

Garey & Johnson, *Computers and Intractability*, from appendix A.

**[ND17] MINIMUM CUT INTO BOUNDED SETS**

INSTANCE: Graph  $G = (V, E)$ , weight  $w(e) \in Z^+$  for each  $e \in E$ , specified vertices  $s, t \in V$ , positive integer  $B \leq |V|$ , positive integer  $K$ .

QUESTION: Is there a partition of  $V$  into disjoint sets  $V_1$  and  $V_2$  such that  $s \in V_1, t \in V_2, |V_1| \leq B, |V_2| \leq B$ , and such that the sum of the weights of the edges from  $E$  that have one endpoint in  $V_1$  and one endpoint in  $V_2$  is no more than  $K$ ?

*Comment:* Remains NP-complete for  $B = |V|/2$  and  $w(e) = 1$  for all  $e \in E$ ...

**[GT11] PARTITION INTO TRIANGLES**

INSTANCE: Graph  $G = (V, E)$ , with  $|V| = 3q$  for some integer  $q$ .

QUESTION: Can the vertices of  $G$  be partitioned into  $q$  disjoint sets  $V_1, V_2, \dots, V_q$ , each containing exactly 3 vertices, such that for each  $V_i = \{u_i, v_i, w_i\}$ ,  $1 \leq i \leq q$ , all the of the edges  $\{u_i, v_i\}$ ,  $\{u_i, w_i\}$ , and  $\{v_i, w_i\}$  belong to  $E$ ?

**[GT19] CLIQUE**

INSTANCE: Graph  $G = (V, E)$ , positive integer  $K \leq |V|$ .

QUESTION: Does  $G$  contain a clique of size  $K$  or more, i.e., a subset  $V' \subseteq V$  with  $|V'| \geq K$  such that every two vertices in  $V'$  are joined by an edge in  $E$ ?

*Comment:* Solvable in polynomial time for graphs obeying any fixed degree bound  $d$ ...

**[SP18] EXPECTED COMPONENT SUM**

INSTANCE: Collection  $C$  of  $m$ -dimensional vectors  $v = (v_1, v_2, \dots, v_m)$  with non-negative integer entries, positive integers  $K$  and  $B$ .

QUESTION: Is there a partition of  $C$  into disjoint sets  $C_1, C_2, \dots, C_K$  such that

$$\sum_{i=1}^K \max_{1 \leq j \leq m} \left[ \sum_{v \in C_i} v_j \right] \geq B$$

?