Convergence Properties of General Network Selection Games

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***Abstract*—We study the convergence properties of distributed network selection in HetNets with priority-based service. Clients in such networks have different priority weights (*e.g.*, QoS requirements, scheduling policies, etc.) for different access networks and act selfishly to maximize their own throughput. We formulate the problem as a non-cooperative game, and study its convergence for two models: *(i)* A purely client-centric model where each client uses its own preference to select a network, and *(ii)* a hybrid client-network model that uses a combination of client and network preferences to arrive at pairings. Our results reveal that: *(a)* Pure client-centric network selection with generic weights can result in infinite oscillations for *any* improvement path (*i.e.*, shows strongly cyclic behavior). However, we show that under several classes of practical priority weights (*e.g.*, weights that achieve different notions of fairness) or under additional client-side policies, convergence can be guaranteed; *(b)* We study convergence time under client-centric model and provide tight polynomial and linear bounds; *(c)* We show that applying a minimal amount of network control in the hybrid model, guarantees convergence for clients with generic weights. We also introduce a controllable knob that network controller can employ to balance between convergence time and its network-wide objective with predictable tradeoff.**

I. Introduction

Heterogeneous networks (HetNets) have emerged as one of the key features for addressing the capacity and QoS demands of future 5G networks. The architecture comprises hierarchical, multi-tier deployment of base stations (BSs)[1] with different footprints, which potentially operate over different access technologies. An important question that arises in such networks is *how should the clients be paired with BSs?*

As WiFi becomes an integral part of cellular operators' strategy to address traffic demand, cellular operators desire more control on how to pair clients with BSs. At the same time, consumer device vendors also demand more control over client-BS pairing (*e.g.*, to give the client control over its preferred network, to provide flexibility for device implementation constraints and propriety algorithms, etc.). For this reason, the client-BS pairing is an actively discussed topic in relevant standard meetings. For example, the Third Generation Partnership Project (3GPP) has been trying to specify interworking solutions between cellular technologies (*e.g.*, LTE, 3G, femto, etc.) and IEEE 802.11 (WiFi) technologies [1], [2], [3].

The following solution candidates for the WLAN-UTRAN/E-UTRAN (UTRAN/E-UTRAN[2] is referred to as "RAN" in the remainder of this paper) access network selection have been recently identified [1]: *(i)* In the first solution, RAN provides assistance information to the clients. A client then uses RAN assistance information, client measurements, and information provided by the WLAN to steer its traffic towards a BS in WLAN or RAN; *(ii)* In the second solution, the offloading rules are in RAN specification. The RAN provides (through dedicated and/or broadcast signaling) thresholds which are then used in the rules. The client then follows RAN rules to steer its traffic towards WLAN or 3GPP; *(iii)* In the third alternative, the traffic steering for the client is fully controlled by the network using dedicated traffic steering commands, potentially based also on WLAN measurements (reported by the client).

In this paper, we study the first two models identified by the standard, *i.e.*, purely client-centric solution, and hybrid client-network solution[3]. We model the network selection problem as a game termed "network selection game" with priority-based weights, and study its equilibria properties. The priority weight is a generic term introduced in this paper that can depend on the specific client, BS, or both; and is introduced to capture several practical issues in HetNets such as queuing and scheduling policy at the BS, QoS requirement of the client, packet size, etc. Next, we propose a generic wireless access throughput sharing model that captures the basic properties of different access networks (*e.g.*, WiFi, 3G, LTE, etc.) while employing different priority weights and transmission rates for clients. Finally, we analyze important properties of equilibria in these games, such as existence of equilibria and convergence time. In particular, we make the following contributions:

- We study the existence of equilibria for the client-centric solution, and provide an example with 3 clients and 3 BSs for which *any* improvement path (*i.e.*, better/best response) can oscillate infinitely [Theorem 1]. Despite this negative initial result, we prove convergence for the following scenarios: *(i)* When weights are selected in a particular manner [Theorems 2, 3], or *(ii)* when clients observe additional constraints when switching their BSs [Theorems 4, 5]. We also show that the particular weights that guarantee convergence in case *(i)* realize different fairness objectives such as max-min fairness, harmonic fairness, etc.
- We provide tight bounds on the convergence time of client-centric solution. While the best bounds from other game theoretic models that can potentially be applied to our

[1]A generic term that denotes AP in WiFi, NB in 3G, eNB in LTE, etc.

[2]Collective terms for the (e)Node Bs and radio network controllers which make up the (evolved) UMTS radio access network.

[3]Both of these are in the spirit of "fog networking:" the cloud descending to client devices for both the data-plane and control-plane of the network.

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problem have exponential computational complexity, we derive a low degree polynomial [Theorems 6, 7, 8] and linear [Theorem 9] bounds on convergence time.

- We study *if* and *how* supervision from network can guarantee convergence to desired equilibria. In particular, we show that minimal network control over clients' decisions to switch can guarantee convergence to equilibria with generic weights, and also guide the system to converge to an equilibria close to the network objective, all with a predictable convergence time. We introduce a controllable knob that can be employed by network to balance between how fast it converges and how close the equilibria are to network operator's objective. We also show how to tune the parameters for several practical network-wide objectives, including load balancing.
- We perform extensive simulations to characterize equilibria properties under different policies. Our results reveal that pure client-centric solutions converge to equilibria polynomially with respect to the number of clients, confirming tightness of our convergence time bounds. We also show that network control policies achieve throughput values that are closer to network objective, and with an average lower time complexity. The results highlight the potential of the hybrid solution for adoption by the industry.

This paper is organized as follows. We discuss the related work in Section II. We present our system model in Section III. In Sections IV and V we investigate the existence of equilibria and convergence time properties of purely client-centric network selection, respectively. We study convergence properties of hybrid client-network model in Section VI. We present the results of our simulations in Section VII. Finally, we conclude in Section VIII.

II. Related Work

Network selection is an actively researched topic in HetNets (for a survey please refer to [4]). We discuss papers which are most relevant to our work.

