STATISTICS FOR ELECTROMIGRATION TESTING*

Harry Schafft and James A. Lechner
National Bureau of Standards
Gaithersburg, MD 20899

and

Babak Sabi, Mike Mahaney, and Ron C. Smith
Intel Corporation
Hillsboro, OR 97124

ABSTRACT

A comprehensive statistical basis is given for the design and conduct of electromigration stress tests that allows for the efficient use of test parts, equipment, and test time. It shows how to select the size of the sample, the required control of the stress conditions, and the number of failures required before halting the test in order to characterize metallization interconnects with a quantifiable level of confidence. The results are applicable to any failure mechanism for which the failure times obey a Normal or a log-Normal distribution.

1. INTRODUCTION

Electromigration is a metallization failure mechanism that continues to be of great concern for the reliability assessment of VLSI-sized microelectronic devices [1]. Accelerated electromigration tests [2] are used to obtain sample estimates of measures that describe the failure distribution. These estimates are used in assessing metallization reliability and in making major decisions for the selection of metallization and processing technologies. It is therefore important that such tests be designed, conducted, and analyzed to provide reliable and timely information that has a quantifiable level of confidence. To that end, this paper describes the use of statistical methods and procedures for designing and interpreting such tests.

Tests for characterizing a metallization's resistance to electromigration failure involve stressing a sample of metallization test lines at high temperature and high current density, and recording the time for each to fail. Experience has indicated that the time-to-fail of the specimens in the sample is empirically described by a log-Normal distribution. Three parameters of the distribution are commonly used to characterize the metallization for electromigration: the median-time-to-failure (t_{50}), the standard deviation of the logarithm of the failure times or sigma (σ), and a lower p-th percentile of the distribution of failure times (t_p).

Test results must be analyzed statistically in order to make a quantitative characterization of the population from a random sample. To illustrate the need for such an analysis, random samplings of ten were made from a continuous log-Normal distribution of failure times. The distribution was characterized by a t_{50} of 1.0 and a sigma of 0.9. One hundred such samplings are shown in Fig. 1. For these, sample estimates of t_{50} range from approximately 0.4 to 2.1 h.

This paper describes how the sample estimates of t_{50}, σ, and t_p and their confidence limits are affected by: (1) the size of the sample used in the test, (2) the sigma of the population, (3) the use of sample censoring, or halting the test before all specimens have failed, and (4) the uncertainties in the mean values of the stress conditions and in the variations about these means during the test.

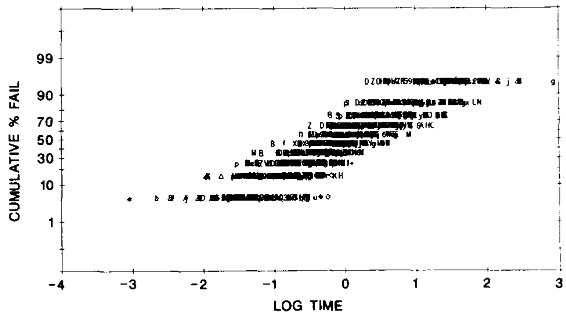

Fig. 1 - Distribution of the log of failure times versus cumulative percent failures obtained from 100 random samplings of a sample size of 10 from a log-normal distribution having a mean of zero (t_{50} = 1.0) and a sigma of 0.9. Different symbols are used to distinguish data of individual samplings.

An underlying assumption used in the paper is that the failure times are log-Normally distributed. Hence, the results are applicable to any failure mechanism for which the times to failure of the parts obey this distribution. The analysis presented applies only to characterizing the metallization at the stress conditions used for the test; it does not deal with extrapolating the results to use conditions.

The existence of freaks [3] in the population and bimodal distributions are not considered in this paper. Their statistical treatment in the context of this paper requires further study. If freaks are encountered and their number represents only a very small percentage of the sample, they should be omitted before the procedures discussed are applied.

2. CONFIDENCE INTERVALS

2.1 Introduction

Confidence intervals are given for t_{50}, σ, and t_p in terms of their respective sample estimates, t_{50S}, s, and t_{pS}, where the failure data of the entire sample is used. The sample estimate for t_p is expressed in terms of t_{50S} and s. There are a number of ways to calculate these sample estimates from the failure data. The best estimators of t_{50} and σ are obtained from the mean of the log of the failure times and the standard deviation of these log times, scaled to remove the bias [4]. The confidence intervals given in this section are based on the use of these estimators.

Sample estimates of t_{50} and σ are typically obtained by plotting the failure times on a log scale versus a Normal probability

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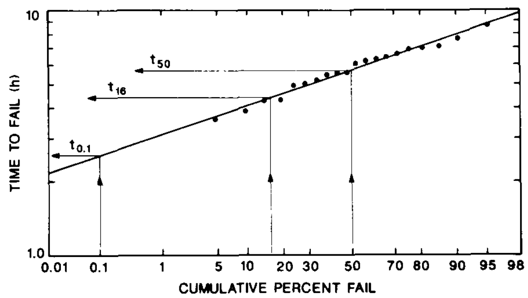

Fig. 2 - Representative plot of failure times on probability graph paper with a least-squares, straight-line fit to the data. The 50, 16, and 0.1 percent sample failure-time estimates are shown.

scale of cumulative percent failed, as shown in Fig. 2. A best straight-line fit to the points is made using an unweighted, least-squares fitting procedure [5] and the intersection of the line with the 50% point defines the sample estimate, t_{50S}, of t_{50}. The sample estimate, s, of sigma is obtained from the difference between the logarithms of t_{50S} and of the percentile that is one standard deviation from t_{50S}, or approximately t_{16S}.

Compared to the best estimators of t_{50} and σ described above, the least-squares method is equivalent for estimating t_{50} but is less efficient for estimating σ. The estimate of σ is also biased. Preliminary results from a study now underway indicates that for a sample size of ten, the mean and the standard deviation of the distribution of s values obtained with the least-squares method are approximately 15% larger. With increasing sample size, the bias decreases and the efficiency increases. The least-squares method suffers because it assumes that the data points are independent and vary about the line with a constant variance. Both assumptions are violated when the points are ordered as they are for probability plots, and when the variance is clearly not constant with the order in which they are plotted, as illustrated in Fig. 1.

