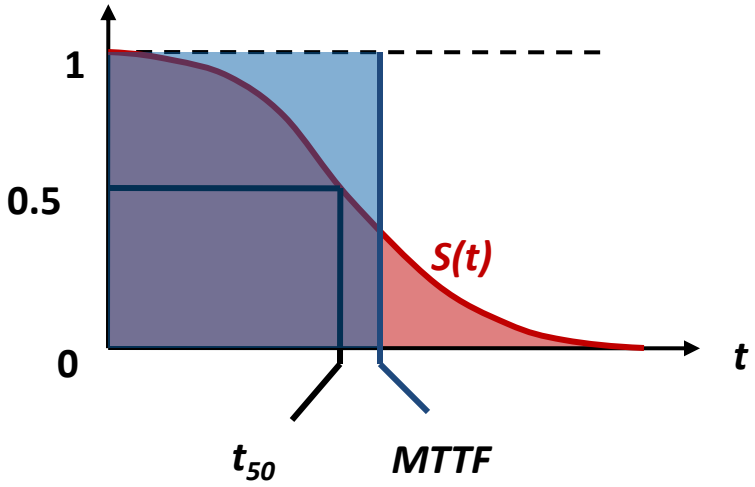
$$F(t) = 1 - \exp[-H(t)]$$

Human Mortality Graphs





Reliability Indicators



- Mean time to failure (MTTF)

$$MTTF = \int_0^\infty t\ f(t)\,dt = \frac{1}{N}\sum_{j=1}^{N} t_N = \int_0^\infty S(t)\,dt$$

- Median time to failure (t_{50}) is the solution of

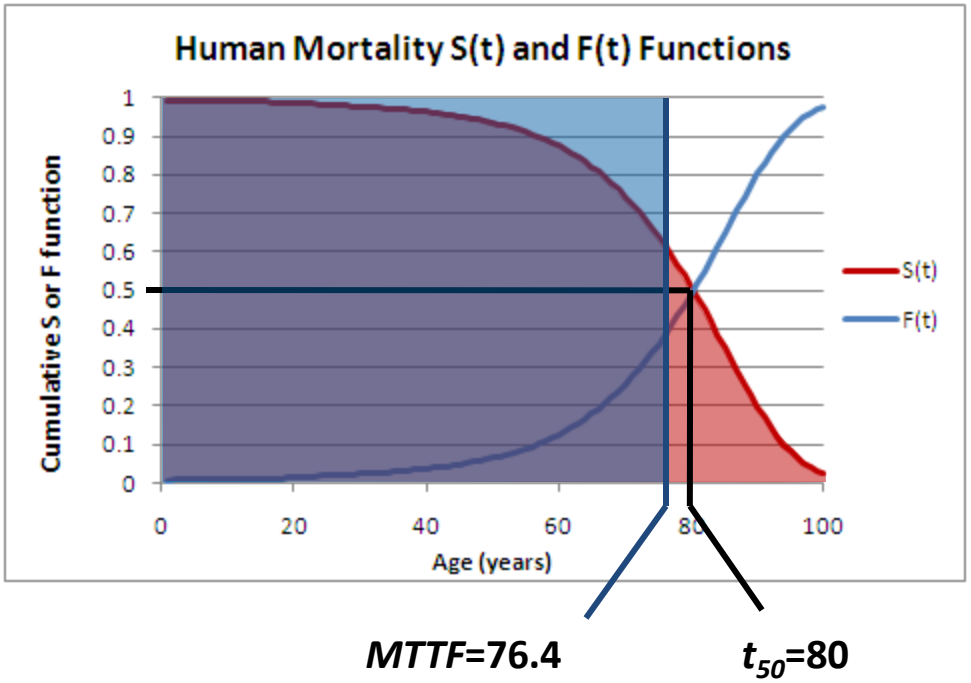
$$S(t_{50}) = 0.5$$

  - Time at which half of the initial population fails

Exercise 3.1b

- Find the mean and median times to failure for the human mortality data set from the last exercise

Exercise 3.1b Solution



- Sum S(t) to get MTTF

Reliability Measures: DPM

- Metric designed for low fail rates
- DPM = **D**efects **P**er **M**illion

% pass	% fail	DPM
99	1	10,000
99.9	0.1	1000
99.95	0.05	500
99.99	0.01	100
99.999	0.001	10

