CS 311 Homework 1

due 16:00, Thursday, 7^{th} October 2010

Homework must be submitted on paper, in class.

Question 1. [20 pts.; 5 pts. each]

Draw state diagrams for DFAs recognizing the following languages over the alphabet $\{0, 1\}$. remember to indicate the initial state and the final state(s), and to label all the transitions.

a. $\{ w \mid w$ contains the symbol 1 at least three times.$\}$.

b. $\{ w \mid w$ has all the 0 symbols precede all the 1 symbols.$\}$. *Hint:* What does the question state about how many times each symbol must appear? (*end of hint*).

c. $\{ w \mid w$ contains the substring 110 exactly once$\}$.

d. $\{ w \mid w$ does not contain 110$\}$.

Question 2. [12 pts.; 4 pts. each]

Given $M = \langle\{0, 1, 2, 3, 4\}, \{\mathtt{a}, \mathtt{b}\}, \delta, 0, \{4\}\rangle$ where δ is given by the table below:

	a	b
0	1	0
1	2	0
2	4	3
3	1	4
4	4	4

a. Draw the state diagram for this DFA.

b. Informally describe the language that M accepts.

c. For each of the following three strings, determine whether the string is accepted. List the sequence of states $r_0, r_1, \ldots, r_n$ (from the formal definition) through which the machine moves as it reads the string. (*Hint*, the final state r_n should correspond to your determination of whether the machine accepts or not!).

- *bbabaabb*,
- *abaabaaa*, and
- *aabab*.

Question 3. [8 pts.]

It is common to use computers to warn us of mistakes we might otherwise miss. (One such mistake to leave unbalanced parentheses (like this!).

It is not possible to determine whether parentheses are balanced using a DFA. Discuss informally why this is so.

Question 4. [10 pts.]

Develop a formal definition for the DFA M, defined as the union of the DFA's described in Question 1a and 1b. Use the 5-tuple notation with a transition table.

Question 5. [10 pts.]

In the lecture, Prof. Black gave you the formal definition of DFA, and also defined how to construct a DFA that represents the union of 2 DFAs. However, in the lecture he did not prove that this construction is correct. Prove that $w \in L(M_a) \Rightarrow w \in L(M_a \cup M_b)$. (This is one piece of the proof of the closure of regular languages under union.)