It is recommended that for complete samples the mean of the log of the failure times and the corrected standard deviation of the log times be used as the estimators of t_{50} and σ, respectively, and that the practice of plotting the data as described above be continued to determine the validity of the assumption of a well-behaved, log-Normal failure distribution.

2.2 Population Median Time To Failure

The variability of the median-time-to-failure, t_{50}, of a population of test lines that can be fabricated from a given metallization, is determined as follows. Assume that when the structures are subjected to an electromigration stress test, the logarithms of the failure times are normally distributed. If t_{fi} is the time for the i-th structure to fail and $Y_i = ln\ t_{fi}$, then Y_i belongs to a population of Y values having a Normal distribution with a population mean μ and a standard deviation σ.

Because the distribution of the population of Y's is Normal, the mean and the median are equal so $\mu = ln\ t_{50}$. Hence:

$$t_{50} = exp\ \mu.$$

If a linear regression analysis is used to estimate the median of a complete sample of Y values, the sample mean and median are equal. Hence:

$$t_{50S} = exp\ \bar{Y}$$

where t_{50S} is the sample estimate of the median of the population of failure times and $\bar{Y}$ is the sample mean.

If σ is known and N test lines are selected at random and stressed to failure, the probability is $1-\alpha$ that $\bar{Y}$ is within the limits:

$$\mu - z(1-\alpha/2)\cdot\sigma/\sqrt{N} < \bar{Y} < \mu + z(1-\alpha/2)\cdot\sigma/\sqrt{N},$$

where $z(\alpha)$ is the 100α percentile obtained from a Normal distribution [6].* The equivalent limits for t_{50S} are:

$$t_{50}\cdot exp(-z(1-\alpha/2)\cdot\sigma/\sqrt{N}\,) < t_{50S} < t_{50}\cdot exp(z(1-\alpha/2)\cdot\sigma/\sqrt{N}\,).$$

This can be rewritten to define the $100(1-\alpha)\%$ confidence interval for t_{50}:

$$t_{50S}\cdot exp[-z(1-\alpha/2)\cdot\sigma/\sqrt{N}\,] < t_{50} < t_{50S}\cdot exp[z(1-\alpha/2)\cdot\sigma/\sqrt{N}\,].$$

If σ is unknown, then σ must be replaced by the sample standard deviation s and the z factors must be replaced by the percentile factors of the t distribution [7]. The $100(1-\alpha)\%$ confidence interval for t_{50} will then be given by:

$$t_{50S}\cdot\ exp[-t(1-\alpha/2, N-1)\cdot s/\sqrt{N}\,] < t_{50} < t_{50S}\cdot\ exp[t(1-\alpha/2, N-1)\cdot s/\sqrt{N}\,]. \quad (1)$$

The limits that define this interval and others in the paper are two-sided limits. One-sided limits are obtained by replacing $t(1-\alpha/2)$ by $t(1-\alpha)$. The t factors differ significantly from the z factors only for N values less than approximately 20 where they increase as N becomes smaller.

The confidence interval for t_{50}, described by eq.1, decreases with increasing sample size and decreasing values of s as shown in Fig. 3. The figure demonstrates that when s is sufficiently small, sample sizes of 10 or less provide relatively small confidence intervals which are smaller than those obtained when s is larger, even though many more samples are used. For example, the confidence interval for a sample size of 7, when s equals 0.3, is approximately as small as the interval for a sample size of 20, when s is 0.6, and for a sample size of 45, when s is 0.9.

2.3 Sigma

The confidence interval for the population sigma, σ, is based on the sampling distribution of $(N\text{-}1)s^2/\sigma^2$, which is the chi-

* For a 68.27% (one sigma) confidence level, the z factor is 1.0 while for a 90% confidence level, the z factor is 1.645.

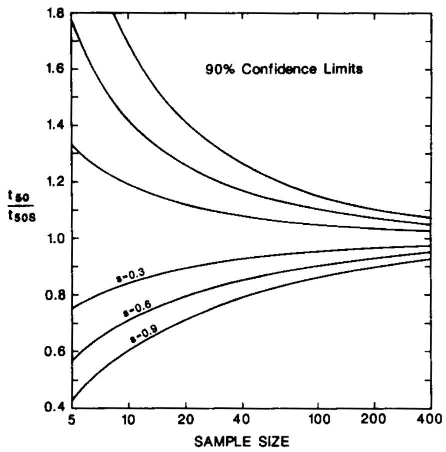

Fig. 3 - Ninety-percent confidence limits for t_{50}, divided by t_{50S}, versus sample size for three sample estimates of sigma.

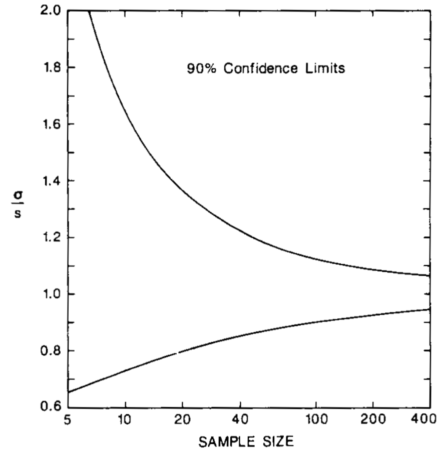

Fig. 4 - Ninety-percent confidence limits for σ, divided by s, versus sample size.

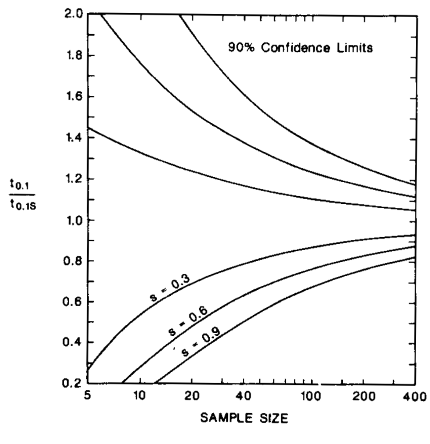

Fig. 5 - Ninety-percent confidence limits of $t_{0.1}$, divided by $t_{0.1S}$, versus sample size for three sample estimates of sigma.

square distribution with N-1 degrees of freedom [8]. The $100(1-\alpha)\%$ confidence limits for σ are given by:

$$s \cdot \sqrt{\frac{N-1}{\chi^2(1-\alpha/2, N-1)}} < \sigma < s \cdot \sqrt{\frac{N-1}{\chi^2(\alpha/2, N-1)}}. \quad (2)$$

The 90% confidence limits for σ, divided by s, are plotted in Fig. 4. They show that the confidence interval decreases with increasing sample size.

