HetNets Selection by Clients: Convergence, Efficiency, and Practicality

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***Abstract*— We study the dynamics of network selection in heterogeneous wireless networks based on client-side control. Clients in such networks selfishly select the best radio access technology (RAT) that maximizes their own throughputs. We study two general classes of throughput models that capture the basic properties of random access (e.g., Wi-Fi) and scheduled access (e.g., WiMAX, LTE, and 3G) networks. Formulating the problem as a non-cooperative game, we study its existence of equilibria, convergence time, efficiency, and practicality. Our results reveal that: 1) single-class RAT selection games converge to Nash equilibria, while an improvement path can be repeated infinitely with a mixture of classes; 2) we provide tight bounds on the convergence time of these games; 3) we analyze the Pareto-efficiency of the Nash equilibria of these games, deriving the conditions under which Nash equilibria are Pareto-optimal, and quantifying the distance of equilibria with respect to the set of Pareto-dominant points when the conditions are not satisfied; and 4) with extensive measurement-driven simulations, we show that RAT selection games converge to Nash equilibria in a small number of steps, and are amenable to practical implementation. We also investigate the impact of noisy throughput estimates, and propose solutions to handle them.**

***Index Terms*— Heterogeneous wireless networks, radio access technology, game theory, convergence time, Nash equilibrium, Pareto optimality.**

I. Introduction

HETEROGENEITY of wireless network architectures (*e.g.*, the coexistence of 2.5G, 3G, 4G, Wi-Fi, femto, etc) is increasingly becoming an important feature of the current and next generation of wireless networks. At the same time, mobile devices are increasingly equipped with multiple radio access technologies (RATs) that can connect to and choose among the different access networks. In such heterogeneous wireless environments, an important question that arises is *how should a client select the best access network at any given time?* We consider this question with client objectives in mind here, and focus on throughput performance, rather than battery, data pricing, or mobility support.

Network selection has been extensively studied in heterogeneous networks (for a survey please refer to [1]), particularly in cases when there is assistance from the network ([2]–[4]), or when the network is able to distribute clients across RATs in order to optimize some notion of system performance ([5]–[8]). There are two possible approaches. The cloud, or network-centric, approach of centralized control of network configuration faces the problem of incompatbility of economic incentive by different network providers and the challenges of signaling latency. The fog approach [cite new paper below], or the client-centric approach, of distributed control plane closer to the users, on the other hand, has to tackle the issues of potential oscillation and inefficiency. In this paper, we consider a client-centric approach, and extend our preliminary analysis and evaluation presented in [9]. In a client-centric approach, clients make decisions to select the appropriate network, without requiring any signaling overhead or coordination among the different access networks. Clients in such networks only strive to maximize their own throughputs without regard for other clients. We can think about cars autonomously switching lanes on a rush-hour highway, which often leads to oscillations, chaos, an reduction in everyone's speed, and realize that the answers may not be straight-forward. We show both why client-centric control of HetNets has desirable features and how it can be designed for convergence and optimality.

The multi-client RAT selection problem is then essentially a non-cooperative game, named as *RAT selection games*, in which clients are the players of the game and the strategies correspond to the selection of RATs.

The main challenge in analyzing the behavior of these games is to incorporate realistic models that (i) capture the multi-rate property of heterogeneous networks (*i.e.*, each client has a distinct transmission rate for each access technology), and (ii) accurately model the impact of each client's decision on other clients' received throughputs.

On the first challenge, we divide the throughput models of different access networks into two general classes. In class-1 throughput models, clients on the same base station (BS) achieve the same throughput, however, different client combinations result in distinct throughput values. This class of throughput models is especially suitable to model *throughput-fair* access networks such as Wi-Fi [10]. In class-2 throughput models, each client receives a client-specific throughput value that depends on the number of other clients sharing the same BS. This class of throughput models is especially

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suitable to model *time/bandwidth/proportional-fair* access networks in 3/4G networks.

On the second challenge, we analyze the most important properties of the equilibria, such as existence of equilibria, convergence time, and Pareto-efficiency, in both single-class and mixed-class games. None of the prior work in game theory has studied the equilibria properties when a mixture of classes is considered. We show that the different fairness metrics coupled with the multi-rate property, cause a vastly different equilibria characteristics in class-1 and class-2 games. We further perform extensive measurement-driven simulations to investigate the performance of distributed RAT selection in practice. The key results are summarized as follows:

- *Existence:* We prove that in single-class RAT selection games, convergence to Nash equilibria is guaranteed [Theorems 1, 2]. When a mixture of classes is considered, we provide an example 2-client game in which an improvement path can be repeated infinitely without reaching an equilibrium. Thus motivated, we introduce a hysteresis mechanism to RAT selection games, and prove that by applying appropriate hysteresis policies, convergence to equilibria can still be guaranteed [Theorems 3, 4].
- *Convergence Time:* We provide tight upper bounds on the convergence time of RAT selection games. We show that in class-1 games, convergence to Nash equilibria occurs within a number of steps that is exponetial in the number of clients [Theorem 5]. However, we show that class-2 games converge to Nash equilibria within a number of steps that is polynomial in the number of clients [Theorems 6, 7]. We also analyze the impact of client dynamics and show that in class-1 games the time to re-convergence could be very high. However, we derive a linear upper bound on re-convergence time for class-2 games [Theorem 8].
- *Efficiency:* We study the optimality of Nash equilibria with respect to the set of Pareto-dominant points. We show conditions under which the Nash equilibria are also Pareto-optimal [Theorems 9, 10]. When the conditions are not met, we introduce a metric termed *average Pareto-efficiency gain* to quantify the distance between the Nash points and the set of Pareto-dominant points. We show that in class-1 games, the distance between a Nash point and Pareto-dominant points can become unbounded [Theorem 9]. However, we provide tight constant approximation bounds for class-2 games [Theorems 11, 12].
- *Practicality:* We perform hundreds of measurements across multiple access technologies (*e.g.*, HSPA, HSPA+, Wi-Fi) to obtain information on the availability and quality of access networks in an indoor environment. With extensive measurement-driven simulations, we show that RAT selection games converge to equilibria with a small number of switchings. We also show that the appropriate selection of switching threshold provides a balance between convergence time and the efficiency of equilibria. Finally, we investigate the impact of noisy throughput estimates and propose solutions to handle them.

This paper is organized as follows. We discuss the related work in Section II. We present our system model in Section III. In Sections IV, V, and VI we investigate the existence, convergence time, and Pareto-efficiency properties of equilibria in RAT selection games, respectively. We present the results of our measurement-driven simulations in Section VII. Finally, we conclude in Section VIII.

II. Related Work

There exist a large number of studies in academia and industry on network selection in HetNets. We highlight the main differences in the models and analysis between this paper and the most relevant samples.

WLAN-3GPP Radio Interworking Standardization Efforts: RAT selection is currently an actively debated topic in industry and standardization efforts related to WLAN-3GPP (Third Generation Partnership Project) interworking [11]–[14]. The following solution candidates for the WLAN-UTRAN/E-UTRAN (UTRAN/E-UTRAN[1] is referred to as "RAN" in the remainder of this paper) access network selection have been identified [11]: (i) In the first solution, RAN provides assistance information to the client through broadcast signaling. The client then uses RAN assistance information, client measurements, and information provided by the WLAN to steer its traffic towards an access point in WLAN or RAN; (ii) In the second solution, the offloading rules are specified in RAN specifications. The RAN provides (through dedicated and/or broadcast signaling) thresholds which are then used in the rules. The client then follows RAN rules to steer its traffic towards WLAN or 3GPP; (iii) In the final solution, the traffic steering for the client is fully controlled by the network using dedicated traffic steering commands, potentially based also on WLAN measurements (reported by the client). In this paper, we focus on a client centric approach (solution (i)) and analyze some of the most important properties of equilibria in this approach.

Theory of Congestion Games: Congestion games [15], [16] model the negative congestion effects when users compete for limited resources. For formal definitions and most important results, please refer to [17]. These games have been extensively leveraged in networking problems such as wireline routing [18], wireless spectrum sharing [19]–[21], wireless access point selection [22], [23], etc. The idea here is that each user i pays a user-specific cost $c_r^i(x)$ when it uses resource r, which depends on the congestion level (x) and the specific preference of the user for r. The congestion impact of a user on a resource r is denoted by a weight. The congestion level (x) of resource r, is then the sum of the weights of the users that select r. Each user in these games aims to minimize its own cost. Over the last few decades several papers have studied the convergence properties of different classes of these games. Majority of these proofs is based on giving *potential functions* [24] (functions in which the gain (loss) observed by any user's unilateral move, is the same as the gain (loss) in the potential

[1]Collective terms for the (e)Node B's and radio network controllers which make up the (evolved) UMTS radio access network.

function). The convergence properties of a subclass of these games with separable preferences and player-independent costs was studied in [25]. Our proof in Theorem 1 is an application of [25] to the class-1 RAT selection games. The convergence properties of congestion games with separable preferences and player-independent weights was studied in [26]. Our class-2 throughput models have similarities to the games studied in [26]. However, unlike [26] (and the majority of convergence proofs in related work such as [15], [24], [25], and [27]), we present a new proof methodology [Theorem 2] that does not rely on potential functions. More importantly, a key issue we must face in RAT selection games is that different technologies have different classes of throughput models. None of the prior work in game theory has studied the equilibria properties when a mixture of classes in considered.