Theory of Congestion Game: Congestion games [5], [6] model the negative congestion effects when users compete for limited resources. For formal definitions and most important results, please refer to [7]. These games have been extensively leveraged in networking problems such as wireline routing [8], wireless spectrum sharing [9], [10], [11], wireless access point selection [12], [13], [14], etc. The priority-based network selection game studied in this paper falls into the generic category of congestion games with client-specific preferences and costs [15], which is not fully understood due to the generic structure of the problem. On the other hand, the notion of priority weights studied in this paper can capture several important characteristics of today's networks (*e.g.*, from BSs implementing different notions of fairness to heterogeneity in users' applications, QoS requirements, and packet sizes). We derive several theoretical results clearly motivated by practical needs for the priority-based network selection problem. Our results provide substantial improvements over known results in congestion games.

Game Theory of Network Selection: Network selection has been studied using Game Theory via several models including non-coopertive [12], [13], [16], evolutionary [17], [18], etc. Evolutionary games assume a very large number of clients where a single client has minimal impact on other clients. This is not the case in our problem in which a single client can have major impacts on other users' decisions. Our introduction of priority-based service generalizes the proposed non-cooperative models [12], [13], [16], whose results are derived assuming a *unity* priority weight across different BSs. By introducing the notion of priority weights, we analyze several important characteristics of today's networks with heterogeneous clients, packet sizes, application QoS requirement, etc. Further, we address the problem of network assistance and show that minimal network control can have major impact on the desirability of equilibia.

III. System Model

In this section, we present the system model and propose a generic BS throughput sharing model that captures the properties of both random and scheduled access networks.

A. Network Model

We consider a heterogeneous wireless network deployment which is composed of a set of BSs $\mathbf{K} = \{1, ..., K\}$, and a set of clients $\mathbf{N} = \{1, ..., N\}$. Each client $i \in \mathbf{N}$ is allowed to choose its preferable BS from a subset of $\mathbf{K}$. We assume that all BSs are interference-free by means of spectrum separation between BSs that belong to different access networks, and frequency reuse among BSs of the same network. Given the current consumer device capability, we assume that each client sends/receives its traffic only through a single BS at any given time. Nevertheless, clients still are capable of probing the spectrum and estimate their throughput on the other BSs (*e.g.* through beacon messages), and switch to a desirable BS with better expected throughput. We consider a priority-based network selection scheme where clients are assigned priority weights on different BSs. The weight of a client on a BS could depend on many factors such as network configuration, QoS mechanism, economical aspects, etc., and can be determined by the client, the BS, or both. For example, 802.11e implements QoS by assigning different contention windows (priority weights) to different clients.

B. Throughput Sharing Model

We define the throughput of client i on BS k ($\omega_{i,k}$) with weight $\phi_{i,k}$ as:

$$\omega_{i,k} = \frac{\phi_{i,k}}{\sum\limits_{j \in \mathbf{N}_k} \frac{\phi_{j,k}}{R_{j,k}}}, \quad \phi_{ik} \in \mathbb{R}^+ \tag{1}$$

Here, $R_{i,k}$ denotes the instantaneous PHY rate of client i on BS k and depends on multiple factors such as modulation and coding scheme, etc. Also, $\mathbf{N}_k$ denotes the set of clients on BS k. Table I denotes the list of notation in this paper.

The throughput representation in Eq. (1) captures a variety of access mechanisms. For example, consider downlink throughput of a client i on a BS k, and assume that the BS transmits L_i bits to each client i in a round-robin manner. The throughput of client i would then be similar to Eq. (1) with $\phi_{i,k} = L_i$. Similarly, uplink throughput of clients on a WiFi BS that employs QoS by assigning different contention windows ($CW_{i,k}$) to clients, can be modeled by Eq. (1) with $\phi_{i,k} = \frac{2}{CW_{i,k}}$ [19], [20].

Eq. (1) also models the throughput of synchronized access schemes (*e.g.*, 3G, LTE, etc.). In order to better understand how Eq. (1) models synchronized access (*e.g.* TDMA) throughput, set $\phi_{i,k} = \lambda_{i,k} R_{i,k}$ in Eq. (1). The throughput of a client then becomes:

$$\omega_{i,k} = \frac{\lambda_{i,k} R_{i,k}}{\sum\limits_{j \in \mathbf{N}_k} \lambda_{j,k}} \tag{2}$$

Eq. (2) denotes the throughput of clients on a TDMA BS that allocates $\lambda_{i,k}$ unit of time to each client. If we set $\lambda_{i,k}$ = 1, then we have $\omega_{i,k} = \frac{R_{i,k}}{N_k}$, *i.e.* a time-fair system which allocates equal access time to each client. With appropriate selection of weights in Eq. (1), a BS can achieve different notions of fairness in different access mechanisms. We will describe this issue in detail in Section IV.

TABLE I
MAIN NOTATION

$\mathbf{N}$: Set of all clients in the network	N: Number of clients
$\mathbf{K}$: Set of BSs	K: Number of BSs
$\mathbf{N}_k$: Set of clients on BS k	N_k: Number of clients on BS k
$\phi_{i,k}$: Weight of client i on BS k	$\omega_{i,k}$: Throughput of i on BS k
$R_{i,k}$: PHY rate of client i on BS k	σ_i: Strategy Profile of client i
η: Switching Threshold	p: randomization parameter

C. Network Selection Games

We model the BS selection problem as a non-cooperative game, in which clients select their BSs in a distributed manner to increase their throughputs. In this game, set of clients ($\mathbf{N}$) represents the players, and the strategy of each player (client) is its selected BS. Let σ_i denote the strategy selected by player i. Then, we define the strategy profile as a combination of chosen strategies of all clients denoted by vector $\sigma = (\sigma_1, \sigma_2, ..., \sigma_N)$. An *improvement path* is a sequence of strategy profiles where at each stage, only a single client changes its BS and strictly increases its throughput by this migration. A path is considered to be the *best response improvement path* if at every step, the migrating client selects a BS which gives *the maximum throughput* among all available BSs. The competition between selfish clients to choose their BSs may lead to a strategy profile where none of the clients can improve their throughput through a unilateral change of their BSs, *i.e.* a Nash equilibrium.