2.4 Population Percentile Failure Time

For the Normally distributed population of Y_i values where $Y_i = ln\ t_{fi}$, the p-th percentile of the population, Y_p, is

$$Y_P = \mu + z_P \cdot \sigma = ln\ t_P$$

and the sample estimate of Y_P is

$$Y_{PS} = \bar{Y} + z_P \cdot s = ln\ t_{PS},$$

where z_P is the p-th percentile of the standard Normal distribution. For example, $z_P = -3.09$ for the $0.1-$th percentile which is used below for Fig. 5.

Values $z'(u)$ and $z'(l)$ are determined such that

$$\bar{Y} + z'(u) \cdot s < \mu + z_P \cdot \sigma < \bar{Y} + z'(l) \cdot s.$$

These values are related to the noncentral t-distribution [9]. The $100(1-\alpha)\%$ confidence limits for t_P are given by

$$t_{PS} \cdot exp\{s \cdot [z'(u) - z_P]\} < t_P < t_{PS} \cdot exp\{s \cdot [z'(l) - z_P]\} \quad (3)$$

where the z' values can be obtained with acceptable accuracy ($< 3\%$ for $N > 10$) from the following approximation [10]:

$$z'(u) = \frac{z_P - \sqrt{z_P^2 - A \cdot B}}{A}$$

$$z'(l) = \frac{z_P + \sqrt{z_P^2 - A \cdot B}}{A}$$

and where

$$A = 1 - \frac{z(\alpha/2)^2}{2 \cdot (N-1)} \ and\ B = z_P - \frac{z(\alpha/2)^2}{N}.$$

The 90% confidence limits for t_P (defined by eq. 3), divided by t_{PS}, are plotted for the 0.1 percentile in Fig. 5 for three values of s. (Note that for the 0.1-th percentile, $z_P = z(.001) = -3.09$ in the above expressions for z'.) The figure shows how the confidence interval decreases with increasing sample size and with decreasing s. The curves are much like those in Fig.

3 for t_{50} except that the interval at a given sample size is considerably larger for t_p.

3. CENSORING OF DATA

3.1 Introduction

This section examines the effect of halting the stress test when only K out of the N lines on test have failed, that is, when the test data are sample (Type II) censored. It discusses the very significant test-time savings that are possible with sample censoring and the care that should be exercised in the selection of estimators for the parameters of the failure distribution. The effect that censoring has on the confidence intervals for t_{50}, σ, and $t_{0.1}$ are reviewed and precautions in combining test results are given with a brief mention of time (Type I) censoring. The subject of data censoring is considerable and only some of the basic issues can be examined in this paper.

3.2 Effects on Test Time

The median time for the K-th test part to fail, out of a total of N on test, can be expressed as follows:

$$t(0.5; K|N) = exp\{\mu + \sigma \cdot z(0.5; K|N)\}$$

where $z(0.5; K|N)$ is the median of the K-th smallest observation out of a sample of N from a standard Normal distribution, which is closely approximated [11] by

$$z(0.5; K|N) = z(\{K - 0.3175\}/\{N + 0.365\})$$

for 1 < K < N. For K = N,

$$z(0.5; N|N) = z(0.5^{1/N})$$

exactly [11]. The time savings of sample censoring, expressed as the ratio of the median time to complete the test with censoring to without censoring (t_C/t), is given by

$$t_C/t = t(0.5; K|N)/t(0.5; N|N), \qquad (4)$$

or

$$t_C/t = exp\ \sigma \left[z\left(\frac{K - 0.3175}{N + 0.365} \right) - z(0.5^{1/N}) \right].$$

This time-savings ratio is plotted in Fig. 6 versus σ for a sample size of 40 for different levels of censoring to illustrate how greatly test times can be shortened, especially when σ is large. For example, the test time can be reduced to one fifth when censoring is 30% and σ is 1.0. Even for censoring as small as 10%, the test time can be reduced by more than one half for populations with a large σ. Increasing N will increase the time savings but this is relative to the test time for all the samples to fail, which increases with increasing N. The net effect of increasing N is to alter the test time only slightly, for a fixed percentage of censoring.

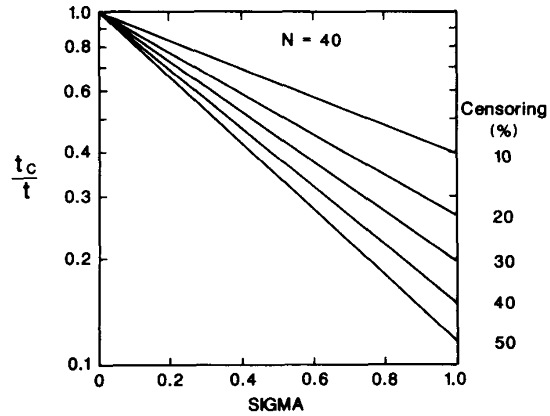

Fig. 6 - Ratio of test time with sample censoring, t_c, to that with no censoring, t, versus sigma for different levels of sample censoring when the sample size is 40.

3.3 Estimators for t_{50} and σ

For censored samples, the mean of the $ln\ t_{f_i}$ values and their standard deviation are not appropriate estimators of μ and σ because of the bias introduced by censoring. (Note: $t_{50} = exp\ \mu$.) The Best Linear Unbiased Estimates, commonly known as BLUE's [12], can be used but tables for their use are available only for sample sizes up to 20. Maximum Likelihood estimators [13] can be used for larger sample sizes.