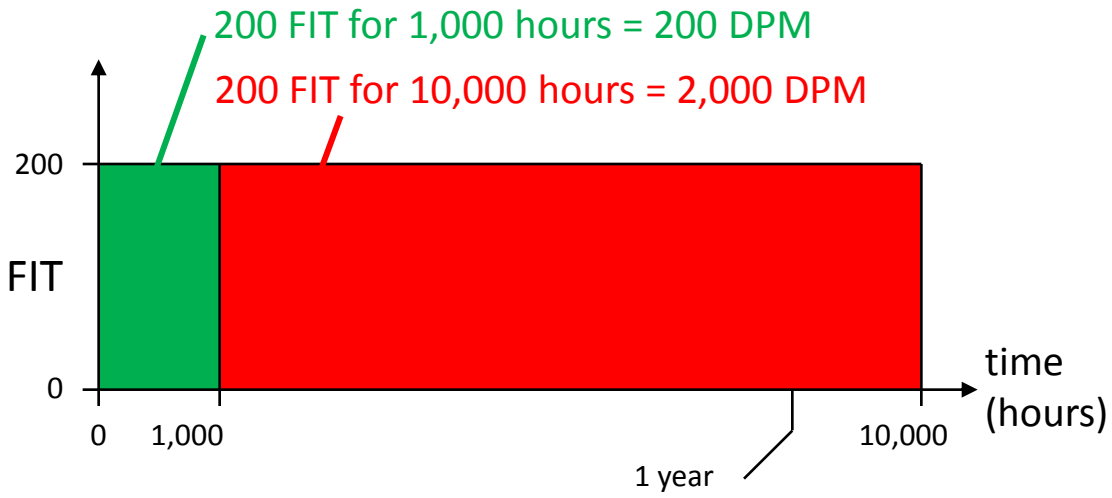
Typical target at end of life

Typical target at t=0

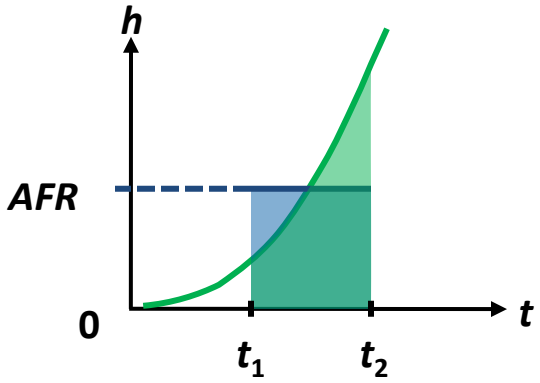
Typical range for semiconductor reliability

Reliability Measures: FIT

- FIT = **F**ailures **I**n **T**ime
- FIT is a fail *rate*, fails per **b**illion device hours
  - FIT = DPM per 1,000 hours
- DPM is a fail total, fails per **m**illion total devices
  - DPM = FIT * hours / 1,000



Reliability Indicators: AFR



- AFR, Average Fail Rate

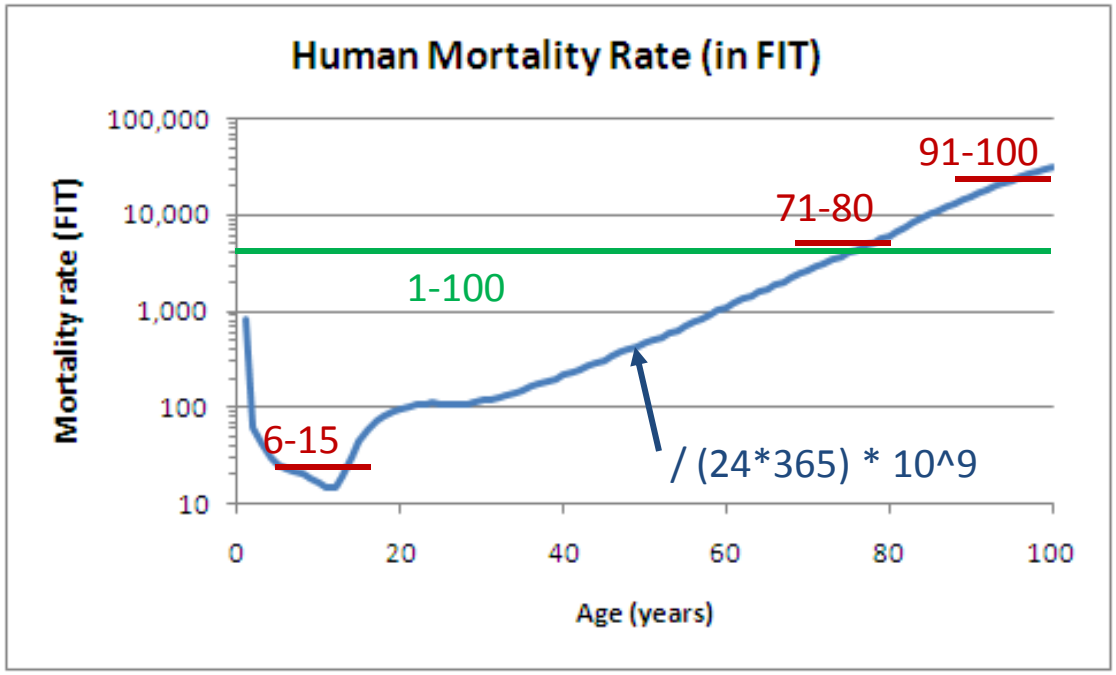
$$AFR(t_1,t_2)=\frac{\int_{t_1}^{t_2} h(t)\,dt}{t_2-t_1}=\frac{H(t_2)-H(t_1)}{t_2-t_1}=\frac{\ln S(t_1)-\ln S(t_2)}{t_2-t_1}$$

