Form and Capacitance of Parallel-Plate Capacitors

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***Abstract*— In basic electrostatics, the formula for the capacitance of parallel-plate capacitors is derived, for the case that the spacing between the electrodes is very small compared to the length or width of the plates. However, when the separation is wide, the formula for very small separation does not provide accurate results. In our previously published papers, we use the boundary element method (BEM) to derive formulas for the capacitance of strip and disk capacitors that are applicable even when the separation is large. In this paper, we present results and formulas for the capacitances of square and rectangular capacitors.**

I. Introduction

THE approximate capacitance of parallel-plate capacitors is derived in simple electrostatics for the case in which the electric charge density on the plates is uniform and the fringing fields at the edges can be neglected [1]. The capacitance $C_0[F]$ is

$$C_0 = \frac{\epsilon S}{d}[F], \quad (1)$$

where $\epsilon[F/m]$ is the dielectric constant, $S[m^2]$ is the area of the plates (assured equal), and $d[m]$ is the separation of the two electrode plates. The total charge $Q_0[C]$ and the uniform surface change density $\sigma_0[C/m^2]$ on the plates are, respectively,

$$Q_0 = C_0V = \frac{\epsilon SV}{d}[C] \quad (2)$$

and

$$\sigma_0 = \frac{Q_0}{S} = \frac{\epsilon V}{d}[C/m^2] \quad (3)$$

where $V[V]$ is the potential difference between the two electrode plates.

Equations (1), (2), and (3) hold when d is far smaller than the plate width. As d becomes large compared to the smallest dimension of the plates, the equations do not provide accurate results. However, for some practical problems the plate separation is wide, and forumlas for the capacitance of capacitors with large plate separation are required [2].

The edge effect of a capacitor can be treated by rigorously solving the Laplace equation. Some papers for edge correction of a strip capacitor [3]–[7] and a disk capacitor [8], [9] have been published. We computed the capacitance of strip and disk capacitors by the boundary element method (BEM) and derived new empirical expressions for the capacitance. The capacitance values of microstrip lines and disk capacitor calculated by the new expression agreed well with results of other analytical expressions and with measured data [10], [11]. In this paper results for the normalized capacitance of the parallel-plate rectangular capacitor are computed by the same method.

In Section II, the BEM for the calculation of capacitance of the parallel-plate rectangular capacitors is presented. In Section III, the charge distribution densities on the electrode plate of parallel-plate square capacitors are computed and compared with those of strip and disk capacitors. In Section IV, capacitance of the parallel-plate square capacitors is computed, and a new empirical expression for the capacitance is derived from the numerical results. The capacitance of a parallel-plate rectangular capacitors is also given in this section. In Section V, a discussion and conclusion concerning the capacitance of the parallel-plate capacitors are presented.

II. Boundary Element Method for Parallel-Plate Rectangular Capacitors

The basic field equation for the calculation of capacitance of capacitors is the Laplace equation for the electrostatic field:

$$\nabla^2 u = 0. \quad (4)$$

The mathematical formulation of the BEM is given in [12] and [13]. We have given the results for the parallel-plate strip capacitor [10] and the parallel-plate disk capacitor [11]. In this paper, the BEM for the calculation of capacitance of parallel-plate rectangular capacitors is presented.

The parallel-plate rectangular capacitors in an infinite space are divided into mn boundary segments with equal area ($= wL/\mathrm{mn}$) in Fig. 1, where w is the width of the rectangular plate and L is the length of the rectangular plate. The identifier of the boundary elements of plates is denoted as $1 \sim 2$ mn. In the general BEM, electrode plates are not always divided into boundary elements with equal area, but in this problem the equal division makes the numerical procedure of calculation easy and efficient. Here, the simplest approximation for the surface density charge in a boundary element is adopted, and it is assumed that $(\mathrm{grad}\ u)^\bullet \vec{n}$ is constant for each element (constant element method). Then, the Green solution [12], [13] is

$$u(\vec{r}) = \frac{1}{4\pi}\int_{\Omega_A} \frac{1}{|\vec{r} - \vec{r'}|} q\, d\Omega_A + \frac{1}{4\pi}\int_{\Omega_B} \frac{1}{|\vec{r} - \vec{r'}|} q\, d\Omega_B, \quad (5)$$

