Just a reminder of various definitions related to transmission lines.

\[
\left(3\right) \quad x \partial_x \frac{0Z}{H} - x \partial_x \frac{0Z}{V} = (x) I \quad \text{and} \quad x \partial_x B + x \partial_x A = (x) \Lambda
\]

uniform trans. lines

\[
\left(2\right) \quad \frac{\partial \gamma \partial + \mathcal{E}}{\mathcal{L} \gamma \partial + \mathcal{H}} = 0Z
\]

characteristic (complex) impedance of the transmission line

\[
\left(1\right) \quad \left[\gamma \partial \partial (x) I\right] \partial H = (\tau, x) I \quad \text{and} \quad \left[\gamma \partial \partial (x) \Lambda\right] \partial H = (\tau, x) \Lambda
\]

voltagex and current along a transmission line
\[ \frac{V}{B} = (0 = x) \quad \text{where} \quad x \frac{C}{\omega} = \frac{x}{\omega} \frac{V}{B} = (x) \]

- Reflection coefficient:
- Electrical length of the line:
- Reflected wave:
- Incident wave:
- Phase velocity:
- Propagation constant:

\[ \frac{C}{\omega l} + \frac{C}{\omega l + C} \frac{V}{l} = x = \frac{C}{\omega} \frac{V}{B} = \frac{C}{\omega} \]
(7) \[
\frac{(p\omega)\tanh TZ + 0Z}{(p\omega)\tanh 0Z + TZ} 0Z = (p)^{\nu_1}Z
\]

(note the change of positive direction)

At input terminals of transmission line, \( \times = d \)

(6) \[
\frac{0Z + TZ}{0Z - TZ} 0 1 = \frac{0 1 - I 0 Z}{0 1 + I 0 Z} = TZ = (0)^{\nu_1}Z \quad 0 = x = d
\]

(5) \[
\frac{x\omega 0 1 - x\omega - \varnothing}{x\omega 0 1 + x\omega - \varnothing} 0Z = \frac{x I}{x A} = \nu_1 Z
\]

Input impedance
\[
(6) \frac{I + \frac{\text{VSWR}}{I}}{I - \frac{\text{VSWR}}{I}} = |I^0| \quad \text{or} \quad \frac{|0_I| - I}{|0_I| + I} = \frac{\text{min}|(x)\Lambda|}{\text{max}|(x)\Lambda|} = \text{VSWR}
\]

(Voltage) standing wave ratio:

Otherwise.

From now on: tr. Lines are lossless and uniform (unless specified)

\[
(8) \quad \frac{(p\psi')\tan^{T}Z\psi' + 0Z}{(p\psi')\tan^{0}Z\psi' + TZ} = (p)^{w}Z
\]

Also, \( \frac{T^\Lambda}{\psi I} = 0Z, f/d_n = \chi, \frac{T^\Lambda}{\psi I} \) lossless transm. Line: 

Circuit representation
\[ \frac{Z}{Z_0} = \left( \frac{V}{\lambda} \right)^{\text{w}} Z = (p)^{w} Z \leftarrow (p)^{w} Z \]

Quarter wave tr. Line (transformer), \( \frac{Z}{\nu} = p \theta \) or \( \frac{V}{\lambda} = p \), see

\[ (p \theta)^{0} \cot \frac{0}{2} Z = (p)^{0} Z \]

Open circuited tr. Line: \( I = 0 \), \( \text{VSWR} = \infty \), \( T Z = \frac{Z}{Z_0} = \infty \), eq. (8)

\[ (p \theta)^{0} \tan \frac{0}{2} Z = (p)^{s} Z \]

Shorted tr. Line: \( I = 0 \), \( \text{VSWR} = \infty \), \( T Z = \frac{Z}{Z_0} = 0 \), eq. (0)

Matched tr. Line: \( I = 0 = \text{VSWR} = 1 \).
This defines a single port network. What about 2-port?

