Yacht For One Or Two

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Overview

- Perfect Yacht
- Retrograde Analysis
- The Rest Of The Story
The Game Of Yacht

- From “Poker Dice” (1800s)
- Idea: Roll 5 dice to fill scoresheet

<table>
<thead>
<tr>
<th>Lower Section</th>
<th>Upper Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aces</td>
<td>Yacht</td>
</tr>
<tr>
<td>Deuces</td>
<td>Four Of A Kind</td>
</tr>
<tr>
<td>Threes</td>
<td>Full House</td>
</tr>
<tr>
<td>Fours</td>
<td>Small Straight</td>
</tr>
<tr>
<td>Fives</td>
<td>Large Straight</td>
</tr>
<tr>
<td>Sixes</td>
<td>Choice</td>
</tr>
</tbody>
</table>

50

30
Yacht Turns

1. Roll 5 dice
2. Select subset of 5 dice to reroll
3. Select subset of 5 dice to reroll
4. Fill in score blank. If dice match no blank, “take a zero”
A Yacht Turn
Yacht Strategy

- Roll for most valuable blank, *but*...
- Save easy blanks for later
- Chance, Aces, Deuces blanks essentially wild
- Yacht blank is very unlikely
Multiple-Player Yacht

- Interesting: “winner takes all”
- May be risk-centric or risk-averse
- Can play by score: still end-effects
Yahtzee

- “History”: E.S. Lowe 1956
  http://www.hasbro.com/consumer/history/yahthist.htm

- Slight variant
  - Slightly different blanks (13 vs. 12)
  - Subsequent Yahtzees wild
  - “Upper Section Bonus”
# Yahtzee Scoring: Upper Section Bonus

<table>
<thead>
<tr>
<th>Lower Section</th>
<th>Upper Section</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aces</td>
<td>Yahtzee</td>
<td>50</td>
</tr>
<tr>
<td>Deuces</td>
<td>Four Of A Kind</td>
<td></td>
</tr>
<tr>
<td>Threes</td>
<td>Full House</td>
<td></td>
</tr>
<tr>
<td>Fours</td>
<td>Small Straight</td>
<td>30</td>
</tr>
<tr>
<td>Fives</td>
<td>Large Straight</td>
<td>45</td>
</tr>
<tr>
<td>Sixes</td>
<td>Three Of A Kind</td>
<td></td>
</tr>
<tr>
<td>Bonus (63)</td>
<td>Chance</td>
<td></td>
</tr>
</tbody>
</table>
Perfect Deterministic Games

- Deterministic 2-player game (e.g., Chess): best result against perfect opponent
- Deterministic 1-player game (e.g., Rubik’s Cube): solve (short soln?)
Perfect Yacht

What is “perfect” Yacht?

- Stochastic 1-player game: make moves with highest expected value!
- Stochastic 2-player game: make moves with highest expected winning chance
- Note: perfect information, alternating, terminating...
Previous Work

- Tom Verhoeff, Eindhoven University of Technology
  1999: Online Optimal Single-Player Yahtzee
  http://wwwpa.win.tue.nl/misc/yahtzee/

- William Tunstall-Pedoe (date?): claimed
Forward Analysis

- To calculate value of a pick:
  - Calculate value of each reroll
  - Weight by roll probability
  - Sum

\[ E(\text{pick}) = \sum_{r \in \text{rerolls}} pr(r)E(r) \]
## Counting Die Rolls

<table>
<thead>
<tr>
<th>dice</th>
<th>rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>252</td>
</tr>
<tr>
<td>4</td>
<td>126</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>
Cost Of Pure Forward Analysis

- Up to 32 possible picks. Overestimate (slight)

\[(252 \cdot 32 \cdot 252 \cdot 32 \cdot 252)^{12} \approx 2 \times 10^{26}\]

- Modern machine: $10^{11}$ insns/hour
State Space Search

- Expected value depends only on state
- State is only scoresheet blanks plus current dice
- Can estimate value of scoresheet just before turn
- Each state: $252^3 \cdot 32^2 \approx 2 \times 10^{10}$
- States: $12! \approx 5 \times 10^8$
- Cost down to $\approx 10^{19}$
Retrograde Analysis

- Previous slide silly
  - There are only $2^{12}$ scoresheet states
  - Each state independent of history!
  - Start with last states

$$E(s) = \text{score}(s) + E(\text{succ}(s))$$
In-Turn Retrograde Analysis

- Can use same trick for turn
- Work from end of turn to beginning
Performance

- Current code: $\approx 1$ sec for complete Yacht
- Yahtzee solution is harder:
  - Extra blank: $\times 2$
  - Lower Section Bonus: $\times 63$
  - Multiple Yahtzees: more complex code
- Current code: $\approx 9$ min (20MB)
- (Answers? Yacht 169.8, Yahtzee 254.6)
Yahtzee “High Score”? 

- Open question (Verhoeven): Perfect solitaire play to beat previous high score?
- Can solve using retrograde analysis on target score
- Target scores in range 0..600 “interesting”
Two-Player Yacht?

- Interesting example of “race game”

- States?

\[
\sum_{k=0}^{11} \frac{1}{2} \binom{12}{k} \left[ \binom{12}{k} + 1 \right] \\
+ \sum_{k=0}^{10} \binom{12}{k+1} \binom{12}{k} \\
= 3850257
\]

- Time? 12.5 days (2500 states/sec, 600 scores)
Why?

Results show the power of

- Retrograde analysis
- Combinatorics and probabilistics
- Modern hardware

Another traditional game nears solution... Next?