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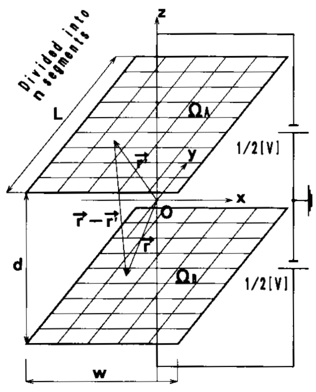

Fig. 1. Model of a parallel-plate rectangular capacitor.

$$u(\vec{r}_i) = \frac{1}{4\pi}\sum_{j=1}^{\mathrm{mn}}\int_{\Omega_j}\frac{1}{|\vec{r}_i - \vec{r}_j'|}q_j d\Omega_j + \frac{1}{4\pi}\sum_{j=\mathrm{mn}+1}^{2\,\mathrm{mn}}\int_{\Omega_j}\frac{1}{|\vec{r}_j - \vec{r}_j'|}q_j d\Omega_j \tag{6}$$

$(i = 1, 2, 3, \cdots, 2\,\mathrm{mn})$.

Here, Ω_j is the area of the jth element and the integration is carried out on Ω_j. q is the surface charge divided by permitivity of medium. The discrete form of (6) is

$$u(\vec{r}_i) = \sum_{j=1}^{\mathrm{mn}} s_{ij}q_j + \sum_{j=\mathrm{mn}+1}^{2\,\mathrm{mn}} f_{ij}q_j \tag{7}$$

where

$$q_j = [(\mathrm{grad}\ u^\bullet \vec{n}]_j \text{ on } \Omega_j. \tag{8}$$

S_{ij} is the boundary integration between the elements on the same rectangular plate, and f_{ij} is the boundary integration between the elements on the opposite rectangular plate. The potential of the plates is set to $1/2[V]$ and $-1/2[V]$, with vanishing potential at infinity. The choice of plate potentials ($1/2[V]$ and $-1/2[V]$) is justified only in the symmetric plate situation. This leads to improved calculation speed and accuracy. By putting $\vec{r}$ on the center of each boundary element and accounting for boundary conditions, the boundary integrations become

$$s_{ij} = \frac{1}{4\pi}\int_p^q\int_g^h \frac{1}{\sqrt{(x-X)^2+(y-Y)^2}}dxdy \tag{9}$$

and

$$f_{ij} = \frac{1}{4\pi}\int_p^q\int_g^h \frac{1}{\sqrt{(x-X)^2+(y-Y)^2+d^2}}dxdy. \tag{10}$$

These integrations can be done analytically. The bounds of integrations are

$$g = \frac{k'-1}{m}w, h = \frac{k'}{m}w, X = \frac{2k-1}{2m}w, \tag{11}$$

$$p = \frac{l'-1}{n}L, q = \frac{l'}{n}L, Y = \frac{2l-1}{2n}L, \tag{12}$$

$$i = m(k-1)+l, j = m(k'-1)+l', \tag{13}$$

$$(k = 1,2,3,\cdots,m), (k' = 1,2,3,\cdots,m), \tag{14}$$

$$(l = 1,2,3,\cdots,n), (l' = 1,2,3,\cdots,n). \tag{15}$$

Equation (7) is expressed as a matrix of order 2mn that is

$$[A]\{q\} = \{u\} \tag{16}$$

and

$$[A] = \begin{bmatrix} S & F \\ F & S \end{bmatrix} \tag{17}$$

where S and F with order mn are the submatrixes of $[A]$. Their elements are presented by (9) and (10). $\{q\}$ is a column of unknown q_j, and $\{u\}$ is a column whose upper mn components are $1/2[V]$ and whose lower mn components are $-1/2[V]$. The charge of each boundary element of plates is given by the solution of (16). The capacitance $C_R[F]$ is presented by considering $V_R = 1[V]$:

$$C_R = \frac{Q_R}{V_R} = \frac{\epsilon w L}{\mathrm{mn}}\sum_{i=1}^{\mathrm{mn}} q_i[F] \text{ on } \Omega_A = -\frac{\epsilon w L}{\mathrm{mn}}\sum_{i=\mathrm{mn}+1}^{2\mathrm{mn}} q_i[F] \text{ on } \Omega_B \tag{18}$$

where wL/mn is the area of each boundary element.

The calculation of the capacitance of the parallel-plate square capacitor $C_S[F]$ is presented by considering $L = w$, $n = m$, and $V_S = 1[V]$:

$$C_S = \frac{Q_S}{V_S} = \frac{\epsilon w^2}{m^2}\sum_{i=1}^{m^2} q_1[F] \text{ on } \Omega_A = -\frac{\epsilon w^2}{m^2}\sum_{i=m^2+1}^{2m^2} q_i[F] \text{ on } \Omega_B. \tag{19}$$

A model of a parallel-plate square capacitor is presented in Fig. 2.

III. Charge Distribution on Plates

A. Charge Distribution of the Parallel-Plate Square Capacitor

The charge distribution on the plates of a parallel-plate square capacitor is computed. To accomplish this, we apply the LU decomposition method [14] to the solution of linear equation (16). In Fig. 3 the normalized charge density on the square plates calculated by the BEM is plotted against the normalized position along the half width of the plate, taking b as the parameter. The normalized charge density σ_{SN} is defined as the charge density divided by σ_{S0}, i.e.,

$$\sigma_{SN} = \frac{\sigma_S}{\sigma_{S0}} = \frac{\sigma_S d}{\epsilon V}, \tag{20}$$

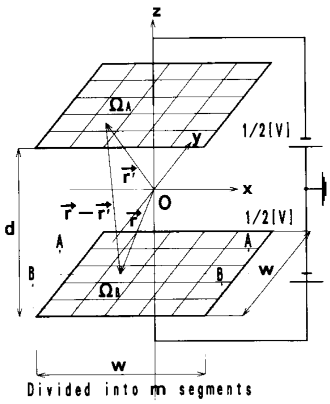

Fig. 2. Model of a parallel-plate square capacitor.

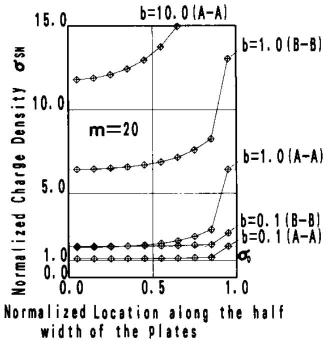

Fig. 3. Normalized charge density on the plates against location normalized by width of the square for $b = 0.1, 1, 10$.

where σ_S is the charge density computed by the BEM for the parallel-plate square capacitor, σ_{S0} is the charge density in simple electrostatics given by (21),

$$\sigma_{S0} = \frac{\epsilon V_S}{d}[C/m^2], \tag{21}$$

and b is the aspect ratio that is given by (22):

$$b = \frac{d}{w} = \frac{\text{plate separation}}{\text{plate width}} = \text{aspect ratio}. \tag{22}$$

In Fig. 3 the normalized charge densities are shown, respectively, for $b = 0.1, 1, 10$, where the solid lines are normalized charge density on the plate along the section A-A in Fig. 2, that is, the center of the square plate, and the dotted lines are the normalized charge density on the plate along the section B-B in Fig. 2, that is, the end of the square plate. In both sections A-A and B-B, the normalized charge density at the edges becomes much larger than that at the center even when b is small. The normalized charge density for $b = 10$ along the section B-B is so large that it can not be presented in Fig. 3. The assumption that the density is uniform does not hold until b becomes very close to zero. The normalized charge density along section B-B is always larger than that along section A-A in same aspect ratio, and the normalized charge density at the edge along section A-A equals that at the center along section B-B in same aspect ratio. As b increases, the normalized density of charge becomes large, though the total charge decreases. And even the normalized density at the center is much greater than σ_0. In the limiting case of $b \to \infty$, σ_S equals the charge density of the one square capacitor in an infinite space.

