Steady-state temperature profiles in narrow thin-film conductors

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Temperature profiles in sufficiently narrow thin-film conductors can be computed in a quasi-one-dimensional model. The model applies when the linewidth is less than a thermal decay length. We discuss the thermal decay length as a function of geometrical and material parameters and find that many practical cases satisfy the narrow stripe criterion. Fringing effects are included in the model. Three specific examples are discussed: (i) the temperature of an isolated stripe, including fringing, (ii) temperature profiles near junctions of narrow stripes, and (iii) temperature profiles at the junction of a narrow tap and a wide stripe. The simplicity of the model makes it convenient for the design of electromigration experiments, for programmable read-only memory fusing current calculations, etc. Examples and graphs are given for the case of passivated aluminum on SiO_2. The graphs are useful for estimation of thermal parameters without calculation.

I. INTRODUCTION

The problem of calculating the steady-state and transient temperature profiles in thin-film conductors has received attention before.[1–4] A few special cases such as cracked or grooved stripes have been treated using a complicated two-dimensional analysis.[1–2] For uniform or one-dimensional cases the solutions are simple.[3,4] In this paper, we will show that there are important practical cases which are not strictly one-dimensional, but which may be solved to a good approximation using a "quasi-one-dimensional" model. The simplicity of the model makes it a practical design tool.

This study was motivated by the need to understand the temperature profiles near voltage measurement taps in electromigration test structures. Wafer-level reliability tests usually involve acceleration of failure by increasing the current density. This inevitably causes hot spots and steep thermal gradients which obscure interpretation of the data. Meaningful interpretation may be possible if physical observation of the failure site is combined with detailed calculation of temperatures and temperature gradients. The model will also be useful in other applications such as the calculation of programmable read-only memory (PROM) fusing currents.

The quasi-one-dimensional model applies to conductor stripes that are narrow compared with a thermal decay length. The heat transport equation, the narrow stripe (NS) criterion, and the thermal decay length are discussed in Sec. II. In Sec. III three applications are discussed.

(i) The temperature rise of an isolated current-carrying stripe of arbitrary width, including fringing effects.

(ii) The temperature profiles near junctions of stripes satisfying the NS criterion.

(iii) The temperature profiles near the junction of a narrow voltage tap and a wide current-carrying stripe.

For the case of aluminum on SiO_2 specific examples are given and important functions are displayed graphically. This makes the paper useful for quick estimates of temperature profiles in many practical situations.

II. THE HEAT TRANSPORT EQUATION AND THE THERMAL DECAY LENGTH

Consider a thin-film conductor separated by a thin dielectric film from a thermally massive and conductive substrate (Fig. 1). The steady-state temperature at a point in the conductor is determined by a balance between joule heating due to electrical current, heat conduction in the plane of the film (horizontally), and heat conduction vertically through the dielectric into the substrate. Radiative and convective processes are negligible. The relative thicknesses and thermal conductivities of the metal and dielectric films are such that there is negligible vertical temperature gradient in the metal film. Nor are there thermal gradients in the substrate. The only vertical temperature drop occurs across the dielectric film. These conditions decouple the horizontal and vertical heat transfer problems, leading to a two-dimensional differential equation in the horizontal coordinates for the horizontal temperature profile. For steady-state conditions, the heat capacities of the materials are irrelevant.

For these conditions, the power balance is written

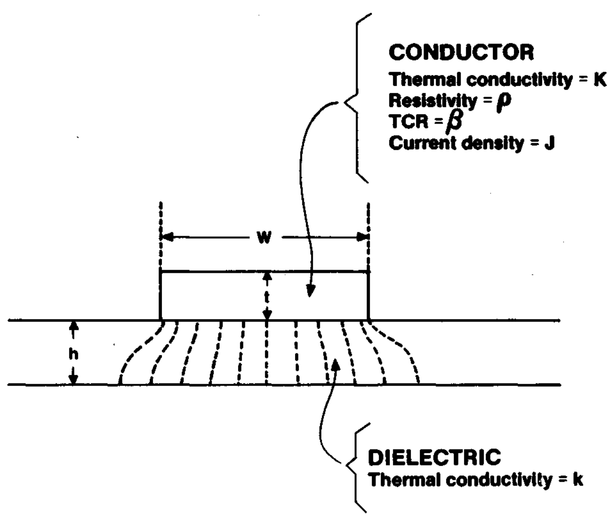

FIG. 1. Schematic cross section of a conductor stripe on a thin dielectric film defining dimensions and material parameters. Schematic heatflow streamlines with fringing effects are shown.

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$$K\nabla^2\theta + J^2\rho - k\theta/th = 0 \quad (1)$$

where the symbols are defined in an Appendix and Fig. 1. Equation (1) is for an arbitrarily-shaped metal sheet, and it ignores edge effects in the heat transport through the dielectric. The physical interpretation of Eq. (1) is as follows: Consider an infinitesimal volume dV in the metal. The net rate of horizontal heat conduction into this volume is $K\nabla^2\theta dV$. The rate of heat production by electrical dissipation is $J^2\rho dV$, where J may be a function of position. By the definition of k, $k\theta/h$ is the rate of heat removal through the dielectric per unit area of metal film, ignoring edge effects. The rate of heat removal per unit volume of metal film is $k\theta/ht$, and for the infinitesimal volume it is $dVk\theta/ht$. Equation (1) is obtained by requiring that these contributions sum to zero in the steady state. The thermal variation of ρ is a large effect and must be taken explicitly into account:

$$\rho = \rho_0(1+\beta T) = \rho_0(1+\beta T_s) + \rho_0\beta(T - T_s)$$

or

$$\rho = \rho_s + \rho_0\beta\theta. \quad (2)$$

Substitution of Eq. (2) into Eq. (1) gives

$$K\nabla^2\theta + J^2\rho_s - (k/th - J^2\rho_0\beta)\theta = 0. \quad (3)$$

This differential equation governs the two-dimensional temperature distribution in an arbitrarily-shaped metal film sheet, with negligible edge effects, and temperature-independent thermal conductivities k and K.