Persson and Rootzen [14] define Restricted Maximum Likelihood (RML) estimators of μ and σ and use these to correct the moment estimators $\bar{Y}$ and s calculated from the logarithms of the observed failure times. These estimators are more easily calculated than are the Maximum Likelihood estimators and represent an option when there is no access to facilities for calculating Maximum Likelihood estimators. They considered the censoring of the lower values, so their formulas must be modified for censoring the higher values. The RML estimators and the corrected moment estimators are described in Appendix A, where it may be seen that they are easier to use than the least-squares estimators because only one z factor needs to be evaluated.

The least-squares method represents a convenient option for analyzing censored data because of experience with its use for complete data. The only publication found which considered its efficacy for censored data is by Gupta [15].* He found that the efficiency of the least-squares fitting is greater than 90% for all degrees of censoring. (Efficiency is defined here as the ratio of the variance of the BLUE's to the variance of these least-squares estimators.) However, only the case for N = 10 was considered.

* Gupta [15] proposed an easy alternative to the BLUE estimators in which he replaces the covariance matrix of the Normal order statistics by the identity matrix which is analogous to assuming that the Normal order statistics are uncorrelated and have the same variance. This approach is identical to least-squares fitting on Normal probability paper, if the plotting positions are chosen to be the expected values of the Normal order statistics rather than the commonly used z(i/(N+1)) values. This difference is expected to be insignificant.

The result of Gupta [15] gives hope that least-squares fitting will provide acceptable estimators for large N as well. Preliminary results, from a study now underway that includes a comparison of the least-squares estimators with those of Persson and Rootzen [14], indicate that to be true. For sample sizes between 20 and 50 and for censoring less than 50%, the least squares estimators for t_{50} and σ are more variable but by no more than approximately 20%. The least-squares estimator for σ is biased, but by no more than 10% in the above range of sample sizes.

3.4 Effects on Measurement Precision

With sample censoring, there is a loss of information, hence the resulting estimates for t_{50}, σ, and $t_{0.1}$ are subject to greater variability than are the estimates from a complete test. Percent increases in the lengths of the confidence intervals for t_{50}, σ, and $t_{0.1}$ are shown in Fig. 7 for increasing levels of censoring. The curves were calculated from the work of Meeker [16] using Maximum Likelihood estimators [13], as described in Appendix B.

The increase in the confidence interval for σ is largest because the loss of information by censoring is most serious for estimating σ. The confidence interval for t_{50} is least affected by small censoring because the data censored is not near $ln\ t_{50S}$ which is, by itself, a relatively efficient estimator of $ln\ t_{50}$ [17]. As censoring increases to 50% , the percent increase in the interval begins to rise sharply, as expected.

It is possible to avoid loss of precision due to censoring by increasing the sample size N. This can be seen in Fig. 8 for estimating t_{50}, where contours of equal variance (equivalently, equal confidence interval) are plotted in the (N,K/N) plane.

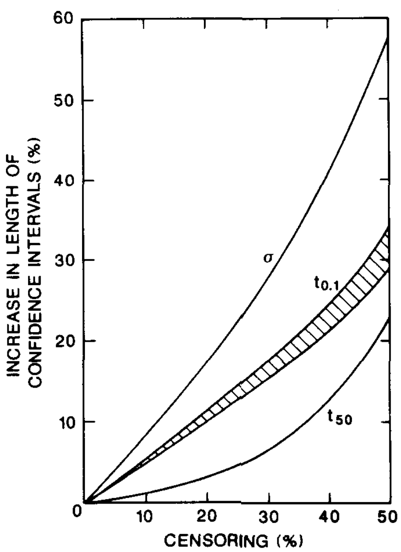

Fig. 7 - Percent increase in the confidence interval lengths for t_{50}, σ, and $t_{0.1}$ versus percent sample censoring. The dependence on s and N of the increase in the length for $t_{0.1}$ is indicated by the band. The upper limit is for s = 1.1 and N = 20; the lower limit is for smaller values of s and larger values of N. The dependence on s and N of the increase in the length for t_{50} is negligible. The increase for σ with censoring is independent of N.

For example, the variance obtained with a sample size of 20, when there is no censoring, is the same as the variance where 30 lines are put on test and the test is halted after 15 have failed.

The time-savings ratio for increasing N and censoring can be determined from the contours of constant time and the top scale in Fig. 8, both for $\sigma = 1.0$. The contours are given in terms of $t(0.5; K|N)/t_{50}$, for varying values of K and N; the top scale is marked off in $t(0.5; N|N)/t_{50}$ for different values of N. The ratio is given by eq. 4. To calculate the time savings for other values of σ, raise the result obtained to the power σ. The time-savings ratio is 0.16 if σ equals 1.0 in the above case where one waits until only 15 fail of the 30 lines on test, instead of testing 20 lines to completion. If σ equals 0.3, the time saving will be only 0.58. This was determined from Fig. 8 as follows.

If $\sigma = 1.0$,

$$t_C/t = \frac{t(0.5; 15|30)}{t_{50}} / \frac{t(0.5; 20|20)}{t_{50}} =\sim 1.0/6.2 = 0.16$$

and if $\sigma = 0.3$,

$$t_C/t = 0.16^{0.3} = 0.58.$$

Contours of constant variance and test time are shown for estimating σ and $t_{0.1}$ in Figs. 9 and 10 respectively, and can be used in the same way as was illustrated for Fig. 8. All three figures are based on the work of Meeker as described in Appendix C.

If the form of the distribution is uncertain, there is another potential benefit when increasing the sample size and the level of

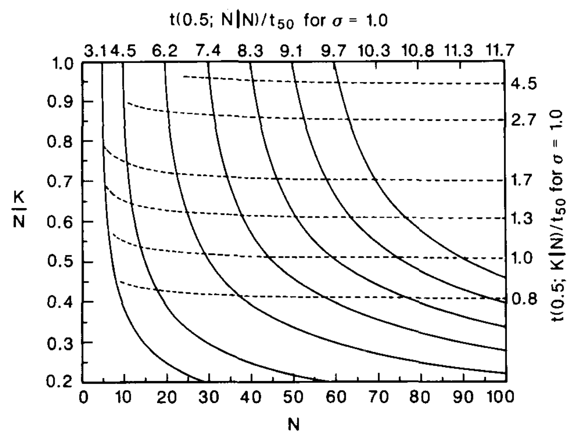

Fig. 8 - Contours of constant variance (solid) for estimating t_{50} and contours of constant median test time (dashed), for K failures out of N. Each variance contour represents the variance, σ^2/N_0, for an uncensored test of N_0 items; from left to right, N_0 = 5, 10, 20, 30, 40, 50, and 60. Each time contour represents the median time as labeled in units of $t(0.5; K|N)/t_{50}$, for σ=1.0. The median uncensored test time for selected values of N is given on top in units of $t(0.5; N|N)/t_{50}$, also for σ=1.0. To obtain median times for other σ's, raise the normalized time to the power σ.