Clients use a distributed algorithm to select their BSs. Consider synchronized slotted time for now. Based on the algorithm, client i migrates with probability p from BS k to BS k' at time t, if $\frac{\omega_{i,k'}(t+1)}{\omega_{i,k}(t)} \geq \eta$ (*i.e.*, a client expects to increase its throughput by a factor equal or more than η). The randomization parameter p is envisioned to reduce the probability of concurrent moves to/from a single BS, which can lead to oscillations.

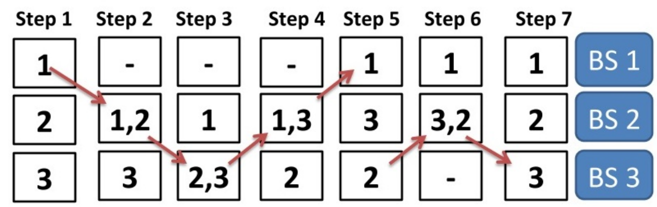

Fig. 1. An example 3 client, 3 BS network selection game, in which only a single improvement path exists and has cyclic behavior.

IV. Existence of Equilibria

Consider the generic throughput sharing model of Eq. (1). Our first result shows that with generic weights, a better/best response improvement path can be repeated infinitely without reaching an equilibrium.

Theorem 1. *There exists an instantiation of Network Selection Games for which any improvement path repeats infinitely.*

Proof: We provide an example with 3 clients and 3 BSs in Fig. 1 to prove the Theorem. Clients 1, 2, and 3, are initially connected to BSs 1, 2, and 3 respectively. Consider a sequence of moves as depicted in Fig. 1. In order to have the cycle, the following inequalities must hold at every step:

Step 1: $(R_{1,1})^{-1} \geq (\phi_{1,2}.(R_{1,2})^{-1} + \phi_{2,2}.(R_{2,2})^{-1})/\phi_{1,2}$
Step 2: $(\phi_{1,2}.(R_{1,2})^{-1} + \phi_{2,2}.(R_{2,2})^{-1})/\phi_{2,2} \geq (\phi_{2,3}.(R_{2,3})^{-1} + \phi_{3,3}.(R_{3,3})^{-1})/\phi_{2,3}$
Step 3: $(\phi_{2,3}.(R_{2,3})^{-1} + \phi_{3,3}.(R_{3,3})^{-1})/\phi_{3,3} \geq (\phi_{1,2}.(R_{1,2})^{-1} + \phi_{3,2}.(R_{3,2})^{-1})/\phi_{3,2}$
Step 4: $(\phi_{1,2}.(R_{1,2})^{-1} + \phi_{3,2}.(R_{3,2})^{-1})/\phi_{1,2} \geq (R_{1,1})^{-1}$
Step 5 : $(R_{2,3})^{-1} \geq (\phi_{2,2}.(R_{2,2})^{-1} + \phi_{3,2}.(R_{32})^{-1})/\phi_{22}$
Step 6: $(\phi_{2,2}.(R_{2,2})^{-1} + \phi_{3,2}.(R_{3,2})^{-1})/\phi_{3,2} \geq (R_{3,3})^{-1}$

The above inequalities can be validated for an infinite number of $R_{i,k}$s and $\phi_{i,k}$s. One example of such selection is:

$$\begin{gathered} \phi_{2,2} = \phi_{3,2} = \phi_{1,2} = \phi_{3,3} = 1, \phi_{2,3} = 2 \\ ,(R_{1,1})^{-1} = 9, (R_{1,2})^{-1} = 7, (R_{2,2})^{-1} = 1, \\ (R_{3,2})^{-1} = 3, (R_{2,3})^{-1} = 5, (R_{3,3})^{-1} = 3 \end{gathered} \tag{3}$$

For the selected values, at each step only the specific client depicted in Fig. 1 can move, and only to the selected BS. Hence, the cycle exists for any improvement path. ■

This negative result shows that in generic network selection games, oscillations can happen infinitely. This motivates us to study the conditions under which convergence can be guaranteed. In particular, we study the following: *(i)* class of weights that can guarantee convergence, and *(ii)* client or network control policies that guarantee convergence.

A. Weights that Guarantee the Existence of Nash Equilibria

Prior results proved convergence properties when the utility (throughput in our context) of a client falls into one of the two categories: *(i)* if utility of each client depends on the specific client and the number of clients reside on the same resource [15], [21], and *(ii)* if throughput of clients that share the same resource depends on the specific client combination, but is the same across the clients on the same resource [15], [22].

Case *(i)* and *(ii)* can be realized in our throughout sharing model of Eq. (1), if we set $\phi_{i,k} = R_{i,k}$ (*i.e.*, $\omega_{i,k} = \frac{R_{i,k}}{N_k}$) and $\phi_{i,k} = 1$ (*i.e.*, $\omega_{i,k} = \frac{1}{\sum_{j\in\mathbf{N}_k}\frac{1}{R_{j,k}}}$), respectively.

Thus motivated, we first set $\phi_{i,k} = (R_{i,k})^\beta$, and study convergence properties for this class of weights. Different β values realize different notions of fairness on a BS. For example, $\beta = 0$ results in all clients achieving the same throughput (*i.e.*, a throughput-fair realization), while $\beta = 1$ results in each client receiving a throughput that is client specific and depend only on the number of clients on the same BS (*i.e.*, a time/bandwidth-fair realization). With $\beta = \frac{1}{2}$, we have $\omega_{i,k} = \frac{1}{\sum_{j\in\mathbf{N}_k}\frac{1}{\sqrt{R_{i,k}R_{j,k}}}}$.

This specific selection of β is particularly interesting and it achieves harmonic fairness. Harmonic-fair share of clients on a BS with throughput model of Eq. (1) is achieved if the *sum of throughput inverse of clients is minimized*. Sum of throughput inverse of the clients is

$$\sum_{i\in\mathbf{N}_k}\frac{\sum_{j\in\mathbf{N}_k}\frac{\phi_{j,k}}{R_{j,k}}}{\phi_{i,k}} = \sum_{j\in\mathbf{N}_k}\frac{\phi_{j,k}}{R_{j,k}}\sum_{i\in\mathbf{N}_k}\frac{1}{\phi_{i,k}} \geq (\sum_{j\in\mathbf{N}_k}\frac{1}{\sqrt{R_{j,k}}})^2 \quad (4)$$

The final inequality is due to Cauchy-Schwarz, and the minimization happens when $\phi_{i,k} \propto \sqrt{R_{i,k}}$, *i.e.* with $\beta = \frac{1}{2}$. Table II shows different notions of fairness that can be achieved for different β values.