B. Form and Charge Distribution of the Parallel-Plate Capacitors

In our previous published paper, the normalized charge distribution on plates of a parallel-plate strip capacitor and a parallel-plate disk capacitor are presented [10], [11]. Here we consider the charge distribution of the parallel-plate strip capacitor, the parallel-plate disk capacitor, and the parallel-plate square capacitor. The normalized charge density of the parallel-plate strip capacitor σ_{PN} and that of the parallel-plate disk capacitor σ_{DN} is defined as the charge density divided by σ_{P0} and σ_{D0}, i.e.,

$$\sigma_{PN} = \frac{\sigma_P}{\sigma_{P0}} \text{ and } \sigma_{DN} = \frac{\sigma_D}{\sigma_{D0}} \tag{23}$$

where

$$\sigma_{P0} = \frac{\epsilon V_P}{d}[C/m^2] \text{ and } \sigma_{D0} = \frac{\epsilon V_D}{d}[C/m^2]. \tag{24}$$

where σ_P and σ_D are the charge density computed by the BEM, σ_{P0} and σ_{D0} are the charge density from simple electrostatics, respectively, for the parallel-plate strip capacitor and the parallel-plate disk capacitor.

The aspect ratio of the parallel-plate strip capacitor is given by (22), and that of the parallel-plate disk capacitor is defined as

$$b = \frac{d}{2R} = \frac{\text{plate separation}}{\text{plate diameter}} = \text{aspect ratio}. \tag{25}$$

In Fig. 4 the normalized charge densities along the half width (for the parallel-plate strip, square and rectangular capacitor) or the radius (for the parallel-plate disk capacitor) of the plates calculated by the BEM are plotted, for $b = 1.0$. The normalized charge density at the edges becomes much larger than that at the center for all three types of capacitors. The normalized charge density of the parallel-plate square capacitor is larger than that of the parallel-plate strip capacitor. Also, the normalized charge density of the parallel-plate disk capacitor is larger than that of the parallel-plate square capacitor along the center (section A-A). However, the normalized charge density of the parallel-plate square capacitor along the edge (section B-B) is larger than that of the parallel-plate disk capacitor.

IV. Capacitance

A. Normalized Capacitance of the Parallel-Plate Square Capacitor

The normalized capacitance of the parallel-plate square capacitor C_{SN} is defined as

$$C_{SN} = \frac{C_S}{C_{S0}} \tag{26}$$

where

$$C_{S0} = \frac{\epsilon S_S}{d} = \frac{\epsilon w^2}{d}[F] \tag{27}$$

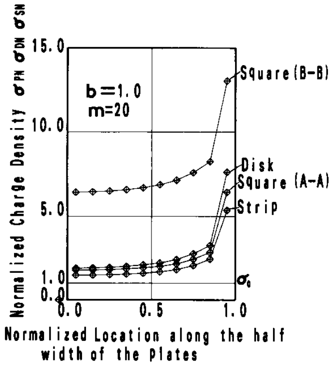

Fig. 4. Normalized charge density on the palates of the parallel-plate strip, disk, and square capacitor.

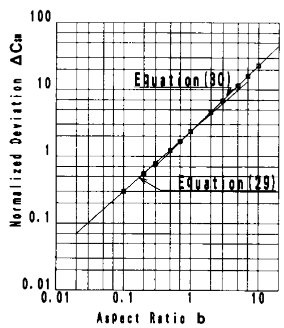

Fig. 5. Normalized deviation of the parallel-plate square capacitor ΔC_{SN} against aspect ratio b.

where C_S is the capacitance computed by the BEM. The accuracy of calculation of C_{SN} improves with the increase of m. However, with increasing m the execution time and required memory for computation rapidly increase [15]. To overcome this difficulty of the BEM, an extrapolation method is applied for $m = 10, 15, 20$. This method has been discussed in our previous papers [10], [11], [15], [16]. Particularly, the errors of extrapolation method were mentioned in [15].