For a current-carrying stripe, Eq. (3) may be modified in several ways.

(i) The Laplacian may be written

$$\nabla^2 = d^2/dx^2,$$

where x is the spatial coordinate along the stripe.

(ii) The current density J may be taken as a known constant.

(iii) Edge effects in the vertical heat conduction may be taken into account. This is done by replacing $k\theta/h$ in Eq. (3) with a more accurate expression for the average heat flux[5] at the metal/dielectric interface.

The problem of finding this average heat flux as a function of θ is mathematically identical to the problem of finding the average surface charge density as a function of the potential difference between the stripe and the substrate; that is, the capacitance. Using the analogy between average heat flux at the metal/dielectric interface and surface charge density, between temperature difference θ and potential difference, and between thermal conductivity and dielectric permittivity we can replace $k\theta/h$ by $k\theta\delta/h$. For the electrical problem, δ is defined as the ratio of the actual capacitance to the capacitance given by the simple parallel-plate formula. The dimensionless factor δ is a function of ratios of dimensions and ratios of dielectric constants (or thermal conductivities) of the dielectrics in the structure and is always greater than unity. That is,

$$\delta = \delta\,(w/h, t/h, \text{ratios of thermal conductivities}) \geqslant 1. \quad (4)$$

The dependence on ratios of thermal conductivities of the dielectrics only occurs when two or more different dielectrics are used in a structure, for example, when a stripe is passivated by a different dielectric than the substrate dielectric film. Formulae for δ have been given by several authors.[6–8] Recent interest[8] in finding simple formulae for δ has been spurred by the need to calculate capacitances in very large scale integrated (VLSI) circuits where fringing effects are beginning to dominate. These considerations also apply in the heat transfer problem in VLSI circuits. It is worth noting, however, that δ in the capacitance problem is not necessarily the same as δ in the heat transfer problem for the identical structure. This is because the ratios of the dielectric constants for two dielectrics is generally not the same as the ratios of their thermal conductivities. If a stripe is passivated by a dielectric with the same thermal conductivity as the substrate dielectric, then an accurate but complicated closed form expression for δ due to Chang[7] may be used. If the stripe is unpassivated, or passivated by a dielectric of negligible thermal conductivity, then the thin plate formula[6] may be used:

$$\delta = 1 + (2h/\pi w)[1 + \ln(\pi w/h)], \; w/h \gg 1 \quad (5)$$

Recently, Yuan and Trick[8] have given a simple formula for δ which may be used for any ratio of thermal conductivities of the dielectrics in the structure. The formula is less accurate than Chang's,[7] but much simpler. For specific examples in this report we shall assume that all dielectrics have the same thermal conductivity and use Yuan and Trick's[8] formula. The results of the formula of Ref. 8 and Eq. (5) are plotted in Fig. 2. Finally, we note that if the stripe does not have a rectangular cross section, t may be replaced by A/w where A is the cross-section area and w is the width of the stripe *at the oxide-metal interface*. For convenience in this paper we shall, however, assume a rectangular cross section.

Making changes in Eq. (3) appropriate to a current-carrying stripe, we write

$$K\theta'' + J^2\rho_s - (k\delta/th - J^2\rho_0\beta)\theta = 0. \quad (6)$$

In Eq. (6) J^2 is a constant, δ is a function of t/h and w/h, and the second spatial derivative along the stripe is denoted by the double prime. Rearranging Eq. (6) we write

$$\Psi'' - \lambda^{-2}\Psi = 0, \quad (7)$$

where

$$\lambda^{-2} = (k\delta/th - J^2\rho_0\beta)/K, \quad (8)$$

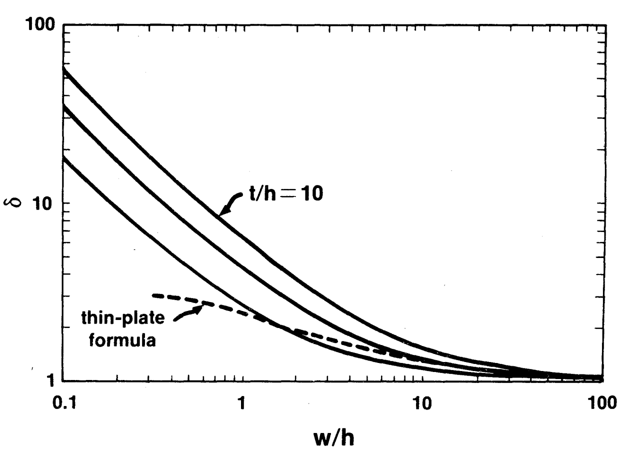

FIG. 2. Parallel plate correction factor as a function of dimensions. Solid lines: Formula of Yuan and Trick (Ref. 8). Top is for $t/h = 10$, middle is for $t/h = 1$, bottom is for $t/h = 0.1$. Dashed lines: Thin plate formula (Ref. 6).

$$\Psi = \theta - \theta_i, \tag{9}$$

$$\theta_i = J^2 \rho_s \lambda^2 / K. \tag{10}$$

Equations (7)–(10) describe the spatial temperature variation in a current-carrying stripe with one-dimensional symmetry.