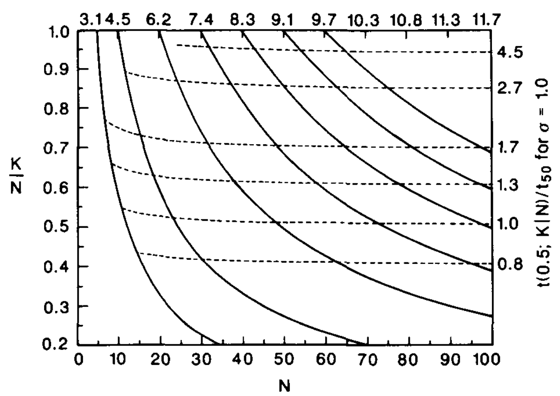

Fig. 9 - Contours of constant variance (solid) for estimating σ and contours of constant median test time (dashed), for K failures out of N. Each variance contour represents the variance, $\sigma^2/(2N_0)$, for an uncensored test of N_0 items; from left to right, N_0 = 5, 10, 20, 30, 40, 50, and 60. For use of graph, refer to the caption of Fig. 8.

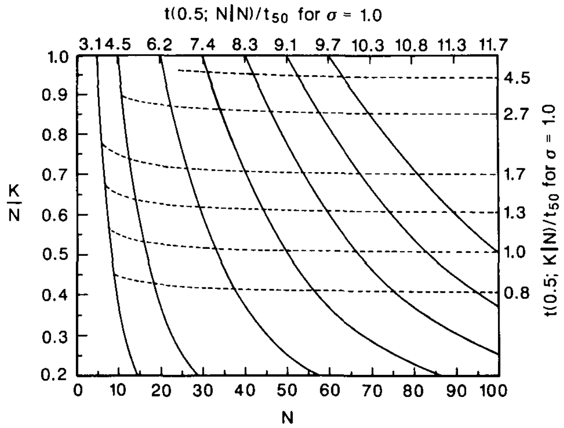

Fig. 10 - Contours of constant variance (solid) for estimating $t_{0.1}$ and contours of constant median test time (dashed), for K failures out of N. Each variance contour represents the variance, $(1+3.09^2/2)\sigma^2/N_0$, for an uncensored test of N_0 items; from left to right, N_0 = 5, 10, 20, 30, 40, 50, and 60. For use of graph, refer to the caption of Fig. 8.

censoring. It provides more information about the early failure character of the distribution, which has important reliability implications. Because the shortest failure time occurs at approximately the 1/(N+1) percentage point of the distribution, there is little that can be learned about the 0.1% point from, for example, a sample size of 20 where the first failure occurs at approximately the 5% point of the distribution. There has been concern whether the early failure distribution actually follows a Weibull distribution rather than a log-normal. Using the log-normal, when the actual distribution is a Weibull, will seriously over-estimate the early reliability of parts. Placing many more lines on test and censoring deeply will provide information about the early-order statistics of the distribution without the use of grossly excessive test times.

3.5 Precautions When Combining Test Data

When test facilities limit the number of lines that can be tested at one time, uncensored data from tests of lines from the same population that are stressed at the same level but at different times may be combined to increase the sample size.* When sample censoring, this procedure leads to more complicated analyses than considered here. The complication arises because of having to sacrifice either the basic assumption of halting the test when K out of N samples have failed (for a predetermined value of K) or the assumption of a single censoring time.

An alternative censoring procedure is to halt the test after a fixed time period, which is called time (or Type I) censoring. In this case, the number K of failures is random in contrast to that for sample censoring, where the number is fixed. The above discussion of the effects of censoring applies approximately to time censoring because it is based on Maximum Likelihood estimators which do not depend on whether the censoring time is random. With time censoring, the fraction observed (K/N) is not known beforehand. Thus, one knows only approximately where to look in Fig. 7 and in the (N, K/N) planes of Figs. 8-10 to estimate the effect of time censoring on the confidence intervals for the test to be performed, and also to estimate a realistic test time.

4. ERROR AND VARIATION IN STRESS CONDITIONS

4.1 Introduction

In establishing the confidence limits for t_{50}, σ, and t_p, it has been assumed that the stress conditions of current density and metallization temperature are accurately known and that each test line is subjected to the same stress conditions. In an actual test, small errors will be encountered in estimating the means of the stress conditions applied to the structures under test. Variations of these conditions about these means for the individual structures will also be encountered.

The effect of these errors and variations on the results of the test are examined for two cases: (1) there is an error in the estimate of one of the two stress conditions; and, (2) there are line-to-line variations in the stress conditions about their means.

The following empirical expression [18] was used to examine the effect of the stress current density J and of the stress temperature T on t_{50} and on t_f:

$$t_{50} = A(1/J)^n \; exp(qQ/kT)$$

where A and n are constants, Q is the activation energy, q is the electronic charge, and k is the Boltzmann constant.

4.2 Errors in the Stress Levels

Only the measurement of t_{50} is affected when the estimates of the mean values for J and T are in error; the measurement of σ

* This procedure is appropriate only if it can be reasonably assumed that the samples have not been altered in the intervening time and so still can be considered to belong to the same population.

is not. The fractional error in t_{50S} for a fractional error e in J is given below, where joule-heating effects on the metallization temperature are included [19].

$$\Delta t_{50S}/t_{50S} = [exp\{(qQ/k)\cdot\ (1/Te - 1/T)\}/(1+e)^n] - 1$$

where:

$$T_e = T_a + \frac{(J(1+e))^2\rho_0}{(K_i/tt_i)[1+0.88\ t_i/w] - (J(1+e))^2\rho_0\beta};$$

$$T = T_e(for\ e=0);$$

and where T_a is the temperature of the silicon substrate; ρ_0, β, t, and w are, respectively, the room-temperature resistivity, temperature coefficient of the resistivity, thickness, and width of the metallization; and K_i and t_i are the thermal conductivity and thickness of the dielectric film between the metallization and the silicon substrate.