Fairness and Pareto-Efficiency: In noncooperative game theory, it is well known that Nash equilibria are frequently Pareto-inefficient, *i.e.*, there exist strategy profiles in which all users can simultaneously increase their payoffs. Pareto-efficiency is a desirable outcome for games. Over the last decades several fairness concepts that achieve Pareto-optima but are not directly related to Nash equilibria have been introduced (*e.g.*, [28], [29]). In contrast, Nash equilibria are achieved through fair competition among the users. Thus, among the Pareto-optima only those that are Pareto-dominant with respect to Nash points (*i.e.*, at least some users increase their payoff and none decreases its payoff) maintain the fair competition property of Nash equilibria. Our metric of average *Pareto-efficiency gain* quantifies the distance of Nash equilibria with respect to such Pareto-dominant sets. Similar concept for Pareto-efficiency has been recently introduced in [30] and [31] for load balancing. However, the techniques in load balancing do not apply to the RAT selection games. Other work introduced the concepts of price of anarchy (PoA) [32] and price of stability (PoS) [33]. PoA bounds the distance of any Nash point with respect to an optimum defined by a social welfare function (*e.g.*, sum of throughputs). PoS bounds the distance of the best Nash from the social optimum. In contrast, the Pareto-efficiency metric is more general and fits the questions about RAT selection better.

Game Theory Applications in Network Selection: Game theoretic techniques have been extensively used to model network selection decisions. For a comprehensive classification of the related game theoretic approaches please refer to the tutorial in [34]. Congestion game based network selection was considered in [35]. However, the model does not capture the multi-rate property of HetNets. As we will show later, due to this defining property in our study, convergence to Nash equilibria cannot be always guaranteed. Other game theory models to study network selection include evolutionary games [36], [37] and Bayesian games [38], among others. In evolutionary games, a group of players form a population, and players from one population may choose strategies against clients from other populations. These games [36], [37] assume a large number of clients in which each of them has a negligible impact on others. This is not the case with RAT selection games in which an individual client has a significant impact on the performance of other clients. In a Bayesian game, each client has a belief (where a belief is a probability distribution over the payoffs for a player) about the payoffs of other clients. These games [38] assume that each client has partial information about the preferences/payoffs of other clients. In contrast, clients in RAT selection games do not require any knowledge about other clients' preferences/payoffs. Finally, the problem of BS selection in IEEE 802.11 based WLANs has been addressed in [39], in which the network consists of only two Wi-Fi BSs. In contrast, our analysis of RAT selection games applies to any number of clients and BSs.

TABLE I
MAIN NOTATIONS

N	Number of clients
M	Number of BSs
n_k	Number of clients on BS k
$\mathbf{N}_k$	Set of clients on BS k
$R_{i,k}$	PHY rate of client i to BS k
R_{min}	minimum $R_{i,k}$ across all clients i and BSs k
R_{max}	maximum $R_{i,k}$ across all clients i and BSs k
$\omega_{i,k}$	Throughput of client i to BS k
σ_i	Strategy profile of client i
η	Switching threshold
p	Randomization parameter
T	Frequency of measurement prior to switching
h	Hysteresis parameter in the switching algorithm

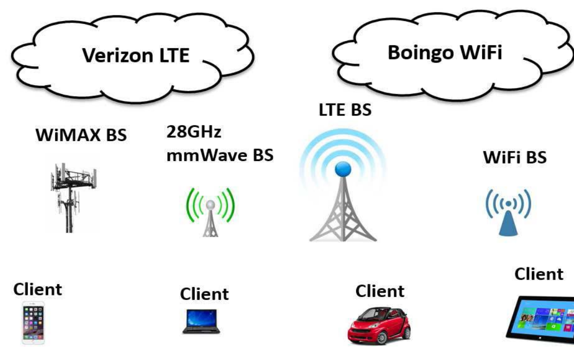

Fig. 1. An example heterogeneous network topology with 2 service providers (Verizon LTE and Boingo Wi-Fi) and 4 access technologies.

III. System Model

In this section, we present the system model and propose a distributed RAT selection algorithm with autonomous actions by each client.

A. Network Model

We consider a heterogeneous network architecture which consists of M base stations (BSs) and N clients. Here, BS is simply a generic term to collectively represent NB in 3G, eNB in 4G, AP in Wi-Fi, femtoBS in femto-cells, etc. The set of BSs and clients are denoted by $\mathbf{M} = \{1, \ldots, M\}$ and $\mathbf{N} = \{1, \ldots, N\}$, respectively. We denote the set of clients connected to BS k by $\mathbf{N}_k$ Summary of main notations used in this paper is shown in Table I. Fig. 1 shows an example of such a heterogeneous network in which BSs consists of multiple access networks (LTE, WiMAX, mmWave, and Wi-Fi).

We assume that all BSs are interference-free by means of spectrum separation between BSs that belong to different access technologies (*e.g.* LTE and Wi-Fi), and frequency reuse among same kind BSs. Each client has a specific number of RATs, and therefore has access to a subset of BSs within the range of each of its RATs. Note that due to the frequency separation between BSs, each RAT can receive beacon signals from *at most one BS*. If a client's wireless interface is able to receive beacon signals from multiple BSs, we model this functionality by assuming multiple RATs for such an interface. For example, an 802.11b wireless card that is able to receive signals from channels 1, 6, and 11 (in 2.4 GHz band), is denoted as a 3-RAT interface. Different access networks in heterogeneous networks have many different characteristics such as packet sizes, physical layer technology, modulation and coding scheme (MCS), etc. Hence, the performance of a client could be very different on different RATs.

In today's consumer wireless devices each RAT has its own radio chip (transmit and receive chain), but only one RAT is used at any given time to route the traffic. The other RATs, however, can still be active and discover neighboring BSs. This is the reason why today's smartphones can seamlessly route the traffic across different RATs (e.g., between Wi-Fi and LTE) based on availability and performance of other RATs. Motivated by today's consumer device capability, we assume that *each client uses only a single RAT at any given time*. Designing appropriate resource allocation algorithms when multiple RATs can be used simultaneously is an important research area of future work, but is out of scope of this paper.

B. Throughput Model

The throughput achieved by a client i on a BS k, denoted as $\omega_{i,k}$, depends on the client's selected access network, the client-specific parameters (*e.g.*, transmission rate) and the other clients that are connected to the same BS. The instantaneous PHY rate $R_{i,k}(t)$ of client i on BS k depends on its selected MCS and the channel conditions at time t. We assume stationary channel conditions without considering mobility.

The different access networks in heterogeneous networks have different medium access (MAC) protocols to share the bandwidth among the clients. We divide the medium access protocols into two classes:

Class-1 Throughput Models: In this class, the throughput of a client i on BS k depends on the specific clients that are connected to k. However, all clients that share the same BS achieve the same throughput, *i.e.*, with abuse of notation

$$\omega_{i,k} = f_k(R_{1,k}, R_{2,k}, \ldots, R_{n_k,k}) \quad \forall i \in \mathbf{N}_k \tag{1}$$

Here, n_k is the number of clients that are connected to BS k. An example of such MAC protocols is the distributed coordination function (DCF) implemented in 802.11, in which a Wi-Fi BS provides fair access opportunity to uplink clients [10], [40]. The throughput of the clients on the downlink depends on the queuing technique implemented on the BS. The most common technique uses a round-robin scheme. Thus, the downlink throughput of a Wi-Fi client can be expressed as

$$\omega_{i,k} = \frac{L}{\sum_{j \in \mathbf{N}_k} \frac{L}{R_{j,k}}} \quad \forall i \in \mathbf{N}_k \tag{2}$$

Here, L is the packet size. Throughput models similar to Eq. (2) are also derived for the uplink [40].

Class-2 Throughput Models: In this class, the throughput of a client i on BS k depends only on the total number of clients that share the same BS (*i.e.*, n_k), instead of the specific client combination. However, the throughput of each client can be different from other clients, *i.e.*,

$$\omega_{i,k} = R_{i,k} \times f_k(n_k) \quad \forall i \in \mathbf{N}_k \tag{3}$$

Time-fair TDMA MAC protocols are an example of class-2 throughput models. Here the wireless medium is time-shared among all the clients such that each client has the same time duration to access the medium. Therefore, the throughput of a client i connected to a time-fair BS k is given by

$$\omega_{i,k} = \frac{R_{i,k}}{n_k} \quad \forall i \in \mathbf{N}_k \tag{4}$$

OFDMA based MAC protocols with fair subcarrier sharing (*e.g.*, WiMAX) are another example of class-2 throughput models. With fair spectrum sharing, clients receive a similar number of sub-carriers. Hence, the throughput of a client i is roughly dependent only on the total number of clients sharing the same BS, and would be similar to Eq. (4).

Another example of Class-2 models is proportional-fair scheduling (PFS) in 3G networks. Here, the PFS algorithm schedules at the next slot, the client that has the highest instantaneous rate relative to its average throughput (for details refer to [41]). With PFS, the closed-form expression of the average throughput of a client with Rayleigh fading is

$$\omega_{i,k} = \frac{R_{i,k}}{n_k} \times \sum_{j=1}^{n_k} \frac{1}{j} \quad \forall i \in \mathbf{N}_k \tag{5}$$

Here, $\sum_{j=1}^{n_k} \frac{1}{j}$ appears due to channel fading [41].