\((x)v(x)I = (x)q \) and \((x)q - (x)v = (x)i\) \((x)q + (x)v = (x)i\)

so that \(0Z/\Lambda = (x)q\) \(0Z/(x)_+,\Lambda = (x)v\) \(0Z/(x)_-,\Lambda = (x)i\) \(0Z/(x)_0\Lambda = (x)\Lambda\) \(0Z/(x)_+\Lambda = (x)i\) \(0Z/(x)_-\Lambda = (x)v\)

Introduction, "normalized" variables: \(v(x)\) \(i(x)\) \(a(x)\) \(b(x)\)

so that \(v(x) = a(x) + b(x)\) \(i(x) = a(x)\) \(b(x) = b(x)\) \(a(x) = a(x)\)

Similarly for current: \(I(x) = I(x)\) \(I(x) = I(x)\)

\(x\in\mathcal{B} = -\Lambda\) \(x\in\mathcal{A} = +\Lambda\)

Scattering matrix
circuit representation

\[ S_{b1}/2 = S_{11}a1 + S_{21}a2 \]

\[ S_{b2}/2 = S_{12}a1 + S_{22}a2 \]

or in matrix form,

\[
\begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a1 \\
  a2
\end{pmatrix}
= \begin{pmatrix}
  q1 \\
  q2
\end{pmatrix}
\]

Each reflected wave \((q1, q2)\) has two contributions: one from the incident wave at the same port and another from the incident wave at the other port. How to calculate them? S-parameters, scattering parameters, scattering matrix. How to calculate these parameters?

2-port figure. Generalize eq. :
Fig. 1.4.2. “matched” termination equal to the tr. line ch. impedance.

\begin{align*}
\text{(14)} & \quad \text{output ret. coeff. with input matched} \\
& \quad 0 = \frac{1}{\bar{Z}_d} \frac{1}{\bar{Z}_q} = \bar{Z}_S \\
\text{(13)} & \quad \text{transmission coeff. with output matched} \\
& \quad 0 = \bar{Z}_d \frac{1}{\bar{Z}_q} = \frac{\bar{Z}_d}{\bar{Z}_q} = \frac{\bar{Z}_S}{\bar{Z}_q} \\
\text{(12)} & \quad \text{reverse trans. coeff. with input matched} \\
& \quad 0 = \frac{1}{\bar{Z}_d} \frac{1}{\bar{Z}_q} = \frac{1}{\bar{Z}_S} \\
\text{(11)} & \quad \text{input ret. coeff. with output matched} \\
& \quad 0 = \bar{Z}_d \frac{1}{\bar{Z}_q} = \frac{1}{\bar{Z}_S}
\end{align*}
Circuit representation

Note:

$Z_{out}$

$I.5.1$ (e.g. by hand).

planes: positions along tr. lines (usually beginning and end). Fig.

why: to "de-embed" the 2-port from tr. lines on I/O. Reference

Shifting Reference planes

— mainly for cascading networks.

Occasional use for chain scatter parameters or $T$-parameters

matrix, transistor does not oscillate.

Advantage: using matched resistive terminations to measure $S$-

$I/O$.

We measure "overall" $S$-parameters, i.e. including the tr. line on

$Z_{0}Z = [T]Z_{0}Z = IZ_{0}Z \\ 0 = Z_{0}Z$ \rightleftharpoons 0 = 0 \\

Sufficient condition: $Z_{0}$ (of 2-port) need not be matched to $Z_{0}Z$

Circuit representation
\[
\begin{bmatrix}
\frac{z_p}{\lambda_p} & \frac{z_{\Theta+1}}{\lambda_{\Theta+1}}
\end{bmatrix}
\begin{bmatrix}
\frac{z_{\Theta+1}}{\lambda_{\Theta+1}} S & (\frac{z_{\Theta+1}}{\lambda_{\Theta+1}})^2 S \\
(\frac{z_{\Theta+1}}{\lambda_{\Theta+1}})^2 S & \frac{z_{\Theta+1}}{\lambda_{\Theta+1}} S
\end{bmatrix}
= \begin{bmatrix}
\frac{z_q}{\lambda_q} \\
\frac{z_q}{\lambda_q}
\end{bmatrix}
\]