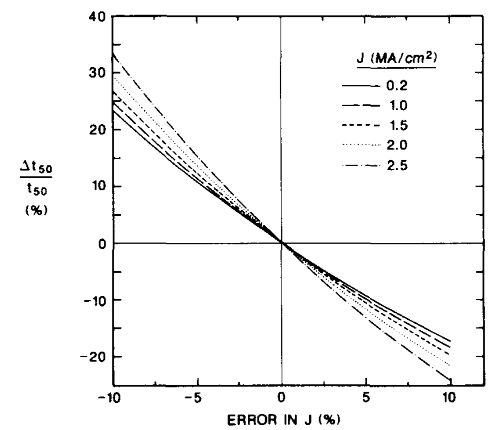

Fig. 11 - Percent error induced in t_{50} by percent error in current-density stress for five current-density levels, and the following conditions: w = 3 μm, t = 1 μm, $\beta = 3.9 \times 10^3$ °C^{-1}, $\rho_0 = 3.14 \times 10^{-6}$ ohm-cm, t_i = 1 μm, and K_i = 0.01 W/cm °C, where the underlying insulator is silicon dioxide.

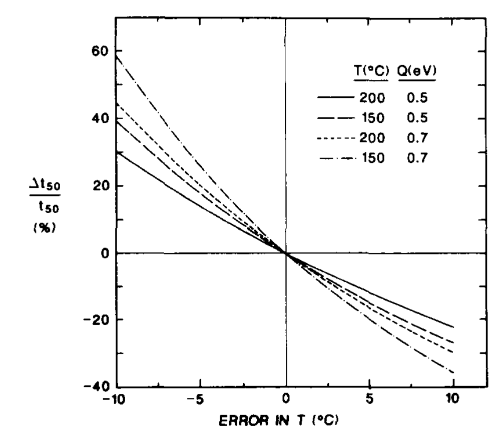

Fig. 12 - Percent error induced in t_{50} by an error in temperature for two different activation energies and two stress temperatures.

The percent error in t_{50S} for a constant T_a is plotted in Fig. 11 versus the percent error in J for a range of J values. The error in t_{50S} increases, as expected, with increasing values of J because of joule heating. For the conditions shown, the induced percent-error in t_{50S} is between two and three times the percent error in J, depending on the level of joule heating.

When there is an error only in the estimate of T, the fractional error in t_{50S} for an error of c °C is given by

$$\Delta t_{50S}/t_{50S} = [exp\{(qQ/k)\cdot\ (1/(T+c) - 1/T)\}] - 1$$

The induced error in t_{50S} is plotted in Fig. 12 versus the error in estimating T for two levels of T and Q. The results show that a 5 °C error in T can introduce as much as a 15 to 20% error in t_{50S}.

4.3 Between-Line Variations in the Stress Levels

When individual test lines are subjected to stress conditions T and J that vary in a random way about the respective means, $\bar{T}$ and $\bar{J}$, individual test lines fail either sooner or later than had they all been subjected to identical stress conditions.

The population of $Y_i = ln\ t_{fi}$ values are normally distributed with a mean of $ln\ t_{50}$ and a variance of σ^2, where t_{fi} is the failure time of the i-th line to fail when subjected to stress conditions $\bar{T}$ and $\bar{J}$.

The effect of variations in the stress conditions can be examined by adding to Y_i independent random variables, v_i, which are normally distributed with a mean of zero and a variance of v^2. The new population of $Y_i + v_i$ values is also normally distributed with a mean of $ln\ t_{50}$ and a variance of $\sigma^2 + v^2$. The failure times, t'_{fi}, of this new population are given by

$$t'_{fi} = exp(Y_i + v_i) = exp(Y_i)\cdot exp(v_i) = t_{fi}\cdot exp(v_i).$$

If the v_i values are small, t'_{fi} is approximated by

$$t'_{fi} = t_{fi}(1+v_i)$$

and the variations appear as fractions of the original time to fail, t_{fi}. Variations described by the random variable v_i model how line-to-line variations in J and T affect Y_i (and thus t_{fi}).

Variations with a standard deviation v, increase the sigma of the population of Y_i's to a new and larger value $\sigma' = \sqrt{\sigma^2 + v^2}$. They do not affect the population mean. The relative effect of the variations on σ is significant only when v becomes comparable to σ. In this case, the percent increase in the confidence intervals for $ln\ t_{50}$ and for sigma will be $100(\sigma' - \sigma)/\sigma$, as an examination of eqs. 1 and 2 will show.

Percent variations of as much as 20% in the t_{f_i} values ($v = 0.2$) can be tolerated before significant increases (20%) in σ' and in the confidence intervals will be observed. This holds as long as σ is 0.3 or larger. For example, an imposed variation of $v = 0.2$ will introduce an increase of 20% if $\sigma = 0.3$, but only 5% if $\sigma = 0.6$.

This implies that individual measurements of, or corrections for, the cross-sectional area of the test lines are not necessary. Small random variations in the temperature of the test lines can be tolerated without significantly degrading the quality of the estimates of t_{50} and σ. The magnitude of these variations, in combination, should not produce random variations in the t_f values of more than approximately 20%. For example, using the results shown in Figs. 11 and 12, variations in J and T of only 5% and 5°C, respectively, could each produce variations of 15% in t_f. The variations in t_{f_i} as a result of the two processes, in combination, will be 21% of the failure times.