The normalized deviation ΔC_{SN} is defined as

$$\Delta C_{SN} = C_{SN} - 1 = \frac{C_{SN} - C_{S0}}{C_{S0}}. \tag{28}$$

In Fig. 5 the normalized deviation is plotted by a solid line against the aspect ratio b.

The fringe field is no longer negligible when $b = 1$. Applying regression analysis to the data of ΔC_{SN}, a simple empirical expression is derived when C'_{SN} is $0.1 \leq b \leq 10.0$:

$$C'_{SN} = 1 + 2.343b^{0.891}(0.1 \leq b < 1.0), \tag{29}$$

$$C'_{SN} = 1 + 2.343b^{0.992}(1.0 \leq b \leq 10.0). \tag{30}$$

The relative errors of (29) and (30) against the numerical value computed by the BEM are at most 1.5[%].

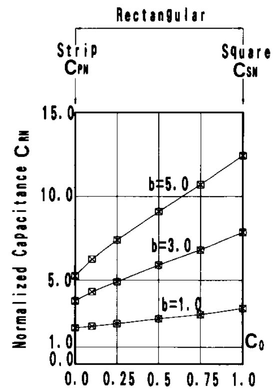

Fig. 6. Normalized cpapcitance of the parallel-plate rectangular capacitor against aspect ration b'.

B. Normalized Capacitance of the Parallel-Plate Rectangular Capacitor

The normalized capacitance of the parallel-plate rectangular capacitor C_{RN} is defined as

$$C_{RN} = \frac{C_R}{C_{R0}} \tag{31}$$

where

$$C_{R0} = \frac{\epsilon S_R}{d} = \frac{\epsilon w L}{d}[F]. \tag{32}$$

$C_R[F]$ is the capacitance computed by the BEM, $w[m]$ is the width of the rectangular plates, and $L[m]$ is the their length. Then the new aspect ratio of the rectangular plates b' is defined as

$$b' = \frac{L}{w}. \tag{33}$$

In Fig. 6 the normalized capacitance C_{RN} is plotted against $1/b'$ taking b as parameter. As $1/b'$ increases, the normalized capacitance C_{RN} becomes large. And as b increases, C_{RN} becomes large. In the limiting case $1/b' \to 0 (b' \to \infty$ or $L \to \infty)$, the normalized capacitance of the parallel-plate rectangular capacitor C_{RN} agrees with that of the parallel-plate strip capacitor C_{PN}. And in the limiting case of $1/b' \to 1 (b' \to 1)$, C_{RN} equals the normalized capacitance of the parallel-plate square capacitor C_{SN}.

C. Comparison of Strip, Disk, Square and Rectangular Capacitors

In this section, we compare the capacitance of parallel-plate strip, disk, square, and rectangular capacitors. The normalized capacitance of the parallel-plate strip capacitor C_{PN} is defined as

$$C_{PN} = \frac{C_P}{C_{P0}} \tag{34}$$

where

$$C_{P0} = \frac{\epsilon w}{d}[F/m] \tag{35}$$

TABLE I
NORMALIZED CAPACITANCE OF PARALLEL-PLATE CAPACITOR FOR GEOMETRICAL FIGURES

Aspect ratio b	Strip Capacitance C_{PN}	Disk Capacitance C_{DN}	Square Capacitance C_{SN}
0.1	1.16983	1.31809	1.29980
0.2	1.29661	1.58007	1.54987
0.3	1.41465	1.83007	1.78426
0.5	1.63226	2.31845	2.23581
0.7	1.83463	2.80352	2.67950
1.0	2.12055	3.53479	3.34336
2.0	2.98619	6.01398	5.58217
3.0	3.77608	8.52857	7.85246
5.0	5.23271	13.59268	12.42957
7.0	6.58852	18.67215	17.02431
10.0	8.50262	26.30143	23.92769