TABLE II

SEVERAL NOTIONS OF FAIRNESS IN NETWORKING CAN BE ACHIEVED BY APPROPRIATE SELECTION OF β. BELOW WE PRESENT β VALUES FOR WHICH THE CORRESPONDING GAMES ARE PROVED TO CONVERGE TO A NASH EQUILIBRIUM.

Value of β	Client Throughput ($\omega_{i,k}$)	Model Name
$\beta \to -\infty$	$\begin{cases}\frac{R_{min}}{L} & \text{if } R_{j,k}=R_{min}{}^1\\ 0 & \text{for other clients.}\end{cases}$	Min. Throughput
$\beta = 0$	$\frac{1}{\sum_{j\in\mathbf{N}_k}\frac{1}{R_{j,k}}}$	Throughput-fair (Max-Min fair)
$\beta = 1/2$	$\frac{\sqrt{R_{i,k}}}{\sum_{j\in\mathbf{N}_k}\frac{1}{\sqrt{R_{j,k}}}}$	Harmonic-fair
$\beta = 1$	$\frac{R_{i,k}}{N_k}$	Time/Bandwidth-fair
$\beta \to +\infty$	$\begin{cases}\frac{R_{max}}{H} & \text{if } R_{j,k}=R_{max}{}^2\\ 0 & \text{for other clients.}\end{cases}$	Max. Throughput

[1] $(R_{min}=\min_{j\in\mathbf{N}_k} R_{j,k},\quad L=$ Num of such $j's)$

[2] $(R_{max}=\max_{j\in\mathbf{N}_k} R_{j,k},\quad H=$ Num of such $j's)$

Convergence properties for two special cases, $\beta = 0$ and 1, have been shown in prior work [15], [21], [22]. We next provide a proof of convergence for $\beta = \frac{1}{2}$, *i.e.*, harmonic fair.

Theorem 2. *Let G be a network selection game with $\phi_{i,k}=\sqrt{R_{i,k}}$ $\forall\ i \in \mathbf{N},\ k \in \mathbf{K}$. Then, G always converges to a Nash equilibrium.*

Proof: Let us define the following potential function for every strategy profile σ:

$$\mathbf{S}(\sigma) = \sum_{m\in\mathbf{K}}\Psi_m(\sigma)$$

where $\Psi_m(\sigma) = (\sum_{j\in\mathbf{N}_m}\frac{1}{\sqrt{R_{j,m}}})^2 + \sum_{j\in\mathbf{N}_m}\frac{1}{R_{j,m}}$ is defined over the strategy profile σ. Consider a selfish step $\sigma_i \to \tilde{\sigma_i}$ of player i from BS k to the BS k' which gives the following inequality:

$$\frac{1}{\sqrt{R_{i,k}}}\sum_{j\in\mathbf{N}_k}\frac{1}{\sqrt{R_{j,k}}} > \frac{1}{\sqrt{R_{i,k'}}}\sum_{j\in\mathbf{N}_{k'}}\frac{1}{\sqrt{R_{j,k'}}} \quad (5)$$

Note that, all BSs $m \in \mathbf{K}\backslash\{k,k'\}$ remain intact after i switch. Thus we have: $\sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\sigma) = \sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\tilde{\sigma})$.

Then by the definition of potential function we have:

$$\begin{aligned} S(\sigma) &= \sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\sigma) + \Psi_k(\sigma) + \Psi_{k'}(\sigma) = \\ &\sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\sigma) + (\sum_{j\in(\mathbf{N}_k\backslash i)}\frac{1}{\sqrt{R_{j,k}}})^2 + (\sum_{j\in(\mathbf{N}_k\backslash i)}\frac{1}{R_{j,k}}) + \\ &\frac{2}{\sqrt{R_{i,k}}}\sum_{j\in\mathbf{N}_k}\frac{1}{\sqrt{R_{j,k}}} + \Psi_{k'}(\sigma) > \\ &\sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\sigma) + (\sum_{j\in(\mathbf{N}_k\backslash i)}\frac{1}{\sqrt{R_{j,k}}})^2 + \sum_{j\in(\mathbf{N}_k\backslash i)}\frac{1}{R_{j,k}} + \\ &\sum_{j\in\mathbf{N}_{k'}}\frac{2}{\sqrt{R_{i,k'}.R_{j,k'}}} + \Psi_{k'}(\sigma) = \\ &\sum_{m\in\mathbf{K}\backslash\{k,k'\}}\Psi_m(\tilde{\sigma}) + \Psi_k(\tilde{\sigma}) + \Psi_{k'}(\tilde{\sigma}) = S(\tilde{\sigma}) \end{aligned}$$

The inequality holds due to (5). Therefore, the potential function monotonically decreases as clients selfishly switch to new BSs. Existence of potential function gives the proof.[6] ■

It is not hard to prove convergence when β = $-\infty$ or $\beta = +\infty$ with the methods proposed in this paper. Nevertheless, whether the existence can be extended to other values of $\beta \in \mathbb{R}$ is still an open problem. Since we have always observed convergence in our simulations for any β value, we pose it as an open problem:

Conjecture 1. *Let G be a network selection game with $\phi_{i,k} = (R_{i,k})^\beta$ $\forall i \in \mathbf{N}, \forall k \in \mathbf{K}$. Then G always converges to a Nash equilibrium for any value of β.*

When $\phi_{i,k} = (R_{i,k})^\beta$, we can extend the results further.

[6]Since the number of BSs and clients is finite, the potential function possesses finite many states. It is also a positive function with a finite maximum and minimum. Further, each client's switch monotonically increase/decrease the potential. Thus, the switchings cannot continue infinitely and have to terminate at an equilibrium, *i.e.*, a Nash equilibrium.