5. INFERENCES ABOUT TWO POPULATIONS

5.1 Introduction

Stress tests are conducted to determine if, for example, a process or material change has affected the resistance of a metallization to electromigration failure. The basis of this determination is a comparison of the sample estimates of the parameters of the failure distribution obtained from the two metallizations. It is assumed below that the sample estimates for t_{50}, σ, and t_p are from tests with no data censoring. The analysis of censored data is usually accomplished by invoking the large-sample Normality of Maximum Likelihood estimators. This approximate analysis can be performed using the formulas below by treating the sample estimators (see 3.3) as Normally distributed, with variances as shown in Figs. 8-10 and calculable from the formulas in Appendix C and the caption of Table B2. Censoring complicates the tests given here in that it is not clear what degrees of freedom to use for the t- and F-distributions. One option is to use K, and then use N for the degrees of freedom. If the conclusions agree, accept them; if not, treat the results as borderline.

5.2 Test to Compare Two Sigmas

If the population sigmas are the same, then the ratio of the sample estimates of the sigmas, $s(1)^2/s(2)^2$, has a sampling distribution called the F-distribution [20]. At a $100\alpha\%$ significance level, $\sigma(1)$ and $\sigma(2)$ are not equal if the ratio s(1)/s(2) exceeds the limits [21]

$$\sqrt{F(\alpha/2; N_1-1, N_2-1)} \text{ and } \sqrt{F(1-\alpha/2; N_1-1, N_2-1)},$$

where N_1 and N_2 are the sample sizes of the two populations.

5.3 Tests to Compare t_{50} Values When Sigmas Are Equal

To test whether the t_{50}'s of the two populations are the same when the σ's of the two populations are equal, the student t statistic is used, where the degrees of freedom is equal to $N_1 + N_2 - 2$. At a $100\alpha\%$ significance level, the t_{50} values for the two populations are not equal if $t_{50S}(1)/t_{50S}(2)$ exceeds the limits [22]

$$exp(\pm t(1-\alpha/2; N_1+N_2-2)\cdot s_p \cdot \sqrt{1/N_1+1/N_2}\,)$$

where $t_{50S}(1)$ and $t_{50S}(2)$ are the sample estimates for the two populations and s_p is the pooled mean-square estimate of σ given by:

$$s_p = \sqrt{\frac{(N_1-1)\cdot s(1)^2 + (N_2-1)\cdot s(2)^2}{N_1+N_2-2}}.$$

A more conservative set of limits is obtained by substituting the larger of s(1) and s(2) for the pooled value, if there is a question about the σ's being equal.

5.4 Tests to Compare t_{50} Values When Sigmas Are Not Equal

To test whether the t_{50}'s of the two populations are the same when the σ's of the two populations are not, the range of the critical limits in 5.3 is increased somewhat because the degrees of freedom, df, is reduced. At a $100\alpha\%$ significance level the population medians are not equal if $t_{50S}(1)/t_{50S}(2)$ exceeds the limits [23]:

$$exp(\pm t(1-\alpha/2; df)\cdot \sqrt{s(1)^2/N_1 + s(2)^2/N_2}\,)$$

where

$$df = \frac{[(s(1)^2/N_1) + (s(2)^2/N_2)]^2}{\frac{(s(1)^2/N_1)^2}{N_1} + \frac{(s(2)^2/N_2)^2}{N_2}} - 2.$$

If $N_1 = N_2 = N$,

$$df = N\{1 + \frac{2\cdot s(1)^2 \cdot s(2)^2}{s(1)^4 + s(2)^4}\} - 2.$$

5.5 Test To Compare Two Percentiles

At a $100\alpha\%$ significance level, the p-th percentiles of the two populations are not equal if the ratio $t_p(1)/t_p(2)$ exceeds the limits:

$$exp(\pm t(1-\alpha/2)\cdot \sqrt{V'})$$

where

$$V' = \left[\frac{s(1)^2}{N_1} + \frac{s(2)^2}{N_2}\right]\cdot\left[1 + z_p^2/2\right]$$

and where, as before, z_p is the p-th percentile of the standard Normal distribution and no assumption is made about whether t_{50} or σ of the two populations are equal. These limits are developed in Appendix D.

6. SUMMARY

The precisions in estimates of the population median-time-to-failure (t_{50}), sigma (σ), and percentile failure time (t_P) are given in terms of sample size and sample estimates of sigma (s). Examples of these precisions are provided in terms of 90% confidence limits for sample sizes of 5 to 400 and for σ equal to 0.3, 0.6, and 0.9. They show that populations with a small σ require relatively few test samples to achieve a given level of precision.

Statistical decision rules to determine from test data if two metallization populations have the same t_{50}, σ, and t_P are provided to use in evaluating the relative effectiveness of metallization processes, treatments, and alloys.

Reductions in test time are achieved when the stress test is halted before all the samples have failed (sample censoring). The time savings increase as the populations σ's increase. Data censoring, however, increases the variability in the estimates for t_{50}, σ, and t_P in ways that are described. Greater time savings are possible without loss of precision if the sample size is increased.

An examination of the effect of errors and uncertainties in the current-density and temperature stresses show that small errors in estimating the stress conditions can lead to relatively large errors in t_{50}. Sample-to-sample variations of the stress conditions, however, have a relatively small effect on σ and on the confidence intervals for σ and t_{50}, especially when the starting σ is larger than approximately 0.4. The estimate of t_{50} is unaffected by these variations.

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APPENDIX A. The Modified Persson-Rootzen Estimators

Persson and Rootzen [14] considered censoring the lower values of a Normal sample. Their formulas have been modified here for censoring the higher values. The modified equations for the Restricted Maximum Likelihood (RML) estimators are:

$$\sigma^*_{RML} = (1/2) \cdot (z(1 - K/N) \cdot (C - \bar{Y})) +$$

$$(1/2) \cdot \sqrt{[z(1-K/N) \cdot (C - \bar{Y})]^2 + (4/K)\Sigma_{i=1}^{K}(C - Yi)^2}$$

and

$$\mu^*_{RML} = C + z(1 - K/N) \cdot \sigma^*_{RML},$$

where C $= Y_K = ln\ t_{fK}$, the maximum observed log failure time.

Using these equations to correct for the bias in the ordinary sample mean and sample standard deviation of the observed $ln\ t_{fi}$, they obtain:

$$\mu^* = \bar{Y} + \sigma^*_{RML} \cdot \alpha^*$$

and

$$\sigma^* = \sqrt{\{(K-1)/K\} \cdot s^2 + \alpha^* \cdot (z(K/N) + \alpha^*) \cdot (\sigma^*_{RML})^2}$$

where $\alpha^* = \{N/(K\sqrt{2\pi})\}\ exp\ -(1/2) \cdot z(K/N)^2$.