This is plugged into first part of eq.

\[
\begin{cases}
\frac{z_{\Theta+1} - \frac{z_p}{\lambda_p}}{\sqrt{\lambda}} = \frac{z_{\Theta+1} - \frac{z_q}{\lambda_q}}{\sqrt{\lambda}} \\
\frac{z_{\Theta+1} + \frac{z_p}{\lambda_p}}{\sqrt{\lambda}} = \frac{z_{\Theta+1} + \frac{z_q}{\lambda_q}}{\sqrt{\lambda}}
\end{cases}
\]

as it travels from \(x = 0\) to \(x = l\) along trajectories of transmission lines. Signal is delayed by \(\Theta\) and electrical lengths of transmission lines.

\[
\begin{bmatrix}
\frac{z_p}{\lambda_p} & \frac{z_{\Theta+1}}{\lambda_{\Theta+1}} S \\
\frac{z_{\Theta+1}}{\lambda_{\Theta+1}} S & \frac{z_{\Theta+1}}{\lambda_{\Theta+1}}
\end{bmatrix}
= \begin{bmatrix}
\frac{z_q}{\lambda_q} \\
\frac{z_q}{\lambda_q}
\end{bmatrix}
\]

Fig. 1.5.1.

Reference planes: positions along transmission lines (beginning and end).
At i-th port (here: 1 or 2):

are real (usually 50 Ohm lines with 50 Ohm terminations).

Properties of S-parameters

\[
\begin{bmatrix}
\mathcal{S}_1 & \mathcal{S}_0 \\
\mathcal{S}_0 & \mathcal{S}_1
\end{bmatrix}
= \begin{bmatrix}
\mathcal{S}_1 & \mathcal{S}_0 \\
\mathcal{S}_0 & \mathcal{S}_1
\end{bmatrix}
\]

and plugging into second part of eq.

Reverse relations obtained by expressing primed variables in eq.

By inspection, first term on RHS must be matrix \( \mathcal{S} \).
V- and I-s are peak values. For sin( ) signal divide by $V$.

\[(18) \quad \left[ (i_x)^2 I_l^0 Z - (i_x)^3 \Lambda \right] \frac{2^0 Z \wedge Z}{I} = \]

\[(i_x)^2 I_l^0 Z \wedge = \frac{2^0 Z \wedge}{(i_x)^2 \Lambda} = (i_x)^2 q \quad \text{and similarly} \]

\[(19) \quad \left[ (i_x)^2 I_l^0 Z + (i_x)^3 \Lambda \right] \frac{2^0 Z \wedge Z}{I} = \]

\[(i_x)^2 I_l^0 Z \wedge = \frac{2^0 Z \wedge}{(i_x)^2 \Lambda} = (i_x)^2 q \quad \text{define} \]

\[(16) \quad \frac{2^0 Z}{(i_x)^2 \Lambda} - \frac{2^0 Z}{(i_x)^3 \Lambda} = (i_x)^2 I - (i_x)^3 I = (i_x)^2 I \]

\[(15) \quad (i_x)^2 \Lambda + (i_x)^3 \Lambda = (i_x)^2 \Lambda \]
Circuit representation

Average power associated with incident wave

\[
P_{\text{incident}} = \frac{1}{2} \left| \left( x \right)^2 \right| = \frac{1}{2} \left| \left( x \right)^2 \right|
\]

and reflected waves anywhere on transmission lines.