  - If t in hours, units are fail fraction per hour
  - Multiply by 10^9 for units of FIT

Exercise 3.1c

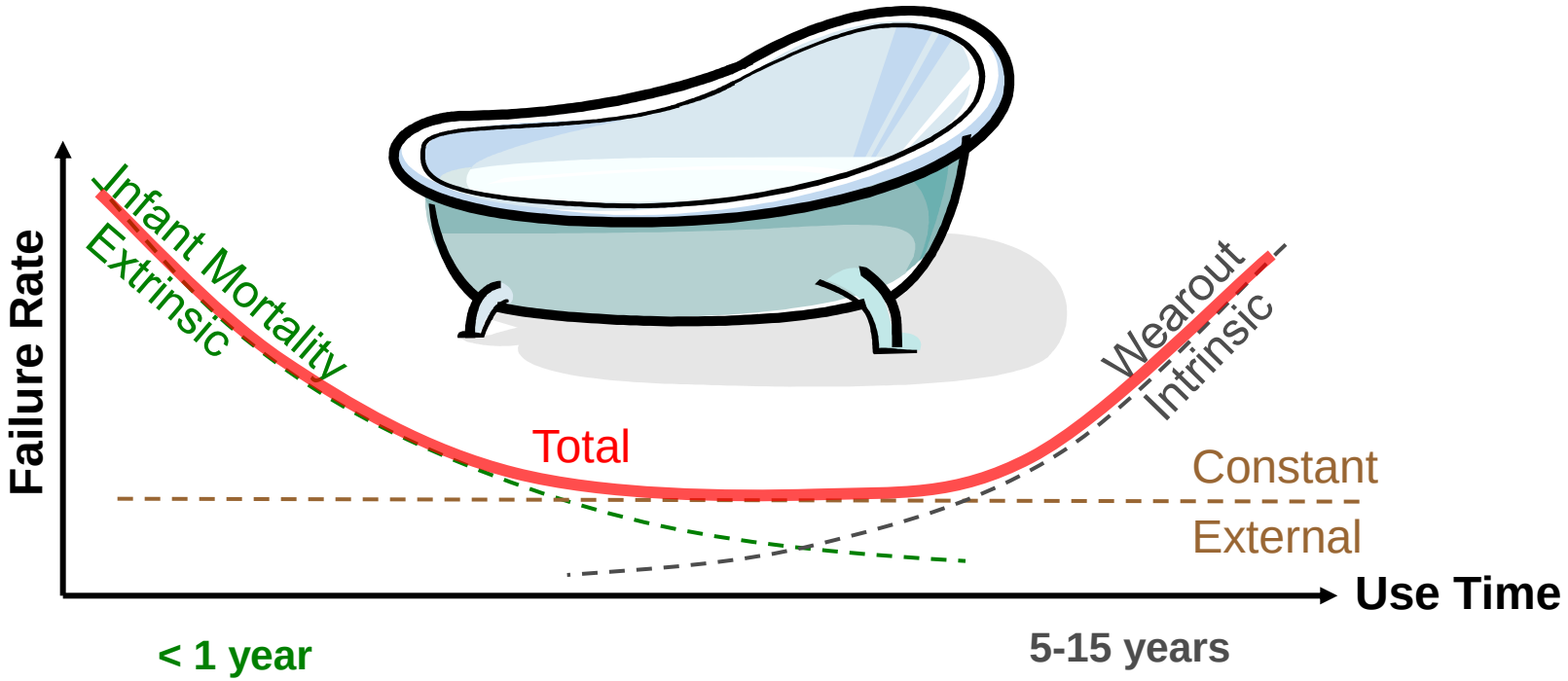
1. Plot the hazard function in FIT
2. Find the AFR (in FIT) for:
   - The 10-year range from ages 6 to 15
   - The 10-year range from ages 71 to 80
   - The 10-year range from ages 91 to 100
   - The entire 100-year range from ages 1 to 100

Exercise 3.1c Solution



Age Range	AFR (FIT)
6-15	22
71-80	4,311
91-100	24,116
1-100	4,270

The Bathtub Curve



- Infant Mortality (IM) from latent reliability defects
- Wearout from reliability mechs like oxide wearout
- Constant from external effects like radiation
- Many versions of this graph – it is a very important concept

The End