They find these latter estimators to be quite close to the Maximum Likelihood estimators for all reasonable degrees of censoring, both asymptotically and for small N.

APPENDIX B. Calculating the Percent Increase in the Confidence Interval for t_{50}, σ, and $t_{0.1}$.

The percent increase in the confidence intervals for the three statistics graphed in Fig. 7 were determined from a Table calculated by Meeker [16] which is an expanded version of one by Gupta [15]. The data from Meeker [16] were used to develop percent increases of the confidence limits due to censoring. The confidence limits for each of the three statistics were calculated by using the values listed in Table B1 in the expressions listed in Table B2 for the level of censoring desired. The confidence intervals were obtained by taking the differences of these limits.

Table B1. Abridged listing, in units of σ^2/N, of the variance of the sample mean of the logarithms of the failure times, the variance of the sample estimate for sigma, and the covariance, when a Normal distribution is fitted by maximum likelihood to singly censored data [16].

Censoring (%)	Var$\{\bar{Y}\}$ / σ^2/N	Var$\{s\}$ / σ^2/N	Cov$\{\bar{Y}$, s$\}$ / σ^2/N
0	1.000	0.500	0.000
10	1.020	0.586	0.041
20	1.063	0.689	0.107
30	1.139	0.820	0.207
40	1.214	0.996	0.362
50	1.517	1.241	0.605

Table B2. Expressions* for the Confidence Limits for t_{50}, σ and t_p. $Var\{\bar{Y} + z_P \cdot s\} = Var\{\bar{Y}\} + z_P^2 \cdot Var\{s\} + 2z_P \cdot Cov\{\bar{Y}, s\}$. For calculating $t_{0.1}, z_P = -3.09$.

Parameter	Limits
t_{50}	$t_{50S}\ exp\left(\pm s \cdot z(1-\alpha/2) \cdot \sqrt{Var\{\bar{Y}\}/N}\right)$
σ	$s\left[1 \pm z(1-\alpha/2) \cdot \sqrt{Var\{s\}/N}\right]$
t_P	$t_{PS}\ exp\left(\pm s \cdot z(1-\alpha/2) \cdot \sqrt{Var\{\bar{Y} + z_P \cdot s\}/N}\right)$

* The use of z factors give adequate estimates of the confidence limits for $N \gtrsim 20$. For smaller N, t factors are more appropriate but introduce the problem of determining the degrees of freedom.

APPENDIX C. Development of Figs. 8, 9, and 10.

Meeker [16] and earlier writers have calculated the asymptotic variances and covariance of the Maximum Likelihood (ML) estimators of $\mu = ln\ t_{50}$ and σ, in units of σ^2/N, for various degrees of censoring. Simple curves which could be easily inverted were fit (within 3 variances. If the fraction observed is K/N, the fitted curves are:

$$2Var\{\bar{Y}\}/(\sigma^2/N) = A + B/(K/N) + (1 - A - B)/(K/N)^2,$$

where A = 0.96132 and B = -0.204312;

$$Var\{s\}/(\sigma^2/N) = A + B/(K/N) + C/(K/N)^2,$$

where A = -0.224197, B = 0.720370, and C = 0.0064277;

$$Var\{ln\ t_{0.1}\}/(\sigma^2/N) = A + B(K/N) + C/(K/N),$$

where A = 7.84667, B = -4.00845, and C = 1.91339.

Each variance contour starts at a point $(N_0,1)$, for sample size $N_0 = 5$, 10, 20, 30, 40, 50, and 60. It continues through points (N,K/N) for which the variance is the same as the variance for an uncensored test of N_0 items. For a given $N > N_0$, the variance for an uncensored test is smaller by the factor N_0/N, so the corresponding K/N is chosen (from Meeker's results) to inflate the uncensored variance by N/N_0.

APPENDIX D. Comparison Test For Failure-Time Percentiles

To compare two failure-time percentiles, consider the p-th percentiles from two Normal populations:

$$Y_P(1) = \mu(1) + z_P \cdot \sigma(1)$$

and

$$Y_P(2) = \mu(2) + z_P \cdot \sigma(2),$$

where $Y_p = ln\ t_p$. No assumption is made about whether the means or the sigmas of these two populations are the same. To test for the hypothesis that the p-th percentiles from the two populations are equal, based on the sample estimates $Y_{PS}(1)$ and $Y_{PS}(2)$, consider:

$$Y_{PS}(1) - Y_{PS}(2) = (\bar{Y}(1) + z_P \cdot c \cdot s(1)) - (\bar{Y}(2) + z_P \cdot c \cdot s(2))$$

or

$$Y_{PS}(1) - Y_{PS}(2) = \bar{Y}(1) - \bar{Y}(2) + z_P(c \cdot s(1) - c \cdot s(2)),$$

where c is the correction factor used to make s an unbiased estimate of σ [4]. The variance V of $Y_{PS}(1) - Y_{PS}(2)$ is given by

$$V = \left[\frac{\sigma(1)^2}{N_1} + \frac{\sigma(2)^2}{N_2}\right] \cdot [1 + z_P^2/2]$$

where N_1 and N_2 are the sizes of the two samples.

The distribution of sample standard deviations s approaches normality as the sample size increases. Hence, for large N the assumption that the distribution of $Y_{PS}(1) - Y_{PS}(2)$ is Normal will be adequate to perform a significance test.

At a $100\alpha\%$ significance level, the p-th percentiles of the two populations are not equal if the difference $Y_{PS}(1) - Y_{PS}(2)$ exceeds the limits:

$$\pm t(1 - \alpha/2) \cdot \sqrt{V'}$$

or if the ratio $t_P(1)/t_P(2)$ exceeds the limits:

$$exp(\pm t(1 - \alpha/2) \cdot \sqrt{V'})$$

where V' is the sample estimate of the variance of $Y_{PS}(1) - Y_{PS}(2)$ obtained by substituting s for σ in the above expression for V.