CS 311 Homework 3

due 16:30, Thursday, 14^{th} October 2010

Homework must be submitted on paper, in class.

Question 1. [15 pts.; 5 pts. each]

Draw state diagrams for NFAs recognizing the following languages:

a. $L = \{w \mid w \in \{\mathtt{0},\mathtt{1}\}^* \text{ and } w \text{ ends with } 00\}$.

b. L is the language represented by the regular expression $(\mathtt{a} + \mathtt{b})^*\mathtt{aa}(\mathtt{a} + \mathtt{b})^*$.

c. $L = \{\mathtt{0101}, \mathtt{101}, \mathtt{1100}\}$.

Question 2. [30 pts.; *a, b* 5 pts.; *c* 20 pts.]

$L^{\mathcal{R}}$ is the language of strings which are the reverse of the strings in the language L. For $L = \{w \mid w \in \{\mathtt{a},\mathtt{b}\}^* \text{ and in each initial substring of } w, |\text{number of } \mathtt{a}\text{s} - \text{the number of } \mathtt{b}\text{s}| \leq 2\}$ complete the following exercises:

a. Construct the diagram of a DFA which accepts strings in the language L.

b. Construct the diagram of an NFA which accepts strings in the language $L^{\mathcal{R}}$.

c. Convert the NFA to a DFA. Be sure to provide all five components of the DFA, show the transition table for δ, and provide a diagram. You may relabel the states for convenience, if you like. If you do, be sure to show the intermediate steps where relabeling takes place.

Question 3. [25 pts.; *a, b, c* 5 pts; *c* 10 pts]

Given languages $L_1 = \{w \mid w \in \{\mathtt{a},\mathtt{b}\}^*, |\mathtt{w}| \geq 3\}$, $L_2 = \{w \mid w \in \{\mathtt{a},\mathtt{b}\}^*, \mathtt{w} \text{ contains } \mathtt{bb}\}$,

a. Construct the diagrams of the DFAs which accept strings in the languages L_1 and L_2.

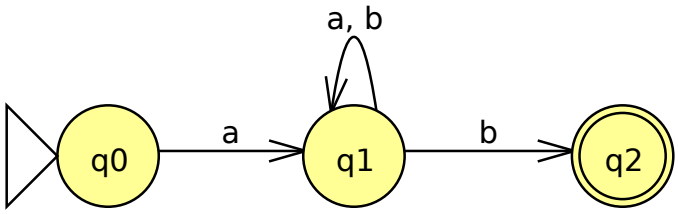
b. Construct the diagram of an NFA which accepts strings in the language $L_1 \cup L_2$.

c. Convert the NFA to a DFA, in the same way as in Question 2*c*.

d. Define a new set of accepting states that will make the DFA accept strings in the language $L_1 \cap L_2$. Explain briefly the difference between this new set of accepting states and the set defined in part *c.* of this question.

Question 4. [25 pts; *a* 7 pts; *b* 18 pts.]

a. Describe the language accepted by the following NFA:



b. Prove that this NFA accepts the language that you described in part (a). In your proof, use induction on the length of the input. Be sure to state your induction hypothesis explicitly. *Hint:* You will use what Hopcroft et al. call "mutual induction" (§1.4.4) (*end of hint*).