The thermal decay length λ defined by Eq. (8) is the characteristic length over which horizontal temperature variations can occur. If a stripe has width less than λ, then T cannot vary across the stripe (i.e., normal to J), even if a noncurrent-carrying (cold) connection is made to the side of the stripe. Such a connection might be a voltage measurement tap. The solution to the thermal problem is greatly simplified in the NS regime defined by $w > \lambda$. In Fig. 3, λ is plotted as a function of the geometrical parameter th/δ for the aluminum/SiO_2 case. The material constants assumed are given in the Appendix. The plot shows that λ deviates from the simple relation

$$\lambda \simeq (Kth/k\delta)^{1/2}, \tag{11}$$

only at impractically high current densities for the practical range of values of th/δ. Moreover, Eq. (11) is a conservative estimate of λ by which to judge whether the NS approximation holds.

Example 1. For a 1-μm-wide, 1-μm-thick aluminum stripe on 1-μm-thick SiO_2, passivated by glass, Fig. 2 gives $\delta = 4.1$ (since $w/h = t/h = 1$). Hence $th/\delta = 2.44 \times 10^{-9}$ cm^2, so from Fig. 3 or Eq. (11), $\lambda = 6.2$ μm. Since $w = 1 \mu m, w \ll \lambda$ is satisfied so the NS criterion holds.

Equation (11) can be used to derive a dimensionless plot by which the validity of the NS approximation may be judged for any combination of materials. Let w_m be the maximum width for the NS approximation to hold, i.e., $w_m = \lambda$. Substitution into Eq. (11) and rearranging gives

$$(w_m/h)^2 \delta(w_m/h, t/h) = (K/k)(t/h), \tag{12}$$

where we have made the functional dependence of δ explicit. This defines a functional relationship between t/h and w_m/h for each value of K/k. If the formula of Ref. 8 is substituted into Eq. (12), solution for w_m/h gives the solid curves plotted in Fig. 4.[9] To show the effect of fringing, Eq. (12) with $\delta = 1$, i.e., ignoring fringing, is plotted as broken lines in Fig. 4. Because the Yuan and Trick[8] formula was used, the solid curves in Fig. 4 strictly apply to the case of a stripe passivated by the same material as the substrate dielectric. This is an extreme case and an intermediate case such as an unpassivated stripe will have a value of w_m/h lying between the solid and the broken line for a particular value of K/k. In the following examples we shall use the solid lines in Fig. 4 to give conservative estimates of w_m.

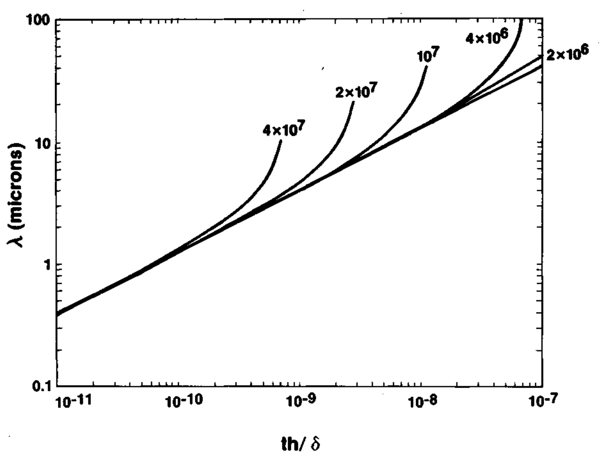

FIG. 3. Thermal decay length as a function of geometrical parameters for the aluminum/SiO_2 system. The current density in A/cm^2 is a parameter.

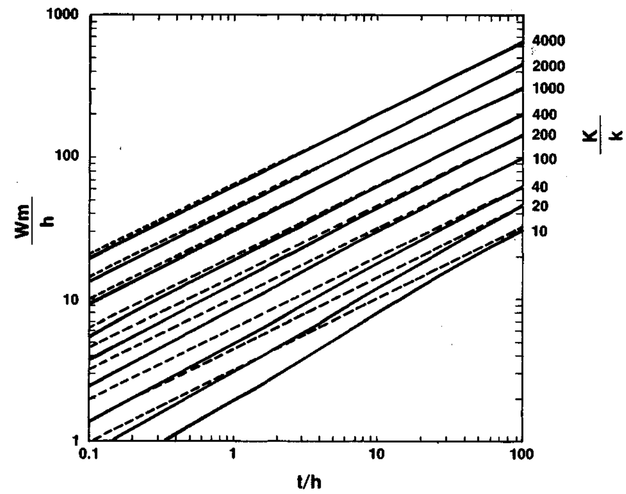

FIG. 4. Universal narrow stripe (NS) criterion plot. The maximum stripe width satisfying the NS criterion is plotted against metal thickness for particular ratios of the metal and dielectric thermal conductivities. Stripe dimensions are in units of the dielectric film thickness. The effect of fringing is shown by the difference between the broken lines, which ignore fringing, and the solid lines. If a stripe's dimensions give a point on this plot below the solid line corresponding to the materials used, then the NS criterion is satisfied.

Example 2. For a 1-μm film of aluminum on 1-μm of SiO_2, $K/k = 151$ and $t/h = 1$. From Fig. 4 we find $w_m/h = 11$, or $w_m = 11$ μm. Therefore, any stripe narrower than 11 μm will satisfy the NS criterion.

Example 3. Same as Example 2, but replace SiO_2 by polyimide. Polyimide is expected to have a thermal conductivity an order of magnitude or so smaller than SiO_2. So we estimate $K/k = 1500$. From Fig. 4. with $t/h = 1$, we find $w_m/h = 38$, so lines narrower than 38 μm satisfy the NS criterion.

Example 4. Same as Example 2 but replace 1 μm of SiO_2 with 100 nm of the same. Now $t/h = 10$ and with $K/k = 151$, Fig. 4 gives $w_m/h = 37$ so $w_m = 3.7$ μm. On thin oxides only very narrow lines satisfy the NS criterion.

III. APPLICATIONS

A. The temperature rise of an isolated stripe

The temperature rise of an isolated stripe far from any voltage taps or contacts to the substrate is easily solved for because the temperature profile is spatially uniform. Moreover, there is no need to invoke the NS approximation since there is no temperature variation horizontally across the stripe. This problem has been solved by, among others, Black[3] but without including the effect of fringing.