and C_P is the capacitance of the parallel-plate strip capacitor computed by the BEM. The normalized capacitance of the parallel-plate disk capacitor C_{DN} is defined as

$$C_{DN} = \frac{C_D}{C_{D0}} \tag{36}$$

where

$$C_{D0} = \frac{\epsilon S_D}{d} = \frac{\epsilon \pi R^2}{d}[F] \tag{37}$$

and C_D is the capacitance of the parallel-plate disk capacitor computed by the BEM. The normalized capacitance of the parallel-plate square capacitor and that of the parallel-plate rectangular capacitor already have been defined, respectively, as (26) and (31).

In Fig. 7 the normalized capacitance of the parallel-plate capacitors are plotted against the aspect ratio b. In Table I the values of normalized capacitance of the parallel-plate capacitor are shown. The aspect ratio of the parallel-plate strip, square, and rectangular capacitors is defined by (22), and that of parallel-plate disk capacitor is defined by (25). As b increases, all normalized capacitances increase. However, the rate of variations is different for the different forms. To make clear the relationship between form and capacitance of the parallel-plate capacitors, in Fig. 8 the capacitance of the parallel-plate disk capacitor $C_D[F]$, that of the parallel-plate square capacitor $C_S[F]$, and that of the parallel-plate rectangular capacitor $C_R[F]$ are plotted against the plate separation $d[m](1/d[1/m])$. In Fig. 8 the area of every capacitor is defined as $S = 1.0[m^2]$, and the dielectric constant is defined as $\epsilon = 1.0[F/m]$. The dotted line in Fig. 8 is the capacitance $C_0[F]$ that is given by (1). The solid lines in Fig. 8 are the calculated capacitance of the parallel-plate disk capacitor $C_D[F]$, that of parallel-plate square capacitor $C_S[F]$, and that of the parallel-plate rectangular capacitor $C_R[F]$. When $1/d[1/m]$ is large ($d[m]$ is small), they are in proportion to $1/d[1/m]$. However, the rate of variations is different for the forms. When $1/d[m]$ is small ($d[m]$ is large), the proportion does not hold. It is important that all of capacitances (C_D, C_S and $C_R[F]$) approach a limiting value as $1/d \rightarrow 0[1/m](d \rightarrow \infty[m])$.

D. Capacitance of the Parallel-Plate Disk Capacitor for Large Aspect Ratio

The value of the capacitance of the parallel-plate disk capacitor in limiting case of $1/d \rightarrow 0[m](d \rightarrow \infty[m])$ is considered. The capacitance of single disk capacitor in an infinite space is analytically derived [17] as

$$C_D^* = 8\epsilon R[F]. \tag{38}$$

When the area of the single disk plate is $S = 1.0[m^2]$, the radius of the disk plate $R[m]$ is

$$R = \sqrt{\frac{S}{\pi}} = \sqrt{\frac{1}{\pi}} \simeq 0.564[m]. \tag{39}$$

The capacitance of a single disk capacitor becomes

$$C_D^* = 8 \times 1.0[F/m] \times 0.564[m] = 4.512[F]. \tag{40}$$

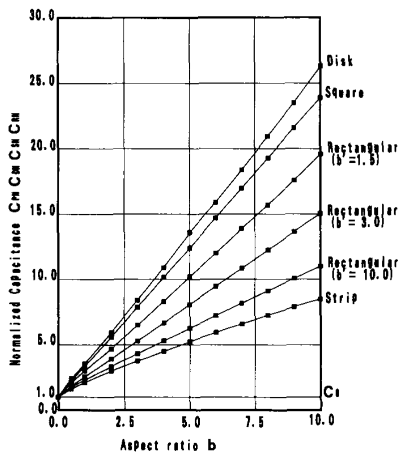

Fig. 7. Form and normalized capacitance of the parallel-plate capacitors against aspect ratio b.