C. RAT Selection Games

We model the RAT selection problem in HetNets as a non-cooperative game, in which clients select RATs in a distributed manner to increase their own individual throughputs. Thus, the player set is the set of clients, *i.e.*, $\mathbf{N}$. Player strategies are the choice of the RATs (or the corresponding BSs).

We denote player i's strategy by σ_i, and its strategy set by $\mathbf{S_i}$. The strategy profile of all clients is denoted by $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_N)$, and the set of all strategy profiles is denoted by $\mathbf{S}$. When each player $i \in \mathbf{N}$ chooses strategy σ_i resulting in strategy profile σ, then player i obtains a utility or payoff. We define the payoff of client i with strategy σ_i as its throughput and denote it by ω_{i,σ_i}.

Let σ_{-i} be a strategy profile of all players except for player i. A strategy profile σ^* is said to be at Nash equilibrium if each player considers its chosen strategy to be the best under the given choices of other players, that is

$$\forall i \in \mathbf{N}, \quad \sigma_i \in \mathbf{S_i} : \omega_i(\sigma_i^*, \sigma_{-i}^*) \geq \omega_i(\sigma_i, \sigma_{-i}^*) \tag{6}$$

Algorithm 1 RAT Selection Algorithm

Input: client i's parameters: η, T, p, h, Set of RATs
Output: Decision to switch, and the selected RAT

1 **for** *each RAT* k' **do**
2   **if** $\frac{\omega_{i,k'}[t+1]}{\omega_{i,k}[t]} > \eta,\ \forall\ t = t - T + 1, \ldots, t$ **then**
3   **if** $class(k') = class(k)$ **then**
4   **if** *rand* $< p^{m_i+1}$ **then**
5   switch to k'
6   **if** *concurrent move* **then** increment m_i
7
8   **else** reset m_i to 0
9
10   **else**
11   **if** $\omega_{i,k'} > h[class(k')]$ **then**
12   **if** *rand* $< p^{m_i+1}$ **then**
13   switch to k', $h[class(k)] \leftarrow \omega_{i,k}$
14   **if** *concurrent move* **then** increment m_i
15
16   **else** reset m_i to 0
17

Therefore, at Nash equilibrium, no client will profit from deviating its strategy unilaterally. Finally, we define a *path* as a sequence of strategy profiles in which each strategy profile differs from the preceding one in only one coordinate. If the unique deviator in each step strictly increases its throughput, the path is called an *improvement path*.

D. Distributed RAT Selection Algorithm

We propose a generic distributed RAT selection algorithm. While the algorithm is simple as we wished, its performance analysis is actually consequently more difficult. Consider synchronized slotted time for now. In RAT selection games, each client uses only one RAT at any given time for communication. However, a client is able to decode the traffic on its other RATs. For example, if RAT j of client i is tuned to BS k, then client i is able to decode the packets transmitted by k, and therefore has the information on the number of clients on k and their rates. Thus, each client can estimate its expected throughput if it decides to use another RAT for communication.

Algorithm 1 summarizes the RAT selection algorithm operated by each client. In order for client i to make a switch at time $t+1$ from BS k to BS k' (by changing its RAT), the expected gain defined as $\frac{\omega_{i,k'}[t+1]}{\omega_{i,k}[t]}$ should be higher than a given threshold (η) for the past T time slots (Line 2).

Here, T corresponds to the frequency of measurement prior to switching. Note that if multiple clients switch to a BS concurrently, their expected throughputs would be different from their achieved throughputs. In order to minimize the number of concurrent switches to the same BS, we assume that clients switch probabilistically with probability $p < 1$ (Line 4). The randomization parameter, p, depends on the congestion in the network and acts similarly to the 802.11 contention window mechanism. Similar to the binary exponential back-off in the 802.11 DCF, we assume that when concurrent migrations to a BS happen, a client sets its randomization parameter to p^{m_i+1} (Line 6), in which m_i is the number of past consecutive concurrent migrations observed by i. Following the principle of "minimization of the number of algorithmic parameters," we can think of Algorithm 1 as defined only by a single scalar parameter η of "RAT-switching aggressiveness" (with T and p fixed) and the hysteresis parameter h discussed next.

Since clients in the RAT selection games selfishly switch their selected RAT to increase their own throughput, it is possible for some of the clients to keep switching without reaching an equilibrium. A system design mechanism to dampen oscillations and guarantee convergence is to employ hysteresis. The hysteresis parameter in the RAT selection games, h, denotes the dependence of the RAT switching to the history of past switches that a client has made. Algorithm 1 shows a hysteresis policy in which a client that changes its class of BSs (*e.g.*, from class-1 to class-2) needs to have an expected throughput higher than its hysteresis value (Lines 8-9). In section IV we define our hysteresis policy in detail and demonstrate how it can guarantee convergence to equilibria in RAT selection games.

IV. Convergence to Equilibria

In this section, we analyze the convergence properties of RAT selection games. We first consider the case in which all BSs belong to the same class of throughput models. Next, we consider the case when a mixture of the two classes exists. The behaviors qualitatively differ, as we will show.

In RAT selection games, different clients can occasionally join and/or leave a *single BS* concurrently. However, due to the presence of the randomization parameter p, such strategies happen infrequently and diminish rapidly when there is network congestion (due to the exponential decrease of p with congestion). For the rest of this section, we ignore these strategies.

A. Single-Class RAT Selection Games

We first consider the case in which all BSs belong to the same class of throughput models. Note that in our model each client has a different rate for each BS. In addition, each BS has a BS-specific model to share the throughput among clients.

Theorem 1: Class-1 RAT selection games converge to a Nash equilibrium.

Proof: Our proof approach is similar in spirit to the one in [25]. Here we apply it to the RAT selection problem and present it for completeness. Denote the throughput of client i by ω_i. For simplicity, assume the following ranking of client throughputs

$$\omega_1 \leq \omega_2 \leq \ldots \leq \omega_N \tag{7}$$

Define a function g on the *ordered throughput values* as

$$g = \omega_1 \times S^{N-1} + \omega_2 \times S^{N-2} + \ldots + \omega_N \tag{8}$$

Here, S is a very large number (*i.e.*, $S \ggg \omega_i, \forall$ possible $\omega_i : i \in \mathbf{N}$). Now assume that client i migrates from BS a to BS b. Note that in class-1 throughput models, all same-BS clients achieve the same throughput. Thus, due to i's migration, the

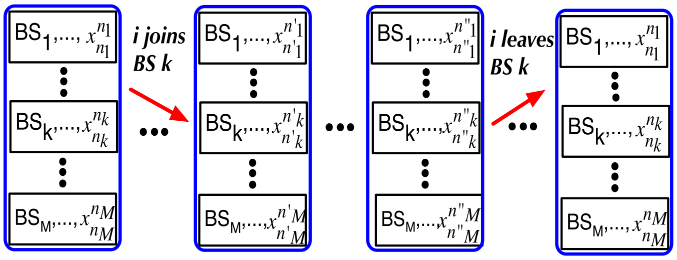

Fig. 2. System state evolution in class-2 RAT selection games. Each system state is composed of a set of tuples. Each tuple includes a BS and the set of clients which are currently on that BS. In the figure above, $x^i_{n_k}$ denotes the i'th client on BS k and n_k denotes the number of clients on BS k. In the state evolution sequence shown above, the beginning and end states are the same, *i.e.* a cycle happens.

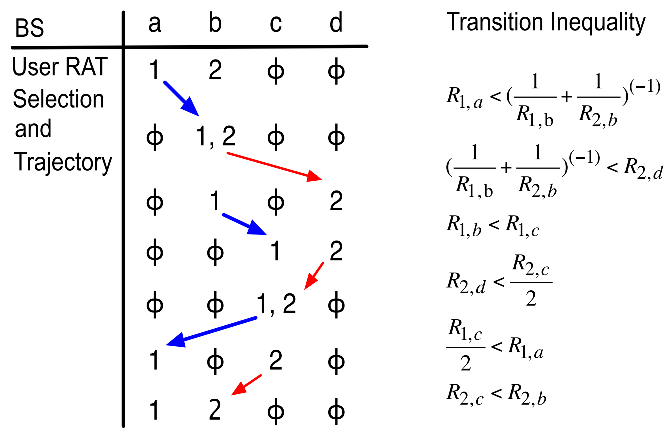

Fig. 3. An example infinite improvement path in a 2-player, 4-strategy RAT selection game with both class-1 and class-2 BSs. BSs a and c are class-2, whereas BSs b and d are class-1. The unique deviator client is shown through arrows in each step. This cyclic path is generated by the six strategy profiles shown, and it can be endlessly repeated. The inequality relevant to each step, *i.e.*, the one that guarantees the RAT switching client strictly increases its throughput is shown on the right. The six inequalities can all be validated for an infinite combination of $R_{i,j}$s. One such example is $R_{1,a} = 7.2$, $R_{1,b} = 9$, $R_{1,c} = 10.1$, $R_{2,b} = 48$, $R_{2,c} = 23.4$, $R_{2,d} = 9$. The selected rates are according to 802.11a for class-1, and 3G HSDPA for class-2.

throughput of all clients on BSs a and b would be affected. The throughput of clients on BS a would increase, since a client has left a. The throughput of clients on BS b would decrease, since a client has joined. However, the throughput of clients on b would be higher than $\omega_{i,a}$, or else client i would not have migrated. Thus, in the new ranking of client throughputs, the value of g in Eq. (8) strictly increases. As the number of clients and BSs is finite, function g cannot increase indefinitely and would terminate at a point. This means that all client migrations would stop at some point. Since no client can increase its throughput by unilateral change of its BS, the termination point is a Nash equilibrium. ■

We next focus on class-2 RAT selection games.