Theorem 3. *Let G be a network selection game with $\phi_{i,k} = (R_{i,k})^{\beta}, \forall i \in \mathbf{N}, \forall k \in \mathbf{K}$ which always converges for the specific $\beta \in \mathbb{R} \setminus \{1\}$. Then, convergence is guaranteed for any new game $\hat{G}$, in which $\hat{\phi}_{i,k} = c_i \cdot (R_{i,k})^{\beta}$ and $c_i \in \mathbb{R}^+$ is a client-dependent constant.*

Proof: Assume client i moves from BS k to k' in the new game. Then we have:

$$\frac{c_i.(R_{i,k})^{\beta}}{\sum\limits_{j \in \mathbf{N}_k} c_j.(R_{j,k})^{\beta-1}} < \frac{c_i.(R_{i,k'})^{\beta}}{\sum\limits_{j \in \mathbf{N}_{k'}} c_j.(R_{j,k'})^{\beta-1}}$$

Let $R'_{il} = R_{il} \cdot c_i^{\frac{1}{\beta-1}}$ for any $l \in \mathbf{K}, i \in \mathbf{N}$ where $(\beta \neq 1)$. Replacing these terms into above inequality yields:

$$\frac{(R'_{i,k})^{\beta}}{\sum\limits_{j \in \mathbf{N}_k} (R'_{j,k})^{\beta-1}} < \frac{(R'_{i,k'})^{\beta}}{\sum\limits_{j \in \mathbf{N}_{k'}} (R'_{j,k'})^{\beta-1}}$$

The new game with rates $R'_{i,k}$ is an instantiation of G and always converges due to the assumption. ∎

The client specific factor (c_i) can model a variety of practical issues, *e.g.*, client specific cost, application weight, etc. For example, the throughput of a client in a game $\hat{G}$ with β = 0 is equal to $\frac{c_i}{\sum_{j \in \mathbf{N}_k} \frac{c_j}{R_{j,k}}}$. Theorem 3 guarantees convergence for this scenario which models a game when BSs serves downlink clients in a round robin manner and client i packet size is c_i.

B. Policies that guarantee the existence of equilibria

In a practical scenario, it may not be possible to limit the BS-client specific weight to specific values. Therefore, in order to stop infinite oscillations and guarantee convergence, we study whether client or network can enforce policies to guarantee convergence. In this section, we study the convergence properties for different policies employed by clients. We will study the optimality of converged points through extensive simulations in Section VII. We defer discussion on network operator's policies to Section VI.

When a policy is employed, a client switches its BS if *(i)* the client expects to increase its throughput by at least a factor of η, and *(ii)* if the policy condition is satisfied. For ease of presentation, we define $\Lambda_k(t) = \sum_{j \in \mathbf{N}_k} \frac{\phi_{j,k}}{R_{j,k}}$ which is the weighted sum of inverse rate of all clients of BS k at time t.

Policy 1: A client $i \in \mathbf{N}$ can migrate from BS k to k' at time t if $\Lambda_{k'}(t+1) < \Lambda_k(t)$.

Convergence is due to the following theorem:

Theorem 4. *Let G be a network selection game where Policy 1 is employed by the clients. Then, G always converges to an equilibrium.*

Proof: Let us define a potential function as below:

$$S(\sigma) = \sum_{k=1}^{K} B^{\Lambda_k(t)}, \quad B >> 0 \tag{6}$$

Consider a selfish step $\sigma_i \to \tilde{\sigma_i}$ of player i from BS k to the BS k' at time t. Then, the change in potential function is:

$$S(\tilde{\sigma}) - S(\sigma) = B^{\Lambda_k(t+1)} + B^{\Lambda_{k'}(t+1)} - B^{\Lambda_k(t)} - B^{\Lambda_{k'}(t)} \tag{7}$$

Based on the second condition of the policy 1 we have: $\Lambda_{k'}(t+1) < \Lambda_k(t)$. Also, we know that $\Lambda_k(t+1) < \Lambda_k(t)$ since client i has already moved out from BS k. Therefore, for a very large value of B, the sum of power terms in first and second term in (7) is always smaller than the third term. Therefore, we have a potential function which monotonically decreases whenever clients switch to a new BS. ∎

We next propose two other policies that guarantee convergence:

Policy 2: When client i switches from BS k to k' at time t, it must expect to achieve the smallest throughput in the destination BS k', i.e., $\frac{\phi_{i,k'}}{\Lambda_{k'}(t+1)} \leq \frac{\phi_{j,k'}}{\Lambda_{k'}(t+1)}$ for all $j \in \mathbf{N}_{k'}$.

Convergence is due to the following theorem:

Theorem 5. *Let G be a network selection game where Policy 2 is employed by the clients. Then, G always converges to an equilibrium.*

Proof: We define the following potential function for the strategy profile σ at time t:

$$S(\sigma) = \sum_{j=1}^{N} B^{(\frac{\Lambda_{\sigma_j}(t)}{\phi_{j,\sigma_j}})}, \quad B >> 0 \tag{8}$$

Consider a selfish step $\sigma_i \to \tilde{\sigma_i}$ of player i from BS k to BS k' at time t. Then, the change in potential function due to the client switch is:

$$S(\tilde{\sigma}) - S(\sigma) = \sum_{j \in \mathbf{N}, \tilde{\sigma}_j = k} B^{(\frac{\Lambda_k(t+1)}{\phi_{j,k}})} + \sum_{j \in \mathbf{N}, \tilde{\sigma}_j = k'} B^{(\frac{\Lambda_{k'}(t+1)}{\phi_{j,k'}})} - \sum_{j \in \mathbf{N}, \sigma_j = k} B^{(\frac{\Lambda_k(t)}{\phi_{j,k}})} - \sum_{j \in \mathbf{N}, \sigma_j = k'} B^{(\frac{\Lambda_{k'}(t)}{\phi_{j,k'}})}$$

Based on the condition in policy 2 we have: $\frac{\Lambda_k(t)}{\phi_{i,k}} > \frac{\Lambda_{k'}(t+1)}{\phi_{i,k'}} \geq \frac{\Lambda_{k'}(t+1)}{\phi_{j,k'}}$ for all j on BS k' at time $t+1$. Since B is considered a very large number, then $B^{\frac{\Lambda_{k'}(t+1)}{\phi_{i,k'}}} > \sum_{j \in \mathbf{N}, \sigma_j = k'} B^{\frac{\Lambda_{k'}(t+1)}{\phi_{j,k'}}}$ (power of a very large number is greater than the summation of power of numbers smaller than the large number). Therefore, we have a potential function which monotonically decreases whenever clients switch to the new BSs. This completes the proof. ∎

Policy 3: When client i switches from BS k to k' at time t, its throughput at time t must be smaller than the throughput of all clients that reside at k' at time $t+1$, i.e., $\frac{\phi_{i,k}}{\Lambda_k(t)} < \frac{\phi_{j,k}}{\Lambda_{k'}(t+1)}$ for all $j \in \mathbf{N}_{k'}$.