Since the temperature profile is spatially uniform, $\Psi'' = 0$, so from Eq. (7) $\Psi = 0$ and from Eqs. (9) and (10)

$$\theta = \theta_i = J^2 \rho_s \lambda^2 / K,$$

independent of spatial coordinates. This demonstrates the physical interpretation of θ_i defined in Eq. (10): It is the temperature difference between the substrate and the metal in a stripe far from any junctions with other stripes or contacts to the substrate. Many of the formulae for the applications to be described below contain θ_i. Substitution of Eq. (8) into Eq. (10) gives

$$\theta_i = J^2\rho_s/(k\delta/th - J^2\rho_0\beta). \qquad (13)$$

Notice that Eq. (13) is independent of the metal thermal conductivity K, but depends on the metal electrical parameters ρ and β. Equation (13) was used to generate plots, given in Fig. 5, of θ_i versus the geometrical parameter th/δ for Al on SiO_2 with a substrate temperature of 25 °C. The temperature variation of k can be (and was in Fig. 5) taken into account by taking the temperature of the dielectric to be $T_s + (1/2)\theta_i$ and iterating Eq. (13). Actually this had negligible effect for $\theta_i < 600$ °C. Thus, for practical situations with Al on SiO_2 none of the parameters in Eq. (13) depends on *stripe* temperature. But ρ_s depends strongly on *substrate* temperature. The denominator of Eq. (13) vanishes when heat is generated in the stripe faster than it can be removed. The slight upward curvature of the lines in Fig. 5 show the onset of this thermal runaway. Runaway occurs for a current density

$$J_m = (k\delta/\rho_0\beta th)^{1/2}, \qquad (14)$$

which is plotted against the geometrical parameter th/δ in Fig. 6 for the Al/SiO_2 case. Figure 6 is useful because many formulae are simpler when $J \ll J_m$, which is often satisfied. For example, when $J \ll J_m$,

$$\lambda = (K/k)^{1/2}(th/\delta)^{1/2} \quad (J \ll J_m) \qquad (15)$$

and

$$\theta_i = (J^2\rho_s/k)(th/\delta)\ (J \ll J_m). \qquad (16)$$

Example 5. Consider an isolated 1-μm-wide, 1-μm-thick, glass-passivated Al stripe on 1 μm of SiO_2 with a current density of 4×10^6 A/cm^2 and a substrate temperature of 25 °C. Since $w/h = t/h = 1$ we find from Fig. 2 that $\delta = 4.1$. So $th/\delta = 2.44\times 10^{-9}$ cm^2. From Fig. 5 using the curve for $J = 4\times 10^6$ A/cm^2 we find $\theta_i = 7.6$ °C so the stripe temperature is $T = 25 + 7.6 = 32.6$ °C. Now consider the

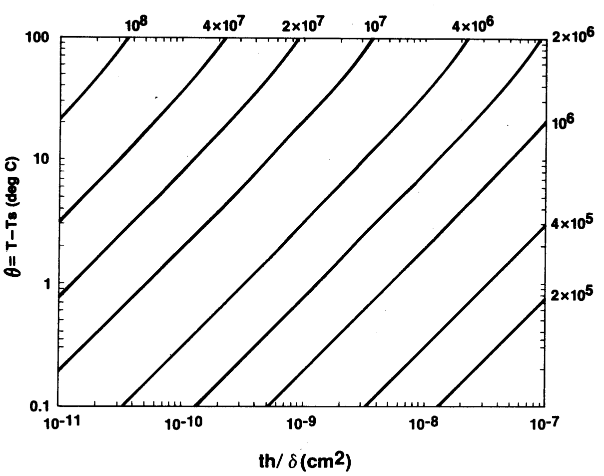

FIG. 5. Temperature rise above the substrate as a function of dimensional parameters and current density. Plotted for Al on SiO_2 with the substrate at 25 °C.

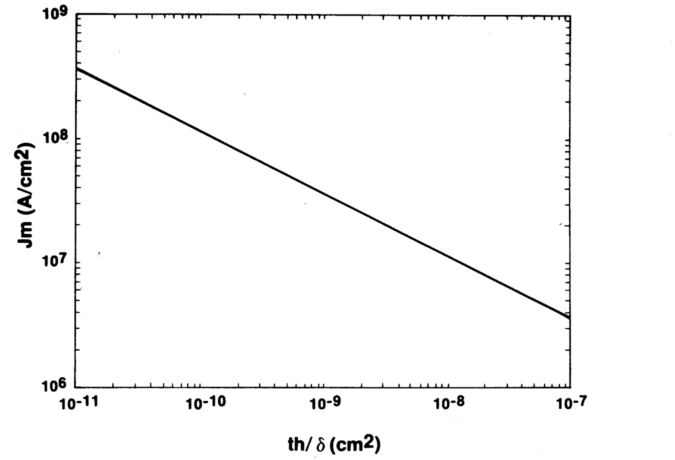

FIG. 6. Runaway current density vs geometrical parameters for aluminum on SiO_2.

same but for a stripe width of 20 μm. From Fig. 2, $\delta = 1.19$ since $w/h = 20$ and $t/h = 1$. So $th/\delta = 8.40\times 10^{-9}$ cm^2 and Fig. 5 gives $\theta_i = 29$ °C. The stripe temperature is, therefore, $T = 25 + 29 = 54$ °C. This example illustrates the large improvement in heat sinking for narrow lines.