Theorem 2: Class-2 RAT selection games converge to a Nash equilibrium.

Proof: Our proof is based on contradiction. Define the system state of the network as the set of BSs and their connected clients.

Now assume that there is a loop in the system, *i.e.* there exists a system-state sequence with identical start and end states, as shown in Fig. 2.

Next, consider the throughput inequalities of the migrating clients between any two consecutive states. As in the definitions of the models in Section III, the throughput of a client i on BS k is equal to $R_{i,k} \times f_k(n_k)$, in which n_k is the number of clients on BS k. For the example depicted in Fig. 2, we have the following inequalities for the intermediate states

$$R_{i,k} \times f_k(n'_k) > \dots \tag{9}$$

$$\dots \tag{10}$$

$$\dots > R_{i,k} \times f_k(n''_k) \tag{11}$$

Next, we multiply all the terms on the right hand sides, and all the terms on the left hand sides of the inequalities. Note that when a client i migrates to BS k, we have an inequality similar to Eq. (9). Similarly, when eventually client i migrates from BS k (in order to have a cycle), an inequality similar to Eq. (11) exists. Therefore, when we multiply the right and left hand sides, all the $R_{i,j}$ terms will cancel each other.

On the other hand, between any two consecutive states, the number of clients on any given BS j goes up or down by 1, each time a client joins or leaves the BS j, respectively. Therefore, whenever the number of clients on BS j becomes equal to n_j by a joining client (*i.e.*, there exists an $f_j(n_j)$ term on the left hand side), an $f_j(n_j)$ term would later appear on the right hand side when a client leaves BS j. Therefore, after multiplying the right and left hand sides of the inequalities, the $f_j(n_j)$ terms will also cancel each other. After all the cancellations we have $1 > 1$, which is a contradiction. Since a cyclic system state sequence can not exist, every class-2 RAT selection games terminates at an equilibrium. At this equilibrium no client can increase its throughput by unilateral change of its BS. This is because if it could, the system state would change and it would not be an equilibrium. Therefore, the resulting equilibrium is a Nash equilibrium. ■

While Theorems 1 and 2 guarantee convergence of single-class RAT selection games to a Nash equilibrium, the resulting Nash equilibrium is not necessarily unique. In Section VII, we show that even for a small number of BSs and clients multiple Nash equilibria can exist.

B. Mixed-Class RAT Selection Games

What happens when there is a mixture of the two classes? We first provide an example 2-player 4-BS game, in which a cycle exists, and therefore convergence to an equilibrium cannot be guaranteed, unlike in the single-class cases. Next, we show how adding appropriate hysteresis policies can guarantee convergence.

Fig. 3 shows an example 2-player RAT selection game in which an improvement path can be repeated infinitely. The BSs are shown as a, b, c, and d, and the players are displayed as 1 and 2. BSs b and d are throughput-fair and belong to class-1 (Eq. (2)), while BSs a and c are time-fair and belong to class-2 (Eq. (4)). The $R_{i,j}$ value of clients on RATs/BSs is shown in Fig. 3.

Initially, clients 1 and 2 are connected to BSs a and b, respectively. During each stage of the game, one of the clients migrates to another BS in order to increase its throughput. In the example depicted in Fig. 3, the improvement path starts

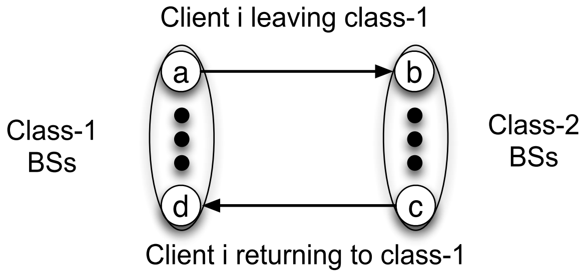

Fig. 4. Hysteresis value of client i in class-1 is its achieved throughput prior to leaving class-1, *i.e.* $\omega_{i,a}$ in the above example.

from (a:1, b:2) strategy profile in which client 1 is connected to BS a, and client 2 is connected to BS b. The path continues as (b:1, b:2), (b:1, d:2), (c:1, d:2), (c:1, c:2), (a:1, c:2), and finally back to (a:1, b:2).

The transition inequalities for the migrating client is also depicted in Fig. 3. All these inequalities hold for the selected $R_{i,j}$ values (and can further hold for an infinite number of client-rate combinations). The existence of such a cyclic improvement path demonstrates that in generic mixed-class RAT selection games, an improvement path can be repeated infinitely.

Infinite oscillation is an undesirable property of a distributed RAT selection algorithm for two primary reasons: (i) it creates a very high amount of signaling and message passing overhead in the operator's network infrastructure, and (ii) it deteriorates user experience by creating constant fluctuations in client's throughput. These emphasizes the need to design system parameters that can stop infinite oscillations by clients and guarantee convergence. We therefore introduce hysteresis, a mechanism that enforces the dependence of the system not only on its current selection, but also on its past selections.

We classify all the BSs according to their throughput class as depicted in Fig. 4. We next define the hysteresis value of a client i in a given class as its last achieved throughput in that class prior to switching to a different class of BSs. For example, if a client switches from a BS a in class-1 to a BS b in class-2, its hysteresis value in class-1 is defined as its throughput on BS a.

Definition 1 (Hysteresis Policy): Assume a client i that has moved from a class of BSs to another class of BSs. In order for i to return to a BS in the previous class, its expected throughput should be higher than the corresponding hysteresis value.

Fig. 4 shows an example client i that has moved from BS a in class-1 to BS b in class-2. It next changes its selected BS in a series of selfish moves within class-2. Now if client i wants to go to BS d in class-1 from its position on BS c, not only $\omega_{i,d}$ should be higher than $\omega_{i,c}$, but also $\omega_{i,d}$ should be higher than $\omega_{i,a}$ (*i.e.*, the hysteresis value).

Our next theorem demonstrates that this type of simple and light-weight hysteresis guarantees convergence to an equilibrium.

Theorem 3: Mixed class-1 and class-2 RAT selection games that use the hysteresis policy described in Definition 1, converge to an equilibrium.

Proof: Our proof is based on contradiction. Define the system state of the network as the set of BSs and their connected clients. Assume there is a loop in the system state evolution that can be repeated infinitely. Consider the second repetition of this loop. Note that in order to have a loop, every client that leaves its class has to return back to its class at a later time. Further, due to the loop repetition, such clients have a history of being in both classes.

For simplicity, assume that the cycle starts when a client leaves a class-1 BS. Fig. 4 shows an example of such a client that leaves class-1, and returns back to class-1 (to form a cycle). Denote the throughput of client i prior to leaving class-1 as $\omega_{i,a}$, and immediately after returning to class-1 as $\omega_{i,d}$.

Now assume that for *every* leave and return by *any* client in class-1, there exists a virtual BS. Each of these virtual BSs (*e.g.*, virtual BS v handling client i), handles only one specific client (e.g., by having zero rates for all other clients), and offers a throughput equal to the average of the client's throughputs before leaving class-1 and immediately after returning to class-1. For example, the throughput of virtual BS v for client i is equal to $\omega_{i,v} = \frac{\omega_{i,a}+\omega_{i,d}}{2}$. Note that due to the hysteresis policy, $\omega_{i,d}$ is greater than $\omega_{i,a}$, and therefore we have the following inequality

$$\omega_{i,a} < \omega_{i,v} < \omega_{i,d} \tag{12}$$

Eq. (12) shows that client i gains by leaving BS a and joining virtual BS v, and also gains later by returning to BS d.