Proof of convergence is based on the potential function in Eq. (8) and uses the same methodology as in the proof of Theorem 5.

V. Convergence Time

Beyond the existence of equilibria, we investigate the convergence time properties of network selection games. Similar to the analysis in Section IV, we assume that at any given time only a single client makes a change. With K BSs and N clients, the number of different configurations is at most K^N. Thus, if G is a network selection game with equilibria, it converges to an equilibrium in at most K^N steps.

By considering potential functions that are similar in spirit to [22], it is possible to slightly improve upon this bound and provide a bound with $O(2^N)$ (assuming $N \gg K$) computational complexity. The bound can be derived when we have $\phi_{i,k}$ = $R_{i,k}^{\beta}$, and β = 0 or 1.

In this section, we significantly improve upon these exponential bounds. We provide polynomial-time bounds for β = 0, $\frac{1}{2}$; and a linear bound for β = 1. We start our analysis by considering the case where $\phi_{i,k} = R_{i,k}^{\beta}$ and β = 0 (*i.e.*, throughput-fair model):

Theorem 6. *Let G be a network selection game with $\phi_{i,k}$ = 1, and let R_{max} and R_{min} denote the maximum and minimum PHY rates of clients in G, respectively. Then, G converges to a Nash equilibrium in at most $(1 + \lceil \frac{NR_{max}}{R_{min} \times \min(1, \eta - 1)} \rceil)^K$ steps.*

Proof: Define $\Lambda_k(t)$ for BS k at time t as the inverse of the throughput of the clients on BS k, *i.e.*,

$$\Lambda_k(t) = \begin{cases} 0 & \text{if } \mathbf{N_k} = \emptyset \\ \sum_{i \in \mathbf{N}_k} \frac{1}{R_{i,k}} & \text{otherwise} \end{cases}$$

Now assume that a client j migrates from BS k to BS k' at time $t + 1$. Therefore,

$$\Lambda_k(t) > \eta \times \Lambda_{k'}(t+1) \geq \Lambda_{k'}(t+1) + \frac{\eta - 1}{R_{max}} \geq \Lambda_{k'}(t) + \frac{\eta}{R_{max}} \tag{9}$$

$$\Lambda_k(t) = \Lambda_k(t+1) + \frac{1}{R_{j,k}} \geq \Lambda_k(t+1) + \frac{1}{R_{max}} \tag{10}$$

From the above equations we have

$$\Lambda_k(t) = \max(\Lambda_k(t), \Lambda_{k'}(t)) \geq \max(\Lambda_k(t+1), \Lambda_{k'}(t+1)) + \Delta \tag{11}$$

in which $\Delta = \frac{\min(1,\ \eta - 1)}{R_{max}}$. The above equation allows us to define an ordering on $\Lambda_i(t)$ values that not only proves convergence, but also provides a bound on convergence time. In order to achieve this, we first discretize $\Lambda_1(t), ..., \Lambda_K(t)$ based on intervals of length Δ, *i.e.*, we define $\hat{\Lambda}_i(t) = L \times \Delta$ if $\Lambda_i(t) \in [L \times \Delta, (L+1) \times \Delta)$. Here L is an integer between 0 and $\lceil \frac{N}{R_{min} \times \Delta} \rceil$ ($\frac{N}{R_{min}}$ is the maximum possible value for $\Lambda_i(t)\ \forall i, t$).

Next, consider an ordering on $\hat{\Lambda}_i(t)$ values, *i.e.*, define $\vec{\hat{\Lambda}}(t) = (\hat{\Lambda}_{i_1}(t), ..., \hat{\Lambda}_{i_K}(t))$ if $\hat{\Lambda}_{i_1}(t) \geq \hat{\Lambda}_{i_2}(t) \geq ... \geq \hat{\Lambda}_{i_K}(t)$. Any client migration strictly decreases the lexicographic order of $\vec{\hat{\Lambda}}(t)$. On the other hand, there are at most (1 + $\lceil \frac{N}{R_{min} \times \Delta} \rceil$ distinct possible values for each $\hat{\Lambda}_{i_k}(t)$. Thus, the number of steps for the game to converge is at most (1 + $\lceil \frac{N}{R_{min} \times \Delta} \rceil)^K$. ■

We next derive a bound on convergence time for $\beta = \frac{1}{2}$ (*i.e.*, harmonic fair model).

Theorem 7. *Let G be a network selection game with $\phi_{i,k}$ = $R_{i,k}^{\beta}$, and $\beta = \frac{1}{2}$. Then, number of steps to converge to a Nash equilibrium is upper bounded by:*

$$\frac{(N^2 + N)R_{max}}{(\eta - 1)R_{min}}$$

Proof: Following the proof of Theorem 2, we have

$$\frac{1}{\sqrt{R_{i,k}}} \sum_{j \in \mathbf{N}_k} \frac{1}{\sqrt{R_{j,k}}} > \eta \frac{1}{\sqrt{R_{i,k'}}} \sum_{j \in \mathbf{N}_{k'}} \frac{1}{\sqrt{R_{j,k'}}} \geq \frac{1}{\sqrt{R_{i,k'}}} \sum_{j \in \mathbf{N}_{k'}} \frac{1}{\sqrt{R_{j,k'}}} + \frac{\eta - 1}{R_{max}} \tag{12}$$

This shows that $S(\sigma)$ decreases by at least $\frac{\eta - 1}{R_{max}}$ at each step. On the other hand, $S(\sigma)$ is bounded by $(\frac{N}{\sqrt{R_{min}}})^2 + \frac{N}{R_{min}}$. Dividing the two yields the bound. ■

Finally, we present a linear time complexity bound for $\beta =$ 1 (*i.e.*, time/bandwidth fair model).