For a strictly one-dimensional case with a nonuniform profile along the stripe, it is not necessary to invoke the NS criterion and it is easy to solve Eq. (7). For a stripe of length l constrained to have a temperature rise above substrate of $\theta_j(1)$ at one end and $\theta_j(2)$ at the other, the temperature profile along the stripe is given by

$$\theta = \theta_i + \{(1/2)[\theta_j(1) + \theta_j(2)] - \theta_i\}\cosh(x/\lambda)/\cosh(l/2\lambda) + (1/2)[\theta_j(1) - \theta_j(2)]\sinh(x/\lambda)/\sinh(l/2\lambda),$$

where x is measured from the center of the stripe and λ and θ_i are given by Eqs. (8) and (13), respectively. Fringing effects are included through the factor δ which appears in expressions for λ and θ_i.

B. Temperature profiles near junction of narrow stripes

A frequently occurring pattern is shown in Fig. 7(a). The horizontal stripe carries current while the vertical tap is a voltage measurement connection carrying no current. Primes denote parameters for the tap, while unprimed parameters are for the current-carrying stripe. We shall derive formulae for the temperature profiles near the junction of the stripe when the NS criterion is satisfied for both the tap and the stripe. This simple, and practical, case demonstrates the physics of the situation and makes it easy to write down general formulae for junctions of many narrow stripes, as shown in Fig. 7(b). The generalization to many stripes, in turn, leads to a theory for calculating current distributions and temperature profiles in a network of narrow stripes.

If the tap and the stripe in Fig. 7(a) satisfy the NS criterion then $w < \lambda$ and $w' < \lambda'$. This means that the temperature cannot vary greatly over the cuboidal volume shown in projection in Fig. 7(a) as $ABCD$. The temperature rise in this region will be taken as uniform with value θ_j. The objective of the theoretical discussion is to solve for θ_j. From Eqs. (7) and (9) the temperature profiles in the tap and stripe are given by, respectively,

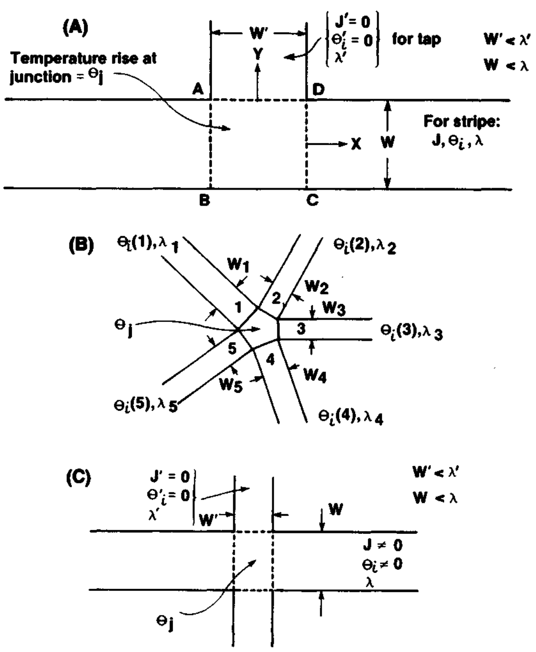

FIG. 7(a). Schematic diagram of tap-stripe contact used in text discussion. Current flows only in the horizontal stripe. (b) Schematic diagram of a general junction of stripes to which Eqs. (26) apply. (c) Schematic diagram for Example 7 in text. Current flows only in the horizontal stripe.

$$\theta = \theta_j \exp(-y/\lambda') \quad (17)$$

and

$$\theta = \theta_i + (\theta_j - \theta_i)\exp(-x/\lambda). \quad (18)$$

In Eq.(17), y is the distance down the tap measured from AD and in Eq. (18) x is the distance along the stripe measured from either AB or CD. In Eq. (17) we have set $\theta'_i = 0$ since $J = 0$ in the tap [see Eq. (13)]. The thermal power flowing across CD into the metal volume $ABCD$ (thickness t) is

$$H_1 = wtK(d\theta/dx)|_{x=0}, \quad (19)$$

or, substituting Eq. (18),

$$H_1 = wtK(\theta_i - \theta_j)/\lambda. \quad (20)$$

This amount also flows into $ABCD$ across AB. Similarly, the thermal power flowing into $ABCD$ across AD is

$$H_2 = -w'tK\theta_j/\lambda'. \quad (21)$$

The thermal power gained through the bottom of the volume $ABCD$ through the dielectric is

$$H_3 = -k\theta_j ww'/h, \quad (22)$$

and the power generated by joule heating within $ABCD$ is

$$H_4 = ww'tJ^2(\rho_s + \rho_0\beta\theta_j). \quad (23)$$

In the steady state,

$$2H_1 + H_2 + H_3 + H_4 = 0. \quad (24)$$

Substitution of Eqs. (20), (21), (22), and (23) into (24) and some manipulation using Eqs. (8) and (10) yields

$$\theta_j[1 + w'\lambda/(2w\lambda') + w'/(2\lambda)] = \theta_i[1 + w'/(2\lambda)].$$

The last terms in the square brackets come from H_3 and H_4 and can be neglected in the NS approximation. That is, in the NS approximation, joule heating and heat loss to the substrate in the junction region are second-order effects. So, in the NS approximation,

$$\theta_j = \theta_i/[1 + w'\lambda/(2w\lambda')], \quad (25a)$$

or from Eq. (15)

$$\theta_j = \theta_i/[1 + (1/2)(w'/w)(\delta'/\delta)^{1/2}] \quad (J \ll J_m). \quad (25b)$$

Notice that for $J \ll J_m$, the factor by which θ_i is multiplied depends entirely on geometrical ratios and not on material constants. An interesting special case of Eq. (25b) occurs when the tap and stripe are of equal width. Then $w = w'$ and $\delta = \delta'$ so $\theta_j = (2/3)\theta_i$ even if fringing is important.