Now, consider the clients that visit class-1 from class-2. Such clients would later return to class-2 due to the existence of the loop. Let j denote an example of such a client that joins BS e on class-1, from a BS in class-2. After a series of migrations in class-1, j returns from a BS f in class-1 to a BS in class-2. Note that since we are considering the second repetition of the loop, j has a prior history of visiting such a class-1 BS (*i.e.* BS f). Thus, we can assume that client j visits class-1 from a virtual BS v'. The throughput of virtual BS v' for client j is equal to $\omega_{j,v'} = \frac{\omega_{j,e}+\omega_{j,f}}{2}$. Note that due to the hysteresis policy, $\omega_{j,e}$ is greater than $\omega_{j,f}$, and hence we have

$$\omega_{j,f} < \omega_{j,v'} < \omega_{j,e} \tag{13}$$

Client j increases its throughput by migrating to BS e in class-1 from BS v', and also gains later by returning to BS v' from BS f in class-1. Thus, for any client that visits class-1 from class-2, we can similarly construct the corresponding virtual BSs Construction of Virtual BSs from client movements is depicted in Fig. 5. Now, note that each virtual BS accommodates only one client, and therefore it can belong to either class-1 or class-2 throughput models (BSs). Thus, we can assume that all virtual BSs belong to class-1. Now by considering class-1 and all the virtual BSs, it follows that the loop is happening within class-1. However, in Section IV-A we proved that single-class RAT selection games do not have cyclic behavior, which is a contradiction. ■

In generic heterogeneous networks, there may be other throughput models that are not captured by our class-1 or class-2 throughput models. For example, consider a throughput model that incorporates both quality of service and admission control by allocating a fixed amount of bandwidth to each

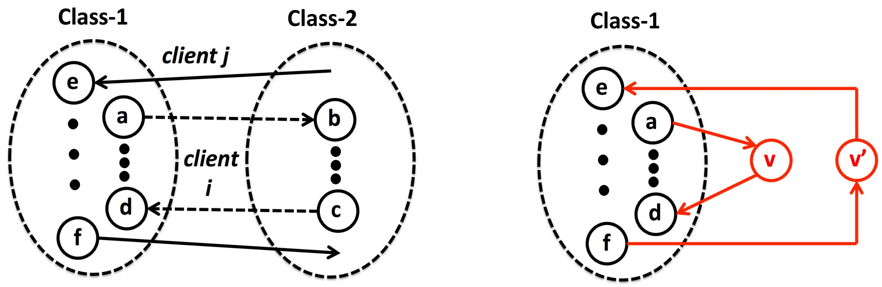

Fig. 5. Construction of virtual BSs (right) from client movements between the two classes (left). For every client i that leaves and returns to class-1, we can construct a virtual BS v that serves only i. For every client j that visits class-1 from class-2 and later returns to class-2, we can construct a virtual BS v' that serves only j. Each virtual BS serves only a single client, and therefore it can belong to either class-1 or class-2. By considering class-1 and all virtual BSs, it follows that the loop is happening within class-1, which is a contradiction. Hence, the hysteresis policy guarantees convergence in mixed class games.

client (irrespective of the other clients), and only admits a certain number of clients. Here, the throughput of a client only depends on its channel quality with respect to the BS and is not affected by the presence or absence of other clients.

It is not hard to prove that if all BSs employ the proposed throughput model, convergence to a Nash equilibrium is always guaranteed. However, as the example in Fig. 3 shows, we may not have convergence if there is a mixture of two or more different classes. Our next Theorem provides conditions under which convergence to an equilibrium is guaranteed for *any number* of classes.

Theorem 4: Let C be a set of throughput model classes and G be a RAT selection game in which all BS classes belong to C. Then, G converges to an equilibrium if the following conditions are met: (i) the hysteresis policy described in Definition 1 *is used, (ii) single class convergence is guaranteed for each class in C, and (iii) we can construct virtual BSs that accommodate only one client with configurable rate and can belong to any class in C.*

Proof: Our proof is based on induction. Define the system state of the network as the set of BSs and their connected clients, and let $|C|$ denote the number of throughput models in C. For $|C| = 2$, Theorem 3 provides a proof of convergence when we have a mixture of class-1 and class-2 models. There, the key step in our proof was to construct a virtual BS that accommodates only one client (by setting the rate for all other clients to zero) and provides a configurable rate to a specific client (by appropriately setting the rate to that client). We also argued that such a BS could belong to both class-1 and class-2 models. Hence, Theorem 3 applies to any two classes of throughput models as long as we have the conditions that are mentioned in Theorem 4.

Induction step: assume that we have convergence for $|C| = c$ classes in C. Now, assume that we add a new class (class-x) to C and that we have a loop in the system state evolution. The loop exists since there is one (or more) client migration to/from a BS in class-x to BSs in the other classes. We can now use the same proof methodology as in Theorem 3. In particular, for any client i that leaves class-x, we can construct a virtual BS that i joins and returns at a later time (to have a loop). Similarly, for any client that joins a BS in class-x, we can construct a virtual BS where the client comes from and returns to, when it eventually leaves class-x. Now since such virtual BSs could belong to any class (condition (iii) in Theorem 4), we can assume that they belong to class-x. Thus by considering class-x and all virtual BSs, it follows that the loop is happening within class-x. However, this is a contradiction since class-x satisfies the convergence property (condition (ii) in Theorem 4). ■

V. Convergence Time

In this Section, we study the convergence time properties of RAT selection games. We first provide upper bounds on the convergence time of these games. Next, we study the impact of client dynamics such as client departure or arrival on the time to re-convergence. Similar to the analysis in Section IV, we assume that the time-slots are small enough that at any given time only a single client makes a change.

A. Bounds on Convergence Time

With M BSs and N clients, the number of different configurations is at most M^N. Thus, the following corollary trivially follows from the proof of Theorems 1 and 2.

Corollary 1: Any class-1 or class-2 RAT selection games converges to a Nash equilibrium in at most M^N steps.

By considering appropriate potential functions, we can significantly improve upon this bound. In this section, we first derive an $O(c^N)$ bound on convergence time for class-1 games, in which c is a constant factor. Next, we derive an $O(N\log N)$ bound on time to convergence for both time (bandwidth)-fair and proportional-fair class-2 games. We proceed by first considering class-1 games.

Theorem 5: Let G be a class-1 RAT selection games with N clients and M BSs. Then, G converges to a Nash equilibrium in at most $\lceil 2^{N\times\Delta\times\frac{R_{max}}{R_{min}}}\rceil$ steps, in which $\Delta = max(1, \frac{1}{\eta-1})$.

Proof: Let $\Delta = max(1, \frac{1}{\eta-1})$ and set $B = 2^{\Delta\times R_{max}}$. Define $\Omega_k(t)$ for BS k at time t as the inverse of the throughput of the clients on BS k, *i.e.*,

$$\Omega_k(t) = \begin{cases} 0 & \text{if } \mathbf{N_k} = \emptyset \\ \sum_{i\in\mathbf{N}_k} \dfrac{1}{R_{i,k}} & \text{otherwise} \end{cases}$$

Define the potential function $\Phi(t) = \sum_{k\in\mathbf{M}} B^{\Omega_k(t)}$. Now assume that a client j migrates from BS k to BS k' at time

$t+1$. Therefore, the change in potential is:

$$\Phi(t+1) - \Phi(t) = B^{\Omega_k(t+1)} + B^{\Omega_{k'}(t+1)} - B^{\Omega_k(t)} - B^{\Omega_{k'}(t)} \tag{14}$$

and we have the following throughput inequalities:

$$\Omega_k(t) > \eta \times \Omega_{k'}(t+1) \geq \Omega_{k'}(t+1) + \frac{\eta - 1}{R_{max}} \tag{15}$$

$$\Omega_k(t) = \Omega_k(t+1) + \frac{1}{R_{j,k}} \geq \Omega_k(t+1) + \frac{1}{R_{max}} \tag{16}$$

From Eq. (15) and since $B \geq 2^{\frac{R_{max}}{\eta-1}}$, we have: $B^{\Omega_{k'}(t+1)} < B^{\Omega_k(t)} \times B^{\frac{1-\eta}{R_{max}}} \leq \frac{1}{2} \times B^{\Omega_k(t)}$. Similarly, from Eq. (16) and since $B \geq 2^{R_{max}}$, we have: $B^{\Omega_k(t+1)} \leq \frac{1}{2} \times B^{\Omega_k(t)}$. Leveraging these inequalities in Eq. (14) yields: $\Phi(t+1) - \Phi(t) < -B^{\Omega_{k'}(t)} \leq -1$.

Thus, the potential drops by at least 1 every step. For the starting configuration we have that $\Phi(0) \leq B^{\frac{N}{R_{min}}} = 2^{N \times \Delta \times \frac{R_{max}}{R_{min}}}$ and clearly, Φ is always positive, yielding the desired result. ■

We next consider class-2 games, and provide bounds on the convergence time for time (bandwidth)-fair and proportional fair throughput models.

Theorem 6: Let G be a class-2 time-fair RAT selection game with N clients and M BSs. Then, G converges to a Nash equilibrium in at most $\lceil \frac{N \times \log \frac{R_{max}}{R_{min}} + \log N!}{\log \eta} \rceil$ *steps.*

Proof: Let $\sigma_i(t)$ denote the BS (RAT) of client i at time t. Next, define the following potential:

$$\Phi(t) = \sum_{i \in \mathbf{N}} \log R_{i,\sigma_i(t)} - \sum_{k \in \mathbf{M}} \log n_k(t)!$$

When client i migrates from BS k to BS k' at time $t+1$, the change in potential is:

$$\begin{aligned}\Phi(t+1) - \Phi(t) &= \log R_{i,\sigma_i(t+1)} - \log n_k(t+1)! \\ &\quad - \log n_{k'}(t+1)! - \log R_{i,\sigma_i(t)} \\ &\quad + \log n_k(t)! + \log n_{k'}(t)! \\ &= \log R_{i,k'} - \log(n_k(t)-1)! \\ &\quad - \log(n_{k'}(t)+1)! - \log R_{i,k} \\ &\quad + \log n_k(t)! + \log n_{k'}(t)! \\ &= \log\left[\left(\frac{R_{i,k'}}{n_{k'}(t)+1}\right)\left(\frac{n_k(t)}{R_{i,k}}\right)\right] \\ &= \log \frac{\omega_i(t+1)}{\omega_i(t)} > \log \eta\end{aligned}$$