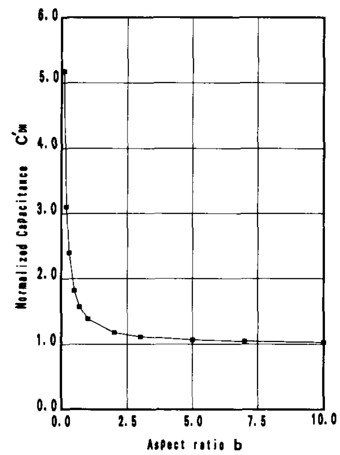

Fig. 9. New normalized capacitance of parallel-plate disk capacitor against aspect ratio b.

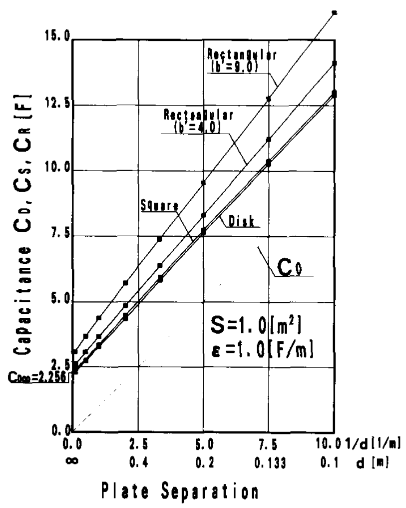

Fig. 8. Form and normalized capacitance of the parallel-plate capacitors against plate separation $d[m]$.

In the limiting case of $1/d \to 0[1/m](d \to \infty[m])$ and for the potential at infinity equal to 0 [V], all field lines are generated between the electrode plate and infinity, and no field lines are generated between the two electrode plates of the parallel-plate disk capacitor. Therefore, the analytical capacitance of the parallel-plate disk capacitor in the limiting case of $1/d \to 0[1/m](d \to \infty[m])$ is

$$C_{D\infty} = \frac{C_D^* C_D^*}{C_D^* + C_D^*} = \frac{C_D^*}{2} = \frac{4.512}{2} = 2.256[F] \tag{41}$$

where C_D^* is the capacitance of a single disk capacitor. This value agrees very well with the numerical capacitance in Fig. 8.

In simple electrostatics the approximate capacitance of parallel-plate capacitors is derived as (1). Many capacitor applications are based on this expression. The exact capacitance of the parallel-plate capacitor becomes important for highly precise measurement of dielectric constant and for calculating of capacitance in LSI and microstrip line, etc. Therefore, in many previous papers and this paper the corrective expression of the capacitance of the parallel-plate capacitor were normalized by closely spaced capacitor C_0. However, for large aspect ratios, the expression would be more clearly normalized by the infinite spacing capacitance.

The new normalized capacitance of parallel-plate disk capacitor C'_{DN} is defined as

$$C'_{DN} = \frac{C_D}{C_{D\infty}} \tag{42}$$

where $C_{D\infty}(= 4\epsilon R)$ is the infinite spacing capacitance of parallel-plate disk capacitor.

The capacitance of parallel-plate disk capacitor is

$$C_D = C_{DN} {}^\bullet C_{D0} = C_{DN} \frac{\epsilon \pi R^2}{d} [F]. \tag{43}$$

The new normalized capacitance C'_{DN} becomes

$$C'_{DN} = \frac{\pi R}{4d} C_{DN} = \frac{\pi}{8b} C_{DN} = 0.3927 \frac{C_{DN}}{b}. \tag{44}$$

In Fig. 9 the new normalized capacitance C'_{DN} is plotted against the aspect ratio b. In the limiting case of $b \to 0$, C'_{DN} becomes infinity, and in the limiting case of $b \to \infty$, C'_{DN} becomes unity asymptotically. When b is larger than 3, the capacitance of parallel-plate disk capacitor is almost determined by the infinite spacing capacitance $C_{D\infty}$.

