CS 311 Assignment 0: The Entrance Examination

Spring 2008

This isn't really an entrance examination. You can't "fail" it, and "passing" it is not required for you to take the class. It's purpose is to let me see how well-prepared you are, so that I can evaluate the base on which I'm building. HOWEVER — and that's a big however — if you can't answer these questions easily, then there is something wrong with your preparation for this class. Have you taken CS 250 and CS 251? Have you reviewed your notes and the textbook from those courses? Are you prepared to do some serious revision to get back up to speed with this material?

I *will* give you a quick reminder of the basic pre-required material before I delve into the details of the relevant parts of this course. I *will not* attempt to provide a thorough background that will obviate doing your own revision.

Question 1: Cardinality of Sets

1. Define the set of natural numbers, $\mathbb{N}$.
2. Which is bigger: $\mathbb{N}$, or $\{n \in \mathbb{N} \mid \text{even}(n)\}$? Give a proof of your answer.
3. Which is bigger: $\mathbb{N}$, or the set of all real numbers $\mathbb{R}$? Give a proof of your answer.

Question 2: Algebras

A is an algebra with carrier S and operations . and $+$; e and f are two of the elements of S.

1. Write down expressions for four elements of S
2. $\forall s \in S. \quad s = e.s \quad \text{and} \quad s = s.e;$ what name do we give to e?
3. $\forall s, t \in S. \quad s + t = t + s;$ what do we call this property of $+$?
4. $\forall s, t, u \in S. \quad (s.t).u = s.(t.u);$ what do we call this property of .?
5. Give a concrete example of an algebra that satisfies the above three properties 2–4. Specify S, e, . and $+$.

Question 3: Morphisms

Suppose that the $\mathsf{binaryNumerals}$ are the binary representations of numbers, and that $+_d$ is the operation of addition of binary numerals, just as you learned it in computer science 101

1. Define a morphism (also called a homomorphism) m between $\mathsf{binaryNumerals}$ and $\mathbb{N}$.

2. Is your m an isomorphism? Why, or why not?

3. Are *all* functions from $\mathsf{binaryNumerals}$ to $\mathbb{N}$ morphisms? Justify your answer.

4. Let A be an alphabet and $f : A^* \to \mathbb{N}$ be defined by $f(x) = \mathrm{length}(x)$. Show that f is a morphism from the algebra $\langle A^*; \mathsf{concat}, \Lambda\rangle$ to the algebra $\langle \mathbb{N}; +, 0\rangle$, where concat is concatentation and Λ is the empty sequence.

Question 4: Relations

Let's define the relation **divides** as follows:

$$a \textbf{ divides } b =_{def} \lfloor b \div a \rfloor \times a = b$$

So, 5 **divides** 15, because $\lfloor 15 \div 5 \rfloor \times 5 = 15$, but $\neg$(5 **divides** 12), because $\lfloor 12 \div 5 \rfloor \times 5 = 10$.

1. Is **divides** reflexive? Justify your answer.

2. Is **divides** symmetric? Justify your answer.

3. Is **divides** transitive? Justify your answer.

4. Is **divides** antisymmetric? Justify your answer.

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