Since $\sum_{k \in \mathbf{M}} n_k(t) = N$ for each t, it is easily seen that:

$$0 \leq \sum_{k \in \mathbf{M}} \log n_k(t)! \leq \log N! \tag{17}$$

so that we may bound Φ from above and below by:

$$\Phi_{\min} = N \times \log R_{\min} - \log N! \tag{18}$$

$$\Phi_{\max} = N \times \log R_{\max} \tag{19}$$

Hence as desired, the time to convergence is at most $\lceil \frac{N \times \log \frac{R_{max}}{R_{min}} + \log N!}{\log \eta} \rceil$. ■

Theorem 7: Let G be a class-2 proportional-fair RAT selection game with N clients and M BSs. Then, G converges to a Nash equilibrium in at most $\lceil \frac{N \times \log \frac{R_{max}}{R_{min}} + \log N!}{\log \eta} \rceil$ *steps.*

Proof: Let $f(n_k) = \frac{n_k}{\sum_{j=1}^{n_k} \frac{1}{j}}$. Next, define the following potential function

$$\Phi(t) = \sum_{i \in \mathbf{N}} \log R_{i,\sigma_i(t)} - \sum_{k \in \mathbf{M}} \log \Pi_{l=1}^{l=k} f(n_l) \tag{20}$$

The bound is derived by applying the same steps as in the proof of Theorem 7 to the potential in Eq. (20). ■

Holding M, η, and $\frac{R_{max}}{R_{min}}$ fixed, Theorems 6 and 7 imply by way of Stirling's approximation that the time to convergence is bounded by $O(N \log N)$ as $N \to \infty$.

B. Impact of Client Departure or Arrival

We now investigate the impact of client dynamics such as client departure or arrival on the system performance. We assume that initially system has reached an equilibrium, and verify the impact of a single client departure or arrival on the time that it takes for the system to re-converge.

In class-1 games, even a single client departure or arrival can create an avalanche of movements across the entire network. Consider a class-1 game with throughput fair model of Eq. (2). Assume that a client with PHY rate R_{min} leaves (joins) BS k. Assume that other clients have PHY rate R_{min} on their current BSs, and R_{max} to BS k. This single client departure (addition) would cause many clients to move towards (away from) BS k.

If $n < \frac{R_{max}}{\eta \times R_{min}}$, then there could be a situation that up to n clients join BS k. This is because even if n clients move to BS k, each client's throughput would still be higher than its initial throughput (*i.e.* before it joined BS k) by a factor of η:

$$\frac{\eta}{\frac{1}{R_{min}}} < \frac{1}{\frac{n}{R_{max}}} \tag{21}$$

Clients that move to BS k can cause new movements on their previous BSs. As a result, the final equilibrium can be very different from the original equilibrium, and in the worst case, the number of re-convergence steps could be similar to the exponential bound derived in Theorem 5.

We next consider class-2 games. We show that when a single client departs the network, these games re-converge to an equilibrium at a very fast timescale. Same analysis can be used to derive a similar bound on re-convergence time for the case that a client joins the network.

Theorem 8: Let G be a class-2 time-fair or proportional-fair RAT selection game with N clients and M BSs. Assume that the system has converged to a Nash equilibrium. Then, if a client leaves the network the total number of steps that it takes for the game to re-converge is at most $\lceil N \times (1 + \log_\eta(c)) \rceil$. *Here, c is a constant equal to 2 and* $\frac{4}{3}$ *in time-fair and proportional-fair games, respectively.*

Proof: Consider the initial Nash equilibrium and let n_k ($k = 1, \ldots, M$) denote the number of clients on BS k. Define a function f for the two throughput models as

$$f(n) = \begin{cases} n & \text{time-fair} \\ \frac{n}{\sum_{j=1}^{n} \frac{1}{j}} & \text{proportional-fair} \end{cases}$$

Assume that a client on BS k leaves the network. As the number of clients on BS k decreases, clients on k would not be interested to change their BS. However, clients on other BSs may decide to move to BS k. Now, if a client from BS k' moves to BS k, then no further client would be interested to join BS k. Instead, clients may be interested to move to BS k', and at most one of them may join BS k'. This is because in class-2 games, each client's throughput only depends on the total number of clients that share a BS, and not the specific client combination. As this process continues, we observe that at each step one of the n_ks is decreased by 1 and the rest remain the same.

Now focus on client i, and assume that i moves l times (from BS $j_1 \to j_2 \to \ldots \to j_{l+1}$) before the system reaches a new Nash equilibrium. Therefore, we have

$$\left.\begin{array}{c} \eta \times \frac{R_{i,j_1}}{f(n_{j_1})} \leq \frac{R_{i,j_2}}{f(n_{j_2})} \\ \ldots \\ \eta \times \frac{R_{i,j_l}}{f(n_{j_l})} \leq \frac{R_{i,j_{l+1}}}{f(n_{j_{l+1}})} \end{array}\right\} \Rightarrow \eta^l \times \frac{R_{i,j_1}}{f(n_{j_1})} \overset{\text{I}}{\leq} \frac{R_{i,j_{l+1}}}{f(n_{j_{l+1}})}$$

Initially, we were at the Nash equilibrium. Hence, we have

$$\frac{R_{i,j_{l+1}}}{f(n_{j_{l+1}}+1)} \overset{\text{II}}{\leq} \eta \times \frac{R_{i.j_1}}{f(n_{j_1})} \tag{22}$$

From I and II we have

$$\eta^{l-1} \leq \frac{f(n_{j_{l+1}}+1)}{f(n_{j_{l+1}})} \Rightarrow l \leq 1 + \log_\eta(\frac{f(n_{j_{l+1}}+1)}{f(n_{j_{l+1}})})$$
$$\Rightarrow l \leq 1 + \log_\eta(\frac{f(2)}{f(1)}) = 1 + \log_\eta(f(2))$$

Thus, the total number of re-convergence steps across all clients is upper bounded by $\lceil N \times (1 + \log_\eta(f(2)))\rceil$. ■

VI. Pareto-Efficiency

Beyond convergence properties, we analyze the Pareto-efficiency of Nash equilibria in RAT selection games. We show that in some cases the Nash equilibria are necessarily Pareto-optimal. When this is not the case, we quantify the improvement of the Pareto-optimal solutions, with respect to the Nash equilibria. In order to do this, we first present the formal definitions of some of the concepts used in this section.

Definition 2: Let G be a game with a set $\mathbf{N}$ *of players. We say that strategy profile* σ' *Pareto-dominates strategy profile* σ *if it holds that*

$$\forall i \in \mathbf{N} : \omega_{i,\sigma'_i} \geq \omega_{i,\sigma_i} \tag{23}$$

Definition 3: Let G be a game with a set $\mathbf{N}$ *of players. We say that strategy profile* σ *is Pareto-optimal, if there is no other strategy profile* σ' *that Pareto-dominates* σ *and has at least a single client* $i \in \mathbf{N}$ *for which* $\omega_{i,\sigma'_i} > \omega_{i,\sigma_i}$.

Definition 4: Let G be a game with N players. Let σ' *denote a strategy profile that Pareto-dominates strategy profile* σ*. We define the average Pareto-efficiency gain of* σ' *to* σ *as*

$$\frac{\sum_{i=1}^{N} \frac{\omega_{i,\sigma'_i}}{\omega_{i,\sigma_i}}}{N} \tag{24}$$

For example, assume that strategy profile σ' has an average Pareto-efficiency gain of α with respect to σ. This means that clients observe an average of α factor increase in their throughputs by changing from strategy profile σ to σ'. We next proceed to analyze the Pareto-efficiency of RAT selection games. We first do this for class-1 throughput models.

Theorem 9: Let G be a class-1 RAT selection game with N clients. Let σ^n *denote a Nash profile and* σ^p *denote a corresponding Pareto-dominant strategy profile. Let* $\gamma = \frac{R_{\max}}{R_{\min}}$ *denote the ratio between maximum and minimum rates across all the clients. Then*

1) G has a Pareto-optimal Nash equilibrium,

2) The average Pareto-efficiency gain of σ^p *to* σ^n *can become unbounded as* $\gamma \to \infty$.

Proof: Part 1. Consider the client-BS profile, σ^1, that maximizes function g in Eq. (8). Since the value of g can not be further increased, σ^1 is a Nash equilibrium. Now assume σ^1 is not Pareto-optimal. Then, there exists a strategy profile σ^2 in which all clients achieve higher or equal throughputs with respect to σ^1, and at least one client achieves a higher throughput. Hence, the value of function g in profile σ^2 would be higher than its value in profile σ^1, which is a contradiction.

Part 2. While the best Nash is always Pareto-optimal, the distance between the worst Nash and a corresponding Pareto-dominant point can be very large. We provide an example for the throughput model of Eq. (2) to prove this. Assume 2 clients and 2 BSs a and b such that

$$\begin{cases} R_{1,a} = 1, & R_{1,b} = \gamma \\ R_{2,a} = \gamma, & R_{2,b} = 1 \end{cases}$$

The profile (1 in BS a, 2 in BS b) is a Nash point in which each client's throughput is equal to 1. On the other hand, the profile (1 in BS b, 2 in BS a) is a Pareto-optimal point in which each client's throughput is equal to γ ($\gamma > 1$). Thus, there exists a Pareto-optimal point in which each client increases its throughput by a factor of γ and has an average Pareto-efficiency gain of γ (in 802.11 a/g γ can go up to 54). ■

We next investigate the Pareto-efficiency of RAT selection games in class-2 throughput models. Specifically, we focus on the time-fair and proportional-fair throughput models of Eq. (4) and Eq. (5), respectively. We first prove that when each client has a similar rate across different RATs (note that different clients can have different rates), all Nash points are also Pareto-optimal. Next, we provide approximations on Pareto-efficiency gains when each client has a distinct rate for each RAT.

