Name: _____

CS 311 Midterm Examination

Time: One hour and fifty minutes
Closed Book

29 Oct 2009

Instructions

This is a closed book exam. Attempt all questions.

Write your answers on the exam paper. You can use the remainder of this sheet and the back side of the exam for scratch space.

Do not turn this page until you are instructed to do so.

Question 1: Languages

Assume we have the alphabet $\Sigma = \{a, b\}$, and consider the following languages over Σ:

- $L_1 = \{a, ab, ba\}$
- $L_2 = \{\epsilon, ab, b\}$
- $L_3 = \mathcal{L}[a^*]$ (i.e., the language represented by the regular expression a^*).
- $L_4 = \{a^n b^n \mid n \geq 0\}$

2 points

1. Write $L_1 \cup L_2$ as a set of strings.

2 points

2. Write $(L_1.L_2)$ as a set of strings.

2 points

3. Write a regular expression for $(L_2)^*$.

2 points

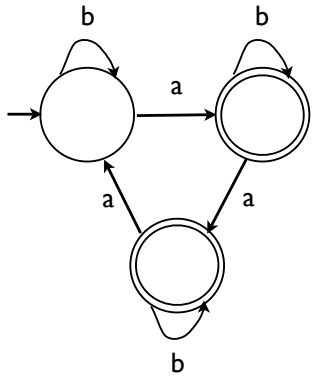
4. Write a regular expression for $\overline{L_3}$.

2 points

5. Write a regular expression for $(L_3.L_4)$.

Question 2: DFA's

Let M be the DFA having alphabet $\{a, b\}$ and the following transition diagram:



3 points

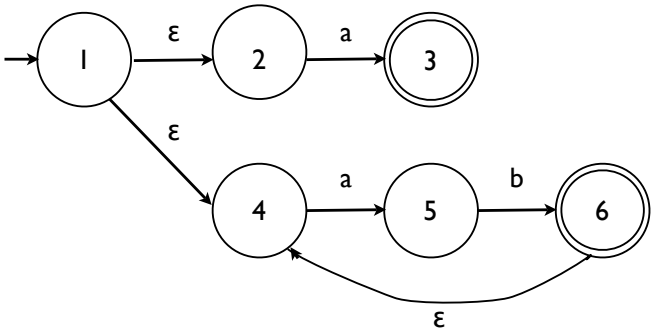
1. Give a simple description of $L(M)$ in English or informal mathematical notation.

7 points

2. Give a regular expression for $L(M)$. Try to keep your expression simple. (Use any method you want.)

Question 3: NFA's and DFA's

Consider the following NFA over the alphabet $\{a, b\}$.



Use the subset construction algorithm to convert this NFA into an equivalent DFA. You may represent your answer using either a state diagram or a transition table. Name each DFA state using the corresponding set of NFA states. Be sure to mark the final states. Be sure to include any error transitions. Be careful!

10 points

Question 4: Regular Expressions and NFA's

Give the state diagram of an NFA recognizing the language represented by the regular expression $a + (a^*b + aab^*)^*$.

10 points

Question 5: Context-free Grammars

Give a context-free grammar generating the language $L = \{a^n b^m \mid n < m \text{ or } n = 2m\}$. (Hint: Think of L as the union of two simpler languages.)

10 points

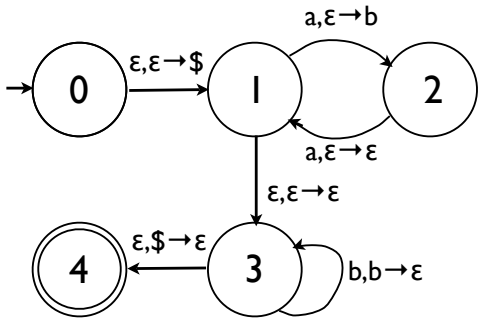
Question 6: Non-regular Languages

Use the Pumping Lemma to show that the language $L = \{w \in \{a, b\}^* \mid w = w^R\}$ (i.e., the language of palindromes) is not regular. Be precise; the details matter.

10 points

Question 7: Pushdown Automata

Let M be the following PDA:



3 points

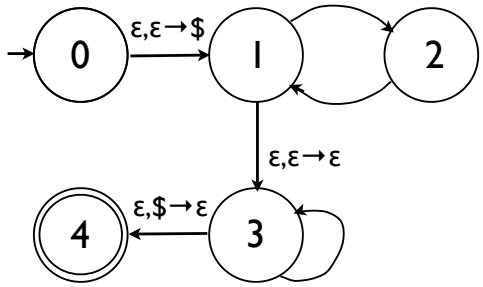
1. Show formally that M accepts the string aab. (Hint: Use instantaneous descriptions.)

2 points

2. Describe $L(M)$ in English or informal mathematical notation.

5 points

3. Show how M can be modified to make a new machine M' such that $L(M') = (L(M))^R$, i.e., M' accepts the reverses of the strings accepted by M. Your modification can *only* change the labels on the transitions from states 1 to 2, 2 to 1, and 3 to 3; the rest of the machine must remain unchanged. Write in your revised transitions below:



Question 8: True or False

Is each of the following assertions true or false? To obtain credit, you *must* briefly explain why the assertion is true or give a counterexample if it is false.

2 points

1. For every pushdown automaton M there is a regular expression describing $L(M)$.

2 points

2. If L is finite then $\Sigma^* - L$ is regular.

2 points

3. If $L^* \neq \emptyset$ then $L \neq \emptyset$.

2 points

4. If L_1 is context-free and L_2 is regular, then $L_1.L_2$ is context-free.

2 points

5. If $L_1.L_2$ is regular then L_1 and L_2 are both regular.