Theorem 8. *Let G be a network selection game with $\phi_{i,k}$ = $R_{i,k}^{\beta}$, and $\beta = 1$. Then, G converges to a Nash equilibrium in at most $\lceil N \times \log_\eta \frac{R_{max}}{R_{min}} + M \times \log_\eta(N) \rceil$ steps, where M = $\min(K, N)$.*

Proof: Define the system state of the network as the set of BSs and their connected clients. Next, consider a sample evolution of the system state until an equilibrium is reached. Denote i_t as the client that makes a switch at time t, k_t as the BS that the client resides prior to switching, and k'_t as the BS that the client joins after switching. Let T denote the total number of switchings until equilibrium is reached. We have the following T throughput inequalities corresponding to T switchings

$$\eta \times \frac{R_{i_t,k_t}}{N_{k_t}} < \frac{R_{i_t,k'_t}}{N_{k'_t}} \quad \forall t = 1, \ldots, T \tag{13}$$

Now, if we multiple all the terms on the right hand sides and all the terms on the left hand sides, and cancel out all the common terms we have

$$\eta^T \times \frac{r_1 r_2 \ldots r_a}{N_1 N_2 \ldots N_b} < \frac{r'_1 r'_2 \ldots r'_a}{N'_1 N'_2 \ldots N'_b} \tag{14}$$

in which r_i and r'_i denote the rate of a client at its first and last BS, while N_j and N'_j denote the number of clients on BSs affected by switchings in the beginning and at the end. Thus, we have the following inequalities

$$R_{min} \leq r_i, r'_i \leq R_{max}, \quad a \leq \min(N, T) \tag{15}$$

$$1 \leq N_j, N'_j \leq N, \quad b \leq \min(K, T) \tag{16}$$

Leveraging the above inequalities in Eq. (14), we have

$$\eta^T < \frac{r'_1 ... r'_a}{r_1 ... r_a} \times \frac{N_1 ... N_b}{N'_1 ... N'_b} \leq (\frac{R_{max}}{R_{min}})^N (N_1 ... N_b) \tag{17}$$

By considering the changes on the number of nodes in each BS, we can show that $N_1 \ldots N_b \leq N(N-1) \ldots (N-M+1)$. The theorem is next proved by taking $\log_\eta(.)$ from both sides of the above inequality. ■

Bounds on convergence time when policies are employed. We now provide bounds on convergence time when weights are generic and client based policies are employed.

Policy 1: Assume $\zeta \cdot \Lambda_{k'}(t+1) \leq \Lambda_k(t)$ where $\zeta > 1$ is another threshold parameter. Then the bounds in Theorem 6 and Theorem 7 can be used. This can be achieved by replacing η with ζ in the corresponding bounds.

Policy 2: Assume $\zeta \cdot \frac{\phi_{i,k}}{\Lambda_k(t)} \leq \frac{\phi_{i,k'}}{\Lambda_{k'}(t+1)}$ where $\zeta > 1$ is another threshold parameter. Then we can show that the proposed potential function is decreased by at least $B^{\frac{\Lambda_{k'}(t+1)}{\phi_{i,k'}}}(B^{(\zeta-1)\frac{\Lambda_{k'}(t+1)}{\phi_{i,k'}}} - N)$ at each selfish step. Set $B = (N+1)^{R_{max}/(\zeta-1)}$. Thus, the potential function is decreased by at least $B^{1/R_{max}} = (N+1)^{1/(\zeta-1)}$ at each step. On the other hand, the potential function is bounded by $N \cdot B^{\frac{\Lambda_{max}}{\phi_{min}}}$, where $\Lambda_{max} = \max_k \sum_i \frac{\phi_{i,k}}{R_{i,k}}$. From these, we observe that the umber of steps in bounded by $N \cdot B^{N\frac{\phi_{max}}{\phi_{min}R_{min}}} / B^{1/R_{max}} = N \cdot (N+1)^{(\frac{\phi_{max}}{\phi_{min}R_{min}}-1)\frac{R_{max}}{\zeta-1}}$.

Policy 3: The bound in policy 2 is also valid for policy 3.

VI. Hybrid Control Mode

In this section, we consider a hybrid model that uses a combination of client and network preferences to guarantee convergence and arrive at a client-BS pairing. We assume a network controller (NC) that has information about the set of clients on each BS, their PHY rates, and their weight parameters. We consider the throughput sharing model of Eq. (1) with arbitrary client-BS weights ($\phi_{i,k}$), and provide several NC policies with guaranteed convergence and low complexity convergence times.

When a client decides to change its BS, the target BS asks the NC whether this migration is allowed or not. The client then switches if NC allows the move. Moreover, in some cases NC can force a client to leave a BS for a period of time when its current association significantly reduces the overall performance of the system.

In our hybrid model, the NC can use a potential function to ensure that selfish BS selection by clients converges to an equilibrium. When a client's decision to switch is inline with the NC's potential function, then it is allowed to move. Otherwise, the client has to choose another BS or remain in its current BS. Hence, clients will keep trying to switch based on their selfish strategy, until no client is interested/allowed to switch its current BS. Note that in the hybrid model, we do not need to define any restriction on weights, neither we need extra client employed policies to guarantee convergence.

Similar to the client based switching threshold η, we can define a switching threshold Δ applied by the NC. This means that the NC allows a client to switch its BS, only if its movement can vary the potential function by at least Δ. We can then easily drive an upper bound on the convergence time of different NC policies by finding an upper bound for the potential function and then dividing it by Δ. The value of Δ must be chosen carefully. A small value will increase the convergence time, while a large value can lead the game to inefficient (low throughput) equilibria.

The efficiency of the hybrid model and its convergence time properties mainly depends on the potential function. We next present some example of potential functions, and provide upper bounds on their convergence time.