Example 6. Consider 1-μm-thick Al on 1-μm-thick SiO_2 and passivated by glass. A 5-μm-wide stripe carries a current density of 2×10^6 A/cm^2. The substrate is at 25 °C. What is the temperature at the junction of a 2-μm-wide voltage tap? For the 5-μm line, $t/h = 1$ and $w/h = 5$, so from Fig. 2, $\delta = 1.53$ while for the 2-μm stripe $t/h = 1$, $w'/h = 2$, so $\delta' = 2.59$. Examination of Fig. 4 shows that the NS criterion is satisfied for both the tap and the stripe. For the 5-μm line, $th/\delta = 6.54\times10^{-9}$ cm^2, so from Fig. 5, $\theta_i = 5.0$ °C. Also, Fig. 6 shows that $J_m = 1.4\times10^7$ A/cm^2 so $J \ll J_m$ and Eq. (25b) can be used. Substitution of the correct values of δ, δ', w, and w' into Eq. (25b) gives $\theta_j = 0.79\theta_i = 4.0$ °C. So the temperature at the junction of the voltage tap is $T_s + \theta_j = 29$ °C, while far ($> \lambda = 10\,\mu$m, from Fig. 3) down the current-carrying stripe the temperature is $T_s + \theta_i = 30$ °C.

It is not difficult to generalize Eqs. (25) to an arbitrary number of stripes, all satisfying the NS criterion, meeting at a junction (of area $\simeq\lambda^2$) as illustrated in Fig. 7(b). In general,

$$\theta_j = [\Sigma\theta_i(n)w_n/\lambda_n]/(\Sigma w_n/\lambda_n), \quad (26a)$$

or, if all current densities are much less than J_m,

$$\theta_j = [\Sigma\theta_i(n)w_n\delta_n^{1/2}]/(\Sigma w_n\delta_n^{1/2}) \quad (J \ll J_m), \quad (26b)$$

where the n^{th} stripe has isolated temperature rise $\theta_i(n)$, width w_n, etc. The sums are taken over all the stripes meeting at the junction, that is, n. Note that $\theta_i(n) = 0$ for an unpowered stripe. The temperature profile is

$$\theta = \theta_i(n) + [\theta_j - \theta_i(n)]\exp(-s/\lambda_n), \quad (27)$$

where s is the distance down the stripe measured from the junction.

Example 7. What is the formula for the junction temperature of a current-carrying stripe contacted by two cold voltage taps as shown in Fig. 7(c)? This can be thought of as four stripes coming into a junction. For this case, Eq. (26a) is written

$$\theta_j = (2\theta_i w/\lambda' + 0 + 0)/(2w/\lambda + 2w'/\lambda')$$
$$= \theta_i/[1 + w'\lambda/(w\lambda')] = \theta_i/[1 + (w'/w)(\delta'/\delta)^{1/2}].$$

Formulae (26) are useful in solving for the electrical current distribution and temperature profiles in a network of stripes satisfying the NS criterion. If each junction in the network is separated by a distance greater than λ, then the temperature of each node is given by Eq. (26). To calculate the values of $\theta_i(n)$, one needs the current density and, therefore, the current flowing in the stripe. The currents will be determined by solution of Kirchoff's equations for the network which requires the total resistance of each line between nodes. This resistance, however, depends on the temperature profile of the stripe and is given by

$$R = (1/wt)\int_0^l (\rho_s + \beta\rho_0\theta)dx \tag{28}$$

for a stripe of length l. Since θ is known analytically in terms of θ_i and the values of θ_j at the ends of the stripe, viz. θ_j (1) and θ_j (2), the integral can be evaluated. For $l \gg \lambda$,

$$R = [(\rho_s + \rho_0\beta\theta_i)l + \rho_0\beta\lambda\,(\theta_j(1) + \theta_j(2) - 2\theta_i)]/wt. \tag{29}$$

The first term in the square brackets is the resistance calculated ignoring the hotter or colder junctions at the ends of the stripe, and the second term corrects this. Thus, one can find the currents and temperature profiles in the network using the following iterative procedure.

(1) Assume resistances of stripes appropriate to the substrate temperature, i.e., $R = \rho_s l/wt$.

(2) Compute the current distribution using Kirchoff's laws.

(3) Compute λ_n via Eq. (8) and $\theta_i(n)$ via Eq. (13) for every stripe in the network.

(4) Compute θ_j for each node in the network using Eq. (26).

(5) Recompute all the resistances in the network using Eq. (29).

(6) Repeat steps 2–5 until convergence obtains.

If any nodes are separated by $l < \lambda$, then Eq. (26) does not hold. The equations for θ_j on the nodes which are closer than λ are coupled. Still, they are linear and can be straightforwardly solved. We shall not pursue this generalization further, since it is probably not so important in practice.

C. Narrow tap with wide current-carrying stripe

A wide current-carrying stripe with narrow high thermal impedance voltage taps is useful in measurements of the thermal conductivity of the underlying dielectric film. Edge effects in the wide stripe can be ignored and the effect of cooling near the tap should be small. The pattern near a tap is shown schematically in Fig. 8. We shall restrict $w' < \lambda'$ and $w \gg \lambda$.

