Pathfinding Search and Adversary Search

CS410/510CS Lecture 6

Bart Massey <bart@cs.pdx.edu>
PSU CS Department
February 15, 2001
Overview

- Pathfinding Search
- Adversary Search
Two Topics

Why these two topics together?

• Amount of material, obviously

• Common themes:
  – Optimization
  – Heuristics
  – BnB pruning
  – Search on graphs
  …
Pathfinding Search

Concerned with finding *path* through graph

- Source: initial node
- Sink: goal node
- Graph: undirected?
Optimal Pathfinding Problem

Find some *shortest* path

- May be many paths
- Combinatorial graph $= \text{big}$
DFS For Pathfinding

Find some path: via DFS

- Requires marking
- In NP: might “get lucky”
- Usual val (not var) order applies
DFS Marking: The Closed List

Two ways to mark (= keep track of visited nodes)

- Mark nodes themselves
- Keep set ("list") of closed (= marked) nodes

Advantages of closed list

- Can unmark trivially
- Can control space usage
DFS Marking: The Open List

DFS uses (implicit or explicit) stack

- Contents is open (= to-do) nodes
- Can represent stack as open list (= priority-queue)
- Disadvantage: explicitly represents siblings

All implicit data in recursive DFS replaced with transparent data structures
Explicit DFS: Example
BFS For Optimal Pathfinding

For optimal (shortest) pathfinding, use BFS

On undirected graph, must decide “parent” vs. “children”
Open and Closed Lists In BFS

- Closed List: Same as DFS
- Open List: Still pqueue—least-recently, not most-recently visited
- BFS, so sizes problematic
- Can reduce open list through ID
- How to reduce closed?
Weighted Optimal Pathfinding

More generally

- Include edge weights
- Normally non-negative
Dijkstra’s Algorithm

Best-First Search: Expand open node on shortest path from source

• Open List: nodes pending expansion

• Closed List: expanded nodes

• Nodes can be reopened! Reopen sibling $v$ of $u$ iff path to $v$ through $u$ is shorter

• Must not stop until goal is closed
Dijkstra’s Algorithm: Example

Keep frontier of nodes on shortest paths until goal closed
Proof: Dijkstra’s Algorithm Works

Consider path from source $s$ to node $v$

- Base case: $v$ is neighbor of $s$; shortest path chosen
- Inductive case: last link of earliest shortest path is $u$

By hypothesis, will find shortest path to $u$: then reopen neighbors of $u$. since earliest, will add $s, \ldots, u, v$
CS350: “Polytime” Pathfinding Algorithms

From CS 350: Dijkstra’s Algorithm is “$O(|E|)$

Is pathfinding NP hard? Yes, for *combinatorial* problems:

- $|E|$ is $O(2^{\text{desc-size}})$
- Is pathfinding *in* NP? In general, no: bounds check might require tracing potentially exponential path!
- Many realistic pathfinding problems have $O(n^k)$ optimal path length bound
Admissible Distance Heuristics

Apply admissible distance heuristic to BFS/Dijkstra? Sure!

Result is $A^*$ (Nilsson, Pearl, Korf)

- Basically BnB on graph
- Because of graph/paths, prune may not be permanent
  - Score on open list is still $f(n) = g(n) + h(n)$
  - Node may be re-opened if shorter $g(n)$ found
A* Example: Dijkstra + BnB

Consider typical lame heuristic \( h(n) \)
A* and The Triangle Inequality

If heuristic is *monotonic* (= *consistent*, obeys *triangle inequality*), need not reopen!

See handout for proofs
**A*: Optimality

No need for fancy proof of correctness. Applying BnB to Dijkstra is safe

Provably time-optimal algorithm! But beware of assumptions

Time behavior in various cases

- DFS-like behavior with perfect heuristic
- BFS-like behavior with 0 heuristic
Iterative Deepening A*

Remove space problem of $A^*$ via ID? Maybe

On trees, can do $IDA^*$

- Performance tradeoff familiar from ID
- BnB still works
IDA* and Loops

Problem: *ID* only reduces open list size. Still must keep complete closed list for systematicity.

*IDA* still *correct* without closed list. But DFS looping problem will dominate with bad heuristic. Tabu?
Tricks For A* and IDA*

- Enforcing monotonicity
- Scaling up heuristic
- Breaking ties: expand best \( g(n) + h(n) \) in order of increasing \( g(n) \)
Pathfinding With Multiple Goals

Everything so far essentially OK with multiple goals. Common real-world case

In CS350, learned “all-pairs shortest-path”. But requires too much memory here
Adversary Search

Search algorithms deal with imperfect information due to computational limits. AI question: “Is the world hostile”?

Consider world with hostile, capable adversary. Successful search there should be robust
Combinatorial Games

“Search in games” could mean many things

- Pathfinding search on game maps
- Realtime search and strategy
- Optimal move planning for e.g. chess or checkers

We take the last meaning. These games are combinatorial: a few simple rules, but many interesting game states
Specific Restrictions

We consider games which are

- Two-player
- “Zero-sum”
- Alternating
- Terminating
- Deterministic
- Perfect Knowledge

Still huge class of problems
The Value Of A Game

*Value* of game (state) $G$ to player $P$ is score $P$ can expect for $G$ given perfect play by $P$ and opponent. Usually

- $1$: $P$ wins
- $-1$: $P$ loses
- $0$: a tie

If for all reachable states of some $G$, $\text{value}(G)$ is tractable, $G$ is “solved”: can make perfect move by maximizing $\text{value}(G)$
Game Trees

Value of a game state can be computed from value of all subsequent possible states: Game Tree
The Minimax Theorem (von Neumann)

In restricted Combinatorial Game, best move minimizes opponent’s expected value
Game Tree Search

Minimax search is

- All solutions (DFS)
- Over combinatorial space
- Intractable

Approximate value? Yes: use heuristic. But hard to build good heuristics...

Soln: depth-limit search (like ID) and use heuristic at leaves!
Game Tree Search Converges

Is depth-limited search with heuristic at leaves better than heuristic at root?

- Intuitively, yes: early mistakes
- Theoretically, yes: proof by Korf
- In practice, yes: e.g. Chess

Searching a “ply” (tree level) deeper uniformly means (in practice) uniformly better idea of value of root
alpha-beta Pruning

Like BnB on depth-limited game tree. But adversary (= all-solutions) limits pruning
Additional Notes On alpha-beta

- Pruning depends on value ordering heuristic quality
  - Perfect pruning = double depth
  - Random pruning = 30

- Other search strategies are available

- Historically, enabled decent play
Game Tree Search and Closed Lists

Game states are graph, not tree! Build closed list, “transposition table”

• Stops looping

• Prevents re-search

• With random replacement, does OK for memory usage
Game Tree Search and Iterative Deepening

Why do ID? Planning to search anyhow...

- “Anytime” property: want to have backed up move when time expires
- Can use backed up values from transposition table for value ordering: dramatically improves $\alpha\beta$ performance
alpha-beta + ID + Transposition Tables

\(\alpha\beta\) works very well with iterative deepening and transposition tables

- Transposition table remembers estimated values
- Get more accurate as search deepens
- Cause more \(\alpha\beta\) pruning
- Increases search depth
More Advanced Methods, Tougher Problems

More Advanced Methods

- “Zero-window” methods
- MTD(f)

Tougher Problems

- Probability
- Hidden Information
- Nonconstant-Sum
- Multiplayer