Name: ____________________

CS 311 Sample Final Examination

Time: One hour and fifty minutes
This is the (corrected) exam from Fall 2009.
The real exam will not use the same questions!

8 December 2009

Instructions

Attempt all questions.

Write your answers on the exam paper. You can use the remainder of this sheet and the back side of the exam for scratch space. If you use your own plain paper for scratch work, please hand in your scratch paper with your exam.

You can bring with you into the exam room a single double-sided "crib sheet" of letter-sized paper with your own notes. No textbooks, or lecture notes, will be permitted. Hand in your crib sheet with your answers.

You probably will not have enough time to answer all of the questions: you are strongly advised to spend the first 10 minutes or so reading through the whole paper, and strategizing where to spend your time. The number of points for each question is given.

Do not turn this page until you are instructed to do so.

Question 1: Regular Languages

3 points

1. Give a regular expression generating the language that consists of all sequences of as and bs whose penultimate character is an a.

4 points

2. Draw a state diagram for an NFA with exactly 3 states that accepts the language of part 1.

8 points

3. Give a DFA that accepts the language of part 1. Remember to specify the start state and the accepting states.

Question 2: Context-free Languages

5 points

1. Let $L = \{(ab)^n(a+b)^{2n} \mid \; n \geq 0\}$.

   Draw the state diagram for a PDA accepting L. (Recall that a PDA state diagram has transition labels of the form "$x, P \rightarrow Q$" where x is an input symbol or ϵ, and P and Q are stack symbols or ϵ.)

6 points

2. If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. For example, language L from part 1 of this question can be written as $(ab)^* \diamond (a+b)^*$. Show that if A and B are *any* two regular languages, then $A \diamond B$ is context-free.

Question 3: Non-Context-free Languages

8 points

1. Let language $L = \{wcw \mid w \in \{a, b\}^*\}$. Use the pumping lemma to show that L is not context-free.

8 points

2. Give the fully detailed state diagram for a Turing Machine M that decides language L from part 1 of this quesiton. (Recall that a Turing Machine state diagram has transition labels of the form "$x \to y, L$" or "$x \to y, R$" where x and y are tape symbols.)

4 points

3. Let $L' = \{ww \mid w \in \{a,b\}^*\}$. Give a high-level description of how to build a Turing Machine M' that decides L' using machine M from part 2 as a sub-component.

Question 4: Computability

Note: Recall that "Turing-recognizable" means the same thing as "Recursively Enumerable (RE)" and "Turing-decidable" means the same thing as "Recursive."

4 points

1. Carefully explain the difference between a Turing-recognizable language and a Turing-decidable language.

14 points

2. Let $A_{TM} = \{\langle M, w\rangle \mid M \text{ is a Turing machine and } M \text{ accepts } w\}$. Give a *careful* proof, from first principles, that A_{TM} is not Turing-decidable.

3. Let $A_{HALT} = \{\langle M, w\rangle \mid M \text{ is a Turing machine and } M \text{ halts on input } w\}$. Show that A_{HALT} is not Turing-decidable. Use a reduction argument; you may assume the result of part 2 and the existence of a Universal Turing Machine that recognizes A_{TM}.

7 points

Question 5: Computational Complexity

5 points

1. Give careful definitions of the classes P and NP.

6 points

2. Let REG be the class of all regular languages. Show that $REG \subseteq P$.

3. Let SAT be the usual Boolean satisfiability problem and FOO be some other problem. Consider the following four possible proofs:

   (a) Proof of a polynomial-time reduction from SAT to FOO.

   (b) Proof of a polynomial-time reduction from FOO to SAT.

   (c) Proof of a $O(100^n)$-time algorithm for deciding SAT.

   (d) Proof of a $O(n^{100})$-time algorithm for deciding FOO.

   Which *two* proofs, taken together, would make their author rich and famous? Explain.

4 points

Question 6: The Language Hierarchy

Let $\Sigma = \{0, 1\}$. Consider the following eight classes of languages over Σ:

- $ALL = \mathcal{P}(\Sigma^*)$
- TR = Turing-recognizable
- TD = Turing-decidable
- NP
- P
- CF = Context-Free
- REG = Regular
- FIN = Finite

Each class is a superset of the next one, and all inclusions except $P \subseteq NP$ are known to be proper. Situate each of the following languages as **low** as you can in the hierarchy (e.g., if a language is in P but is not context-free, the answer is P).

2 points

1. $\{0^n 1^n 0^n \mid n \geq 2\}$

2 points

2. $\{0^n 1^n \mid n \geq 2\}$

2 points

3. $\{0^n 1^n \mid n \leq 2\}$

2 points

4. $\Sigma^* - \{\epsilon\}$

2 points

5. $\{\langle M \rangle \mid M \text{ is a TM that accepts a regular language}\}$

2 points

6. $\{\langle M, w \rangle \mid M \text{ is a TM that does not halt on input } w\}$

2 points

7. $\{\langle M \rangle \mid M \text{ is a TM}\}$