Since $w \gg \lambda$ we may ignore the effect of the boundary FG and solve as if the current-carrying stripe forms a semi-infinite sheet. The solution will therefore be independent of w. Since $w' < \lambda' \ll \lambda$, the temperature will be approximately uniform across the semicircular area $OBCD$ with a value we shall call θ_j. Since it is uniform across $OBCD$ it is uniform along the semicircular arc BCD. So, to a good approximation, outside the semicircle $OBCD$, θ will be given by the radially symmetrical solution to Eq. (3). In cylindrical coordinates with origin O, Eq. (3) is written

$$\frac{1}{r}\frac{d}{dr}\left(\frac{r\,d\Psi}{dr}\right) - \frac{\Psi}{\lambda^2} = 0, \tag{30}$$

where we have assumed no angular dependence for Ψ. The solution to Eq. (30) is a zeroth-order Bessel function of the second kind,[10] $K_0(r/\lambda)$. The solution is, therefore,

$$\theta = \theta_i + (\theta_j - \theta_i)K_0(r/\lambda)/K_0(w'/2\lambda), \tag{31}$$

where λ is given by Eq. (8) with $\delta = 1$. To derive Eq. (31) we have required that $\theta = \theta_j$ at $r = w'/2$. The total thermal

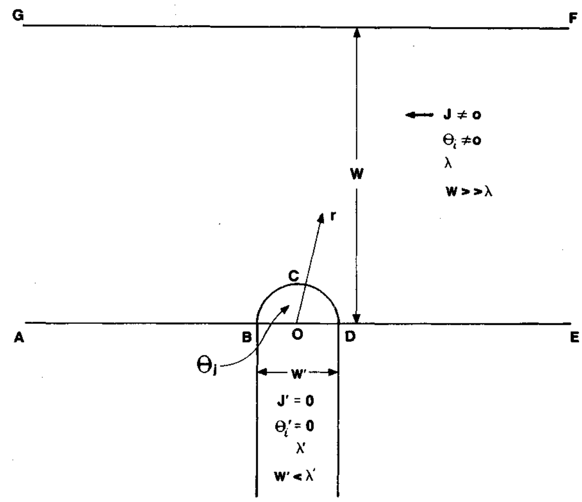

FIG. 8. Schematic diagram of a narrow voltage tap contacting a wide current-carrying stripe. The tap carries no current. Used in text discussion.

power flowing across BCD into the semicircular prism is

$$H = -\pi w' t K\,(d\theta/dr)|_{r=w'/2}$$
$$= (\theta_i - \theta_j)w' t K \lambda^{-1} K_1(w'/2\lambda)/K_0(w'/2\lambda), \tag{32}$$

where we have used the property $dK_0(x)/dx = -K_1(x)$, where K_1 is the first-order Bessel function of the second kind.[10] The Bessel functions are tabulated by Abramowitz and Stegun,[11] who also provide convenient analytical approximations for use on digital computers. On the other hand, the thermal power entering (actually leaving since the sign is negative) the prism through BD is

$$H_2 = -w' t K\theta_j/\lambda'. \tag{33}$$

The derivation of Eq. (33) is the same as Eq. (20). Ignoring the second-order effects due to dissipation within $OBCD$ and heat loss to the substrate through the base of the prism, the power budget for the prism is $H_1 + H_2 = 0$. Substitution of Eqs. (32) and (33) into this condition and solution of the resulting equation for θ_j gives

$$\theta_j = \theta_i/[1 + (\lambda/\lambda')F(w'/2\lambda)] \tag{34a}$$

or

$$\theta_j = \theta_i/[1 + (\delta')^{1/2}F(w'/2\lambda)] \quad (J \ll J_m), \tag{34b}$$

where $F(x) = K_0(x)/K_1(x)$. Equations (31), (34), and (17) (for the tap) give complete details of the temperature profile near the tap. The functions $F(x)$ and $K_0(x)$ are plotted in Fig. 9.

Example 8. Consider a 1-μm-thick aluminum film on 1 μm of SiO_2 with a substrate temperature of 25 °C. The film is patterned into a very wide (say 5 mils) stripe carrying a current density of $J = 2\times 10^6$ A/cm^2. It is contacted by a long 2-μm-wide voltage tap. What is the temperature at the point of contact? For the wide stripe $w/h \to \infty$, so $\delta = 1$ and $th/\delta = 10^{-8}$ cm^2. So Fig. 3 gives $\lambda = 12.5$ μm and Fig. 5 gives $\theta_i = 8$ °C. From Fig. 9 we find $F(w'/2\lambda) = F(0.08) = 0.225$. For the narrow stripe, $t/h = 1$, $w'/h = 2$ so $\delta' = 2.57$. Inspection of Fig. 6 at $th/\delta = 10^{-8}$ cm^2 shows that $J_m = 1.1\times 10^7$ A/cm^2 so $J \ll J_m$ and Eq. (34b) may be used. Substitution of δ', etc., into Eq. (34b) gives $\theta_j = \theta_i \times 0.74 = 5.9$ °C.

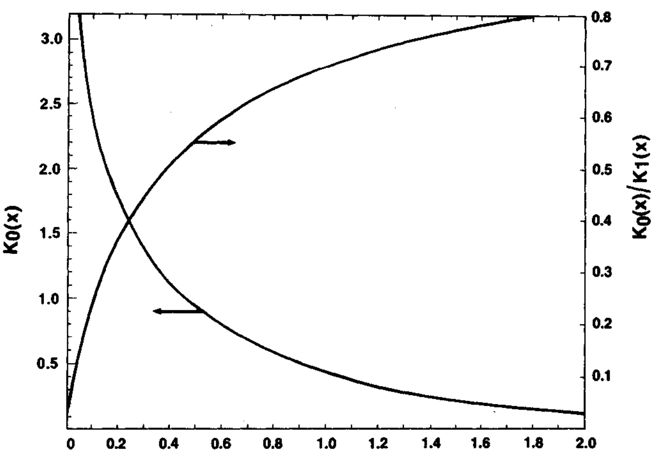

FIG. 9. Functions used in the theory of a narrow tap contacting a wide current-carrying stripe.

IV. SUMMARY

We have demonstrated that a simple quasi-one-dimensional model, the NS approximation, can be used to calculate steady-state temperature profiles near junctions of stripes of narrow thin-film conductors. The approximation takes full account of fringing effects which can be large. For strictly one-dimensional situations the model gives a full account of the temperature profiles, including fringing effects, without invoking the NS approximation. Many practical two-dimensional cases arising in integrated circuit technology satisfy the NS criterion.