Theorem 10: Let G be a class-2 time-fair (or proportional-fair) RAT selection game with N clients. If each client has the same rate across different RATs, then any Nash equilibrium is also Pareto-optimal.

Proof: Assume the contrary. Let $s(i)$ denote the selected BS of client i in the Nash outcome and n_k denote the number of clients on BS k in the Nash outcome. Further, let q_i denote the selected BS of client i in the Pareto outcome and p_j denote the number of clients on BS j in the Pareto outcome. Assume a time-fair throughput model (similar argument holds for proportional-fair model).

From the definition of Pareto-optimality, for each client i we have that $\frac{R_i}{p_{q(i)}} \geq \frac{R_i}{n_{s(i)}}$. Therefore, at least for one client j we have $n_{s(j)} > p_{q(j)}$, and for the rest of the clients (*i.e.*, $\forall$ k $\in$ **N** and $k \neq j$) we have $n_{s(k)} \geq p_{q(k)}$. These inequalities show that each BS in the Pareto-point has a smaller (or equal) number of clients than in the Nash point (with at least one BS having a smaller number). However, the total number of clients across all BSs is equal to N, which is a contradiction. ■

Theorem 11: Let G be a time-fair RAT selection game with N clients and M BSs. Let σ^n denote a non-Pareto-optimal Nash profile and σ^p denote a Pareto-dominant profile with respect to σ^n. Then, the average Pareto-efficiency gain of σ^p to σ^n is bounded by

$$\begin{cases} 2 & \textit{if } N \leq M \\ \dfrac{N+M}{N} & \textit{if } N \geq M \end{cases}$$

Proof: We use the same notation as in the proof of Theorem 10. From Pareto-dominancy definition we have $R_{i,q(i)} \times f_{q(i)}(p_{q(i)}) \geq R_{i,s(i)} \times f_{s(i)}(n_{s(i)})$. From Nash equilibrium property we have $R_{i,s(i)} \times f_{s(i)}(n_{s(i)}) \geq R_{i,q(i)} \times f_{q(i)}(n_{q(i)}+1)$. Thus, by replacing f with the corresponding value in Eq. (4), the improvement factor of client i is

$$\begin{aligned} \frac{R_{i,q(i)} \times f_{q(i)}(p_{q(i)})}{R_{i,s(i)} \times f_{s(i)}(n_{s(i)})} &\leq \frac{R_{i,q(i)} \times f_{q(i)}(p_{q(i)})}{R_{i,q(i)} \times f_{q(i)}(n_{q(i)}+1)} \\ &\leq \frac{n_{q(i)}+1}{p_{q(i)}} \end{aligned} \tag{25}$$

Next, the sum of improvement factors of all clients is

$$\begin{aligned} \sum_{i=1}^{N} \frac{n_{q(i)}+1}{p_{q(i)}} &\leq \sum_{p_k, p_k \neq 0} \frac{n_k+1}{p_k} \times p_k \\ &\leq \begin{cases} 2 \times N & \text{if } N \leq M \\ (N+M) & \text{if } N \geq M \end{cases} \end{aligned} \tag{26}$$

The average gain is derived by dividing the above by N. ■

Note that a game G may have multiple non-Pareto-optimal Nash profiles (σ^ns). Further, each such Nash profile may have multiple corresponding Pareto-dominant profiles (σ^ps). However, the upper bound that we have derived in Theorem 11 is valid across all combinations (*i.e.*, between any σ^n and any of its corresponding σ^ps).

Theorem 11 provides a tight bound on the average Pareto-efficiency gain of time-fair RAT selection games. In order to observe this, consider a 2 player example with 2 BSs a and b with the following rates

$$R_{1,a} = 1, \quad R_{1,b} = 2-\epsilon, \quad R_{2,a} = 2-\epsilon, \quad R_{2,b} = 1 \tag{27}$$

The profile (1 in BS a, 2 in BS b) is a Nash profile, in which each client's throughput is 1. The profile (1 in BS b, 2 in BS a) is a Pareto-dominant profile, in which each client's throughput is $2-\epsilon$. The average Pareto-efficiency gain is equal to $2-\epsilon$, which can become arbitrarily close to 2 as $\epsilon \to 0$, demonstrating a reasonable worst-case bound on efficiency of client-centric HetNets control.

Theorem 12: Let G be a proportional-fair RAT selection game with N clients and M BSs. Let σ^n denote a non-Pareto-optimal Nash profile and σ^p denote a Pareto-dominant profile with respect to σ^n. Then, the average Pareto-efficiency gain of σ^p to σ^n is bounded by

$$\begin{cases} 2 \times (1+\ln(N)) & \textit{if } N \leq M \\ \dfrac{N+M}{N} \times (1+\ln(N)) & \textit{if } N \geq M \end{cases}$$

Proof: We use the same steps as in the proof of Theorem 11. By placing the proportional-fair throughput model of Eq. (5) in Eq. (25), the improvement factor of client i is

$$\leq \frac{n_{q(i)}+1}{p_{q(i)}} \times \frac{\sum_{k=1}^{p_{q(i)}} \frac{1}{k}}{\sum_{k=1}^{n_{q(i)}+1} \frac{1}{k}}. \tag{28}$$

The bound is next achieved due to the following inequality

$$\frac{\sum_{k=1}^{p_q(i)} 1/k}{\sum_{k=1}^{n_q(i)+1} 1/k} \leq \sum_{k=1}^{N} 1/k \leq (1+\ln(N)). \tag{29}$$

■

VII. PERFORMANCE EVALUATION

Thus far we have proved convergence and provided bounds on convergence time and efficiency of client-controlled RAT selection. Now we study the corresponding performance through measurement-driven simulations. We first perform hundreds of measurements to obtain SNR statistics of multiple wireless access technologies. The measurements are performed in several indoor buildings spread across a university campus. We next analyze the performance of these games in realistic environments.

Measurement Driven Simulations: We use the *field test* application in the iPhone to obtain information on the number of wireless towers, their frequency of operation and technology, and the received SNR at the receiver. Our measurements were conducted over AT&T's cellular network. We randomly select 100 locations spread across three floors of a large university building. The measured SNR value across all locations is between -68 dBm and -104 dBm. Each client in these locations has access to UMTS/HSPA, while many locations also have access to HSPA+. The average number of towers observed across the clients (locations) is 4.

In addition to cellular statistics, we also measure the received SNR of the Wi-Fi BSs, their frequency of operation and technology (802.11 a/b/g). The average number of Wi-Fi BSs observed across all the clients is 5. These SNR values are then converted to a data-rate based on the SNR-Rate table of the corresponding technology, and are fed to our simulation.

Equilibrium Analysis: Figs. 6(a) and 6(b) correspond to the number of equilibria and their Pareto-optimality, respectively. With M BSs and N clients, there exists M^N system states, defined as the set of BSs and the clients connected to them. We consider 9 clients each with 3 RATs: 2 Wi-Fi RATs and a 3G RAT. Thus, the total number of system states is $3^9 = 19683$. We randomly select these 9 clients from our database of 100 clients (locations), and repeat this selection for 20 times. For each realization, we consider all system states