Lines are lossless if and only if

\[
\begin{align*}
(l)^- + l &= (0)^- + l & \text{and} & & (l)^+ + l &= (0)^+ + l \\
0 &= x^- + l & \text{and} & & 0 &= x^+ + l
\end{align*}
\]

and for reflected power

\[
\begin{align*}
\left| \frac{\zeta}{(0)^{-2}q} \right| &= \left| \frac{\zeta}{(0)^{+2}q} \right| = \left| \frac{Z_0 \zeta}{(0)^{-2}A} \right| = (0 = x)^{-2} l \\
\left| \frac{\zeta}{(0)^{-2}d} \right| &= \left| \frac{\zeta}{(0)^{+2}d} \right| = \left| \frac{Z_0 \zeta}{(0)^{+2}A} \right| = \left[ \ast \left( (0)^{+2} + 1 \right) \right] \left( \text{Re} \frac{\zeta}{1} \right) = (0 = x)^{+2} l
\end{align*}
\]

Average power associated with incident wave...
2-port network.  

Independent of the input impedance of the port 1. Called $P_{ad}$. Since the line is lossless it is also power available at port 1 and since tr. $\text{Line}$ represents the power available from the source.

\[
\frac{\overline{I} \overline{Z}}{\overline{E}} = (0) \overline{I} + \overline{I} = \overline{I} \overline{Z} \Rightarrow \overline{P} = (\overline{I}) \overline{N}
\]

Resulting in (use eq. 17)

\[
(0) \overline{I} \overline{Z} - \overline{E} = (0 = \overline{I} \overline{Z} \overline{A} : 0 = \overline{I} \forall
\]

\[
0 = (0 = \overline{Z} \overline{I} \overline{Z} \overline{A} : 0 = \overline{Z} \overline{I} \forall
\]

Ports match terminated.

Consider Fig. 1.6.2: Port 1 excited by generator and
power delivered to port 1 is \( \frac{1}{2} \). Net otherwise, some is reflected back to generator (input matched to line) then reflected power = 0.

If \( Z_{0} \) (input impedance of input impedance \( Z \)).

\[ Z_{0}Z = I_{Z} \]

Summary: Generator send available power \( \frac{1}{2} \)(0) | \( Z_{0} \) toward input power delivered to the load \( \frac{1}{2} \) that represents the power \( \frac{1}{2} \). Similarly, \( I_{1}P_{1} - s_{1} = \frac{1}{2} |(0)I_{1}| \) is reflected power from port 1 to port which represents power delivered to port 1 (or \( I_{1} \)). Call it \( P_{1} \) so

\[ \left((0)^{I}_{1}A\right) \frac{Z}{I} = \left(\frac{1}{2} |(0)I_{1}| - \frac{1}{2} |(0)I_{1}|\right) \frac{Z}{I} \]

After simple derivation (see book):
at port 1.

\[ \frac{s_{ad}}{d - s_{ad}} = \frac{0 = (\bar{z}_1) \bar{z}_d}{(1) \bar{z}_d} = \frac{z}{z} | ^{11} S | \]

Also, note that

match terminated.

\[ ^{11} S \] is reflection coefficient of port 1 when port 2 is

\[ \frac{I_0 Z + I Z}{I_0 Z - I Z} = \frac{0 = (\bar{z}_1) + \bar{L}}{(1) + \bar{L}} = \frac{0 = (\bar{z}_1) \bar{z}_d}{(1) \bar{z}_d} = \frac{z}{z} \]

use definition of \(^{11} S\). Note this to use this to calculate S-parameters? See Fig. 1.6.3 and
Circuit representation


\[
0 = (\frac{1}{Z}) (I_1) + I_0 \frac{Z^\wedge}{(\frac{1}{Z}) \omega} \Rightarrow \frac{1}{Z} = \frac{I_2}{S}
\]

ensures that, i.e., the total current at \( I \) consists only of \( I \) (matching condition).