E. New Empirical Expression for the Normalized Capacitance of Parallel-Plate Disk Capacitor

The estimate for the capacitance of parallel-plate capacitor

$$C \simeq C_0 + C_\infty \tag{45}$$

and that of parallel-plate disk capacitor

$$C_{DN} = \frac{C_D}{C_{D0}} \simeq 1 + \frac{8}{\pi} b = 1 + 2.5465b \tag{46}$$

TABLE II
NORMALIZED CAPACITANCE OF EMPIRICAL EXPRESSION FOR PARALLEL-PLATE DISK CAPACITOR AGAINST ASPECT RATIO

Aspect ratio *b*	Computed by the BEM	Expression (46)	Relative error of (46) (%)	Expression (47)	Relative error of (47) (%)
0.1	1.31809	1.25465	-4.813	1.32132	0.245
0.2	1.58007	1.50930	-4.479	1.58640	0.401
0.3	1.83007	1.76394	-3.614	1.83342	0.183
0.5	2.31845	2.27324	-1.950	2.29779	-0.891
0.7	2.80352	2.78254	-0.748	2.80636	0.101
1.0	3.53479	3.54648	0.331	3.56381	0.821
2.0	6.01398	6.09296	1.313	6.06441	0.840
3.0	8.52857	8.63944	1.300	8.54138	0.150
5.0	13.59268	13.73240	1.028	13.45302	-1.027
7.0	18.67215	18.82535	0.820		
10.0	26.30143	26.46479	0.621		

are very interesting expressions for simplification and covering a wide area of b. The capacitance of parallel-plate disk capacitor which is computed by the BEM was compared with the previous results [11]. The results of the BEM agree well with the previous results. Applying the regression analysis to the data of the BEM, a simple empirical expression

$$C_{DN} = 1 + 2.367b^{0.867}(0.005 \le b \le 0.5)$$
$$C_{DN} = 1 + 2.564b^{0.982}(0.5 \le b \le 5.0) \qquad (47)$$

was derived in our published paper [11]. The relative errors of this expression against the numerical values computed by the BEM are at most $1[\%]$ $(0.05 \le b \le 5)$. In Table II, the results of (46) and (47) are compared with the results of the BEM. Expression (46) provides for a wide range of b, with a few errors. The relative errors of (46) against the numerical values computed by the BEM are at most $5[\%]$ $(0.1 \le b \le 10)$. This decreases with the increase of b.

V. CONCLUSION

In this paper the relationship between the form and the capacitance of parallel-plate capacitors is considered. When $1/d[1/m]$ is large ($d[m]$ is small), the capacitance of the parallel plate is in proportion to $1/d[1/m]$. However, when $1/d[m]$ is small ($d[m]$ is large), the proportion does not hold. The value of the capacitance of the parallel-plate disk capacitor in limiting case of $1/d \to 0[m](d \to \infty[m])$ is given. For a given area and spacing, the capacitance of the parallel-plate disk capacitor is the smallest of that for any parallel-plate capacitor. This arises because a disk has the smallest ratio of edge length to plate area.

The normalized capacitance of the parallel-plate square capacitor and that of the parallel-plate rectangular capacitor are calculated by the BEM. Also, a "formula" for the normalized capacitance of the parallel-plate square capacitor is presented. The empirical formula, which is applicable even when the aspect ratio becomes far larger than unity, is derived for the normalized capacitance of the parallel-plate square capacitor. In the limiting case of $1/b' \to 0(b' \to \infty \text{ or } L \to \infty)$, the normalized capacitance of the parallel-plate rectangular capacitor becomes that of the parallel-plate strip capacitor.

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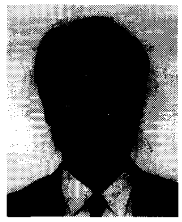

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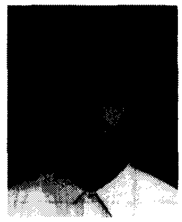

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