# CS 311 Homework 7

due 16:40, Tuesday, $9^{th}$ November 2010

Homework must be submitted on paper, in class.

## Question 1. [40 pts.; 20 pts each.]

Prove that the following languages are not context free using the pumping lemma. Pay careful attention to the structure of your proof, in particular the alternating quantifiers. (# is just a symbol)

*a.* $\{0^n\#0^n\#0^n \mid n \geq 0\}$.

*b.* $\{t\#w \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$

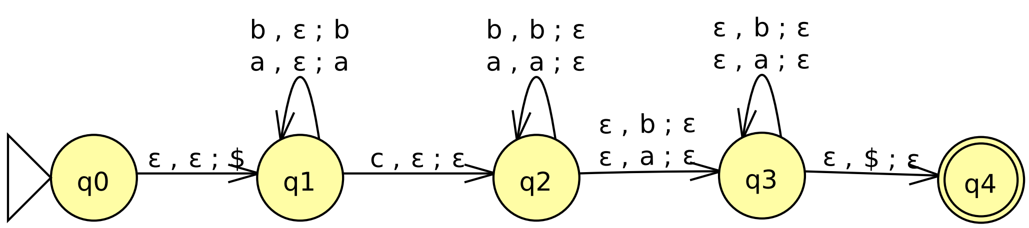
## Question 2. [20 pts.]

Convert the following CFG into an equivalent CFG in Chomsky normal form. Be sure to show intermediate steps as you apply the rules of the conversion. The start symbol is $A$.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow 00 \mid \varepsilon \end{aligned}$$

## Question 3. [20 pts.]

Convert the following pushdown automata into a context free grammar. Use the construction from the lecture of 21st October, slides 15 onwards. This construction is also described in *Sipser* Lemma 2.27




Initially, list all the productions that describe possible transitions of the PDA. You do not need to list productions for transitions that obviously won't happen. For example, it is obvious that the machine will never move from $q2$ to $q1$, even indirectly, so please don't bother to write the rules for $A_{21}$.

Use the following alphabets, and use the diagram to determine the transition function and states.

$$\Sigma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$$
$$\Gamma = \Sigma \cup \{\$\}$$

When constructing the grammar, use variables of the form $A_{01}$ to generates all the strings that take the machine from state $q0$ with an empty stack to state $q1$ with an empty stack. You may augment the stack alphabet, $\Gamma$, as needed to complete the construction. Note that this diagram *does not* meet the requirements for the construction (every transition must either push exactly one symbol or pop exactly one symbol) and you will first have to modify it in one small way!