The total current at \( I \) consists only of \( I \) (matching condition).

\[
0 = (\frac{1}{Z}) (I_1) + I_0 \frac{Z^\wedge}{(\frac{1}{Z}) \omega} \Rightarrow \frac{1}{Z} = \frac{I_2}{S}
\]

What about \( S_{21} \) and \( S_{12} \)? Take the definition:

fig. 1.6.4

Completely analogous situation for \( S_{22} \), just change indices (see

Circuit representation
Port 1 to Port 2:

\[ S_{12} \text{ is forward voltage transmission coefficient from } \]

\[
\begin{align*}
H_{11} & \frac{I_{1}}{E} \left| \frac{Z_0}{Z} \right| E = \frac{(10Z\tilde{Z})/H_{11}}{\frac{Z_0}{Z}/(\tilde{Z})^2}Z_0^{Z} = S_{12}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{(10Z\tilde{Z})}{H_{11}} \right) & = (1)_{1} \left| \frac{I_{1}}{E} \right| \left( \frac{Z_0}{Z}/(\tilde{Z})^2 \right) = (1)_{1} \left| \frac{I_{1}}{E} \right| \left( \frac{Z_0}{Z}/(\tilde{Z})^2 \right)
\end{align*}
\]

Also:

\[
(1)_{0} \left( \frac{Z_0}{Z}/(\tilde{Z})^2 \right) = (1)_{0} \left( \frac{Z_0}{Z}/(\tilde{Z})^2 \right)
\]

Furthermore, \( (1)_{0} \)
Do example 1.6.1 for S-parameters calculation and another ex.

Also: S\textsuperscript{12} is transducer power gain (note the loading conditions).

\begin{equation}
\frac{p_{\text{delivered to load}}}{p} = \frac{(10Z|8|/2H|L|1/2|G|)}{Z/|2| |2| |A| |2| |T|} = |S|_{12}^{12}
\end{equation}

Also:
For $S_{22}$ and $S_{12}$ just interchange ports (terminals).

Transducer power gain is ratio of power delivered to load to available power available from source, from forward transmission coeff.:

$$
\frac{s_{ad}P_1}{P_0} = \left| \frac{Z_0}{Z} \right|^2 \frac{Z_0}{Z} = \frac{Z_0}{Z}.
$$

Port to port matched output match terminated transducer power gain:

$$
\frac{P_L}{P_0} = \left| \frac{s_{ad}d}{d - s_{ad}d} \right| = \frac{1}{12} S.
$$

Input match condition (tr. line matching condition) with reflection coeff. with:

$$
0 = \left( \frac{1}{12} \right) \frac{P_0}{P_1} = \frac{1}{12} S.
$$
Circuit representation

Generalized S-parameters

So far: 50 Ohm lines and 50 Ohm terminations. How about n-port networks with general terminations?

So far: 50 Ohm tr. lines and 50 Ohm terminations. How about Generalized S-parameters?
Circuit representation

\[
\begin{bmatrix}
  u^0Z & \cdots & 0 & 0 \\
  0 & \ddots & 0 & 0 \\
  0 & \cdots & Z^0 & 0 \\
  0 & \cdots & 0 & Z^0
\end{bmatrix}
= [\hat{\nu}^0Z]
\]

\[(36) \quad ([I] \cdot [\hat{\nu}^0Z] - [\Lambda])\begin{bmatrix}
  \frac{\nu^0}{I} \\
  \frac{\nu^0}{I}
\end{bmatrix} = [q]
\]

\[(37) \quad ([I] [\hat{\nu}^0Z] + [\Lambda])\begin{bmatrix}
  \frac{\nu^0}{I} \\
  \frac{\nu^0}{I}
\end{bmatrix} = [p]
\]

how to generalize this for any impedance?
As before, expressing \( V \) gives

\[
(39) \quad \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} = \mathbb{I} a = \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} = \mathbb{I} a \leq \mathcal{A}^1_{\mathbb{H}} Z - \mathbb{I} E = \mathbb{I} \Lambda
\]

\[
(38) \quad (I^{10} Z - \mathbb{I} \Lambda) \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} = \mathbb{I} q
\]

\[
(37) \quad (I^{10} Z + \mathbb{I} \Lambda) \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} = \mathbb{I} \mathcal{A}
\]