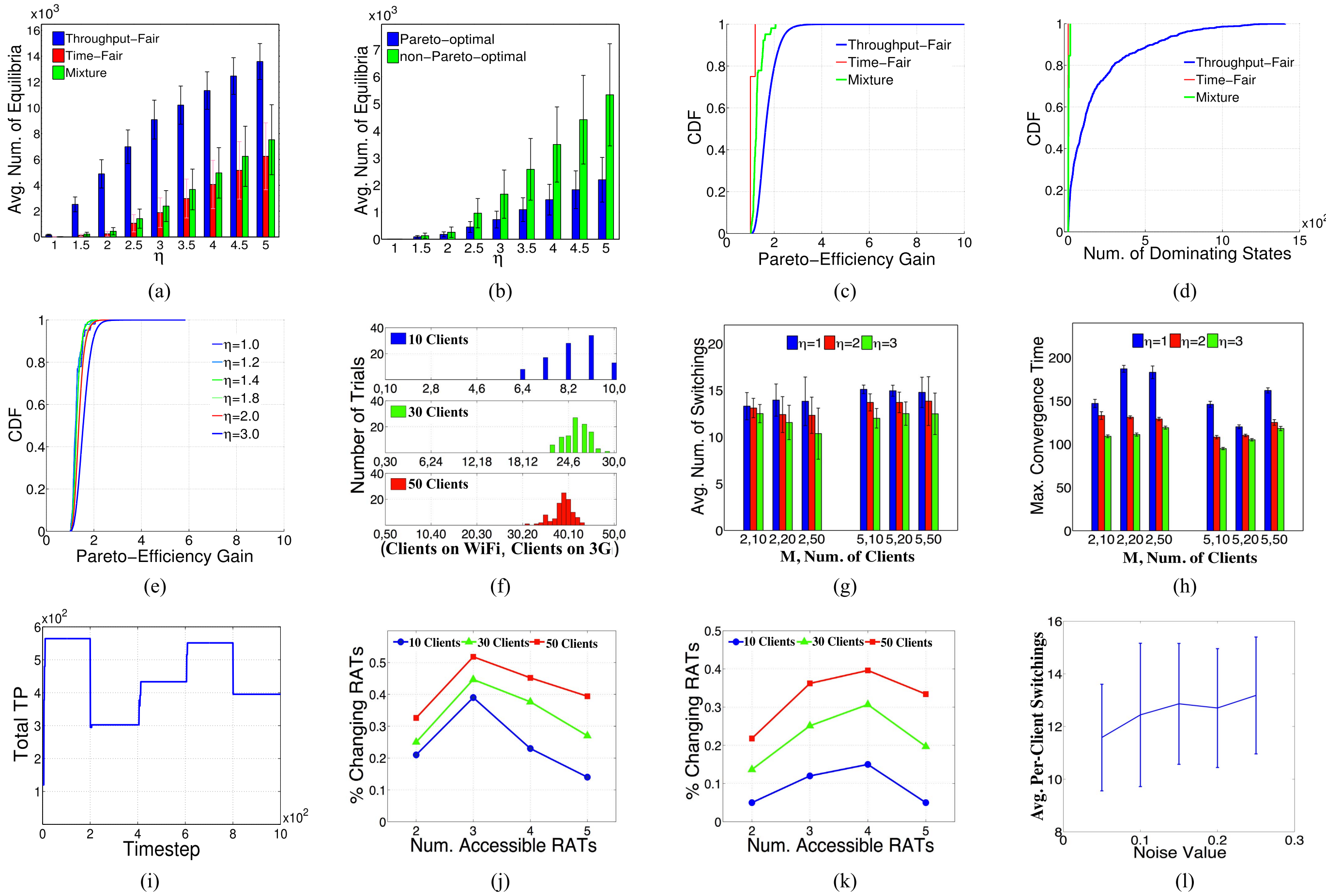

Fig. 6. (a) Average number of equilibria with 9 clients and 3 RATs, (b) Pareto-optimal/non-Pareto-optimal equilibria, (c) CDF of Pareto-efficiency gain; (d) CDF of the cardinality of Pareto-dominant sets; (e) Impact of η on Pareto-efficiency gain; (f) PDF of client distribution across WiFi and 3G for 10, 30, and 50 clients; (g) Average number of per-client switchings with varying clients/RATs/η; (h) Maximum convergence time with varying clients/RATs/η; (i) Sample evolution of aggregate throughput variation with simultaneous multi-client arrival/departure; (j) Average fraction of clients switching RATs due to simultaneous multi-client arrival; (k) Average fraction of clients switching RATs due to simultaneous multi-client departure; (l) Impact of noisy throughput estimations on the average number of per-client switchings.

and count the number of Nash equilibria, and their Pareto-optimality. Note that while the number of Nash equilibria is dependent on η in our RAT selection algorithm, the number of Pareto-optimal points is not, and averages to 6033 across all realizations.

Fig. 6(a) depicts the total number of equilibria as a function of η for 3 different throughput models. With throughput-fair, the throughput model of *all RATs* is according to the relationship in Eq. (2), while in time-fair, the throughput model of *all RATs* is according to the relationship in Eq. (4). In the mixture mode, all Wi-Fi RATs are throughput-fair, while the 3G RATs are time-fair. With $\eta = 1$, there is an average of 200, 4 and 8 Nash equilibria in the throughput-fair, time-fair, and mixture models, respectively. Thus, only a very small number of states form the equilibria in these games. As η increases, the number of equilibria increases rapidly, and the gap between time-fair and throughput-fair models decreases.

Fig. 6(b) depicts the number of Pareto-optimal and non-Pareto-optimal equilibria as a function of η in the mixture model. We observe that by varying η, the ratio between Pareto and non-Pareto equilibria remains similar, while the individual values increase. Since increasing η can significantly increase the number of equilibria, it has the potential to reduce convergence times without compromising Pareto-efficiency gains, shown later.

Average Pareto-Efficiency Gain: We next evaluate the Pareto-efficiency gains of Pareto-dominant points with respect to Nash equilibria. We consider prior configuration setup with 9 clients and 3 RATs. For each Nash equilibrium, we consider the set of Pareto-dominant points and measure the average Pareto-efficiency gain for each Pareto-dominant point, as well as the cardinality of the Pareto-dominant set. Figs. 6(c) and 6(d) depict the corresponding CDF plots across all Nash points. We observe that the average Pareto-efficiency gain in the time-fair model is close to 1, suggesting that in time-fair models Nash points are mostly close to the Pareto-dominant points. The situation in the throughput-fair model is quite the contrary, in which for a small number of Nash points (less than 1% in Fig. 6(c)) the average Pareto efficiency gain can be as high as 10 with a large number of Pareto-dominant points.

Fig. 6(e) depicts the impact of increasing η on the average Pareto-efficiency gains of the mixture model. As η increases, the number of equilibria increases rapidly. However, Fig. 6(e) shows that limiting η to less than 2 only slightly increases the average Pareto-efficiency gains.

Distribution of Clients: Next, we investigate the distribution of clients on WiFi (802.11) and cellular (3G) at equilibrium. In Fig. 6(f), we show such distribution of clients on WiFi and 3G for 10, 30, and 50 total number of clients. For a given number of clients, we randomly select the client locations from our database and instruct the clients to perform our distributed RAT selection algorithm, and therefore, converge to an equilibrium state. The number of clients on WiFi and 3G at equilibrium were then recorded, and tallied. The simulation was repeated 100 times for any given number of clients, and the PDF of client distribution was plotted in Fig. 6(f). From these simulations, we observe that the majority of clients clearly prefer to associate to WiFi networks, but as the number of clients increases past 30, more clients begin to associate with 3G networks over WiFi, due to the saturation of existing WiFi networks by a large number of clients.

Convergence Time: Figs. 6(g) and 6(h) depict the impact of system parameters (number of clients/RATs/η) on the average number of per-client RAT switchings and the maximum convergence time in the mixture model. Here we randomly select a given number of clients from our client database and execute our RAT selection algorithm. The randomization parameter (p) and the frequency of measurement prior to switching (T) are set to $\frac{1}{2}$ and 4, respectively. The simulation is repeated for 300 initialization points.

Fig. 6(g) shows that increasing the number of RATs from 2 to 5, slightly increases the average number of per-client switchings. Similarly, the number of clients has a small impact on the number of per-client switchings. Fig. 6(h) shows a similar trend on the maximum convergence time. Figs. 6(g) and 6(h) also show that by increasing η to 2, the average number of per-client switchings decreases by 1. Thus, the average number of per-client time-slots to reach convergence decreases significantly. Note that a small increase in η does not cause serious degradation in average Pareto-efficiency gains (as observed in Fig. 6(e)), and therefore one can select an appropriate η value for a given network to balance between *convergence time* and *the desirability of the equilibria*.

Impact of Client Departure and Arrival: We now investigate the impact of client dynamics such as client departure and arrival on convergence time, and changes in client-RAT associations. We consider the mixture model.

Fig. 6(i) plots the variation in aggregate throughput of 10 clients over time due to client dynamics. Here, 5 clients leave the game at time-step t=200, 5 clients enter the game at time-step t=400, 2 clients enter at time-step t=600, and finally 3 clients leave the game at time-step t=800. The set of clients that leave or enter at each time-step is selected randomly. In each case where clients depart or arrive, convergence to a new stable equilibrium state happens quickly, occurring over a very small number of time-steps.

This quick convergence occurs because when a new client leaves or joins the RAT selection game, relatively few pre-existing clients switch RATs as a response to the network dynamics. In Figs. 6(j) and 6(k), we show the average fraction of clients that switch their RATs due to simultaneous departure or arrival of 5 clients, respectively. In both cases, the actual number of clients that respond to either type of change decreases with increasing number of clients.

Impact of Noisy Measurements: Since the RAT selection algorithm relies on correct throughput prediction on RATs, sensitivity to noisy estimates can become a bottleneck. In Fig. 6(l) we plot the impact of such noise on the average number of switchings for the mixture model. We model the noise by assuming that the predicted throughput is according to a Gaussian distribution in which the mean is equal to the actual throughput and the standard deviation is equal to the product of the noise value and the actual throughput.

Fig. 6(l) shows that increasing the noise power increases the average number of switchings. Further, it is possible for some of the clients to keep on switching without reaching convergence. This problem can be addressed by adapting the η value according to the noise power. By increasing the η value, a client requires higher throughput values to make a change, compensating for noisy throughput estimates.

VIII. Conclusions

We studied the dynamics of RAT selection games by clients in heterogeneous wireless networks. We investigated the convergence properties of these games and introduced hysteresis as a system parameter that can guarantee convergence. We also provided tight bounds on the convergence time and on the average Pareto-efficiency gains of RAT selection games. Finally, through measurement-driven simulations we showed that RAT selection games converge to Nash equilibria within a small number of switchings. In future work, we will explore how limited amount of network assistance can be combined with client-centric RAT selection, possibly for operator objectives.

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