This paper provides a prescription for convenient calculation of temperature profiles near junctions, etc., of narrow current-carrying stripes. The prescription is summarized as follows.

(1) Write down the dimensions, resistivity, thermal coefficients of resistance, and thermal conductivities involved.

(2) Check from Fig. 4 whether the NS criterion is satisfied for the stripes that are considered narrow. If not, then the approximate theory cannot be used and the prescription ends here. This check is unnecessary if the problem is strictly one-dimensional.

(3) Find the δ's for each stripe from dimensional and thermal conductivity ratios. One can use, for example, Fig. 2 if the thermal conductivities of the substrate dielectric film and the passivating dielectric are the same.

(4) Compute J_m for each stripe using Eq. (14).

(5) Compute θ_i and λ for each stripe using Eqs. (15) and (16) if $J \ll J_m$ (the usual case) or Eqs. (8) and (10) in general.

(6) Find the junction temperature θ_j by substitution of θ_i's, λ's (or δ's), and w's into appropriate formulae such as Eqs. (25), (26), or (34).

(7) Find the temperature profiles by substitution of θ_i, θ_j, and λ into appropriate formulae such as Eqs. (17), (18), (27), or (31).

For the case of Al/SiO_2 many of the above calculations are unnecessary, since the numbers can be looked up in the figures in this paper.

This paper gives a prescription for calculating θ, the temperature rise above the silicon substrate temperature. The substrate temperature can be calculated from the *total* power dissipation on the die, provided the junction-case and case-ambient thermal resistances are known. Alternatively, T_s may be found by measuring the resistance of an unpowered stripe, calibrated with a thermocouple, etc. In any case, T_s may be found by well-known methods.

Finally, we point out the need for measurements of thermal conductivities especially of the various dielectric films used in integrated circuit processing. There is little data for thermal SiO_2 and none, as far as the author knows, for polyimide films, or plasma-enhanced silicon oxides or nitrides or for glasses.

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APPENDIX I: MATERIAL CONSTANTS USED IN EXAMPLES

Aluminum

$$\rho = \rho_0[1 + \beta T\ (^\circ\mathrm{C})];$$

$$\rho_0 = 2.42 \times 10^{-6}\ (\Omega\ \mathrm{cm});$$

$$\beta = 4.752 \times 10^{-3}\ (^\circ\mathrm{C}^{-1});$$

$$K = 2.18(\mathrm{W}/^\circ\mathrm{C\ cm}), \quad \text{independent of } T.$$

Note: In general, there is a rough relationship between ρ and K: $K\rho = LT$, where T is in °K and $L = 2.45 \times 10^{-8}$ W Ω/°K^2. This is the Wiedeman–Franz law.[12]

SiO_2

A curve fit to data for vitreous quartz[3] gives

$$k\ (\mathrm{W}/^\circ\mathrm{C\ cm}) = 1.43 \times 10^{-2} + 3.84 \times 10^{-6} T + 2 \times 10^{-8} T^2,$$

where T is in °C. k varies slowly in the range of most temperatures of interest.

APPENDIX II: GLOSSARY OF SYMBOLS

β	thermal coefficient of resistance (°C^{-1}).
$F(x)$	$K_0(x)/K_1(x)$.
δ	parallel-plate capacitor fringing correction factor.
H	total heat flow, thermal power (W).
h	dielectric film thickness (cm).
J	current density (A/cm^2).
J_m	maximum possible, i.e., runaway, current density (A/cm^2).
K	thermal conductivity of metal (W/°C cm).
$K_n(x)$	nth-order Bessel function of second kind.
k	thermal conductivity of dielectric (W/°C cm).
λ	thermal decay length (cm).
l	length of stripe (cm).
Ψ	$\theta - \theta_i$, deviation from temperature of isolated stripe (°C).

ρ	metal resistivity (Ω cm).
ρ_0	metal resistivity at 0 °C (Ω cm).
ρ_s	metal resistivity at T_s (Ω cm).
T	local metal temperature (°C).
T_s	substrate temperature (°C).
t	metal film thickness (cm).
θ	$T - T_s$ (°C).
θ_i	temperature rise above substrate of an isolated stripe (°C), i.e. far from junctions of stripes.
θ_j	temperature rise above substrate of a junction of stripes.
w	width of stripe (cm).
w_m	maximum stripe width satisfying NS criterion (cm).
s, x, y	spatial coordinates (cm).

[1] Y. Chang and H. L. Huang, Jpn. J. Appl. Phys. **14,** 267 (1975).
[2] Y. Chang and H. L. Huang, J. Appl. Phys. **47,** 1175 (1976).
[3] J. R. Black, in *20th Annual Proceedings of International Reliability Physics Symposium*, San Diego, California, March 1982, pp. 300–306.
[4] F. M. d'Heurle and P. S. Ho, in *Thin Film-Interdiffusion and Reactions,* edited by J. M. Poate, K. N. Tu, and J. N. Mayer (Wiley, New York, 1978), pp. 243–303.
[5] Average heat flux = total thermal power/area of interface.
[6] H. B. Palmer, Trans. AIEE **56,** 363 (1937).
[7] W. H. Chang, IEEE Trans. Microwave Theory Tech. **MTT-24,** 608 (1976), and correction, **MTT-25,** 712 (1977).
[8] C. P. Yuan and T. N. Trick, IEEE Electron Device Lett. **ED-3,** 391 (1982).
[9] This is easy for the Yuan and Trick formula (Ref. 8) since the left-hand side of Eq. (12) becomes quadratic in wm/h.
[10] F. Bowman, *Introduction to Bessel Functions* (Dover, New York, 1958).
[11] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).
[12] C. Kittel, *Introduction to Solid-State Physics*, 3rd edition (Wiley, New York, 1966).