\[
\begin{bmatrix}
\frac{\mathcal{A}^1_{\mathbb{H}}}{Z} & (\mathcal{W}^{10}_{\mathbb{H}} R \mathcal{E}) & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 \\
0 & \cdots & \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} & (\mathcal{W}^{10}_{\mathbb{H}} R \mathcal{E}) & 0 \\
0 & \cdots & 0 & \frac{\mathcal{A}^1_{\mathbb{H}}}{Z} & (\mathcal{W}^{10}_{\mathbb{H}} R \mathcal{E})
\end{bmatrix}
= \begin{bmatrix}
\mathcal{A}^1_{\mathbb{H}} \\
\mathcal{A}^1_{\mathbb{H}} \\
\mathcal{A}^1_{\mathbb{H}} \\
\mathcal{A}^1_{\mathbb{H}}
\end{bmatrix}
\]
and can even be complex. Normalizing impedance. Usually 50 Ohms, but can be anything.

Define it via S-matrix depends on the choice of.

Generalized scattering matrix

\[
[S] = [q]
\]

is completely absorbed by \( Z \). If not matched, then power absorbed is

\[
|\text{power absorbed to port } 1| = \left| I \right| \left( Z \right) = {1 \over \left| I \right|}
\]

If port 1 is matched (so that

\[
Z = \left( I \over I \right)
\]

Also (as before)

\[
\text{power available from source at port } 1. = \text{Re} (I \cdot Z)
\]

\[
= \left| I \right| - \left| q \right|
\]
Circuit representation

Calculating $S_{ij}$:

$$S_{ii} = \frac{b_i}{a_i},$$

$$S_{ki} = \frac{b_k}{a_i}, \quad k \neq i,$$

$$S_{ij} = \frac{V_i}{Z_0} + Z_i I_i, \quad i \neq j,$$

$$\frac{V_i}{Z_0} = \frac{V_i}{Z_0} + Z_i I_i,$$

$$\frac{Z_i}{Z_0} = \frac{Z_i}{Z_0} + Z_i I_i.$$

Parameter conversions: see tables. They all describe the same matched.

transducer power gain from $i$ to $k$ with ports other than $i$.

$$w^{\ldots, i} = \gamma_{i}'z_i' \neq \gamma_0' = \gamma_d, \quad \frac{\gamma_d}{\gamma_q} = \gamma_S.$$

$$\frac{\gamma_d}{\gamma_q} = \gamma_S.$$
S-parameters for transistors

Most common way of specifying transistor performance. Usually S-parameters for transistors

\[
S_{12} \text{ exhibits opposite behavior (why?)}.
\]

\[
\frac{\text{dB/octave}}{f^s \text{is its cut-off frequency}}.
\]

\[
\text{the magnitudes vs. freq. in Fig. 1.9.7. } |S_{21}| \quad S_{12}
\]

\[
\text{and } S_{12} \text{ usually exhibit opposite behavior as RLC circuits (Fig. 1.9.3).}
\]

For S-chart presentation see Fig. 1.9.2. S_{11} and S_{22} typically behave as RLC circuits.

Simple R-C circuit S-chart in Fig 1.9.1

\[
Y_{CB} \rightarrow Y_{CE} \rightarrow Y_{CE}
\]

\[
\leftarrow S_{21} \rightarrow S_{12}
\]

\[
\text{parameters has to go through y or z-parameters. } S_{21} \leftarrow S_{CE} \text{ conversion of } S-
\]

\[
\text{but can be converted to CB. } CE \rightarrow CB \text{ conversion of } S-
\]

\[
\text{Most common way of specifying transistor performance. Usually S-}
\]
If no choice of ground terminal has been made  

Note: sum of the rows and columns in indefinite S-matrix is unity!  

\[
\begin{align*}
0 = \varepsilon_0 \rightarrow & \frac{1}{\Pi D} = \Pi S \\
\frac{1}{\Pi S} & = \text{No choice of ground terminal has been made} \\
\text{If no choice of ground terminal has been made} & = \text{Indefinite S-matrix is unity!}
\end{align*}
\]