1- Aggregate Throughput (AGG-TH): Aggregate network throughput can be considered as an increasing potential function. This means a client i is allowed to move from BS k to BS k' (at the same time the throughput of other clients may increase on BS k and decrease on BS k') if the potential function is increased by at least Δ.

$$\mathbf{S}(\sigma) = \sum_{i \in \mathbf{N}} f(\omega_{i,\sigma_i}(t))$$

where $f(.)$ is an increasing function. For example, if we set $f(.)$ to be identity function and $\Delta = R_{min}/N$, the convergence time will be $O(NK\frac{R_{max}}{R_{min}})$ (since $\mathbf{S}(\sigma) \leq K \cdot R_{max}$).

2- Aggregate Weighted Rates: This model take into account the workload on each BS and can be designed to achieve load balancing. The potential is:

$$\mathbf{S}(\sigma) = \sum_{k \in \mathbf{K}} f\left(\sum_{i \in \mathbf{N_k(t)}} \phi_{i,k} R_{i,k}\right)$$

where $f(.)$ is a concave increasing function. This potential function grows when the throughput of clients increases and the workload is evenly distributed among the BSs. For example if $\beta = -1$, we set $f(x) = \log(x)$ (to achieve balanced number of clients on each BS) and $\Delta = 1/N$. It is straightforward to show that $\mathbf{S}(\sigma) \leq M \log(N/M)$ where $M = \min(K, N/e)$. Thus, convergence time is $O(NM\log(N/M))$.

3- Aggregate Inverse Throughput (INV-TH): The sum of throughput inverse of the clients can be defined as a decreasing potential function aimed at achieving harmonic fairness, *i.e.*

$$\mathbf{S}(\sigma) = \sum_{i \in \mathbf{N}} f(1/\omega_{i,\sigma_i}(t))$$

where $f(.)$ is an increasing function. For example, if we set $f(.)$ to be identity function and $\Delta = 1/(R_{max}N)$, then $0 < \mathbf{S}(\sigma) \leq K/R_{min}$, and the convergence time will be $O(NK\frac{R_{max}}{R_{min}})$.

4- Aggregate Weighted Inverse Rates: This model aims at achieving harmonic fairness while balancing workload on each BS. The potential is:

$$\mathbf{S}(\sigma) = \sum_{k \in \mathbf{K}} f\left(\sum_{i \in \mathbf{N_k(t)}} \frac{\phi_{i,k}}{R_{i,k}}\right)$$

where $f(.)$ is an increasing convex function. The potential function decreases when the throughput of clients increase and the workload is evenly distributed among the BSs. For example if $\beta = 1$, we set $f(x) = x^2$ and $\Delta = 1$. Since $0 < \mathbf{S}(\sigma) \leq N^2$ the convergence time will be $O(N^2)$.

VII. Performance Evaluation

In this Section, we present the results of extensive simulations to study the equilibria properties of network selection games.

Setup. We simulated a network composed of several wireless clients and BSs. The number of nodes ranges from 20 to 200. Nodes are randomly and uniformly distributed across an area of 100 by 100 (m^2). We placed a total number of 16 BSs on the field with equal distance from each other. The clients are assumed to be fixed and the PHY rate of each client is calculated based on the distance to its associated BS. When employing generic rates, unless otherwise specified, we select the weight of each client on a BS randomly from an integer set of $\{1, ..., 10\}$. For each setting we ran 200 trials each from a different starting point, implemented different BS selection mechanisms, and calculated the average value of the metric under consideration.

Convergence Time. Figs. 2(a)-(c) show the convergence time properties of different schemes. Fig. 2(a) shows the average convergence time to equilibria when $\phi_{i,k} = R_{i,k}^{\beta}$. The results show a small convergence time which is polynomial with number of clients across various β values, confirming that our proposed bounds are indeed very tight. Further, $\beta = 0$ and $\beta = -1$ have the maximum and minimum convergence times, respectively.

Fig. 2(c) presents the average convergence time of equilibria after imposing client based policies (Policy 1 to 3) and when Network Control (NC) with aggregate throughput potential is employed. Results show that policy 2 has the fastest convergence time. This is because of the strong restriction that this policy imposes on client switchings. Recall that under this policy, a switching client must achieve the minimum throughput on the destination BS. On the other hand, the convergence time of other policies has a similar growth rate to the NC with aggregate throughput. We observed same convergence time behavior for NC with aggregate inverse throughput potential.

Throughput Efficiency. Next, we study the throughput efficiency of different schemes by calculating the average aggregate throughput of clients at equilibria. Fig. 2(d) compares the average aggregate throughput of clients at equilibria with client and NC policies. Results show that NC with aggregate throughput potential (NC (AGG-TH)) achieves the highest average throughput at equilibria. Similarly, NC with aggregate inverse throughput potential (NC (INV-TH)) outperforms client policies as shown in Fig. 2(e). The results in Figs. 2(d) and 2(e) show that applying minimal supervision through NC, can substantially benefit network's objective.

In Fig. 2(f) we show how far the set of clients' throughput at equilibria is spread out by calculating the variance in different schemes. As the number of clients increases, the variance sharply decreases and their throughput values get closer to each other. Further, with small number of clients NC has higher throughput variance than other policies since NC encourages clients to move in order to achieve a higher aggregate throughput.

Equilibria Disparity. In order to characterize the quality of equilibria, we introduce a performance metric denoted by γ. This metric gives the ratio of aggregate throughput of the best equilibrium (in terms of aggregate throughput) over the worst equilibrium. The value of γ aims to provide an insight into the dispersion of average aggregate throughput across all equilibria. The simulation seeks all existing equilibria across the strategy space of a game by enumeration and finds the profiles with the maximum and minimum aggregate throughput to calculate the γ factor. Due to the exponential growth of strategy profiles for increasing number of clients, in this setting we only consider 9 clients and 4 BSs.

Fig. 2(g) shows the γ ratio on the right side of the plot while the average aggregate throughput is presented on the left side. The results show that increasing β, on average increases the aggregate throughput, while it decreases γ. This is because with increasing β the clients that have the highest PHY rates get more priority compared to other clients. As a result, the aggregate system throughput increases. Moreover, there would be very few equilibria with close aggregate throughput values, reducing γ. However, all these benefits come at the cost of fairness between the clients. This is because low rate clients get lower and lower priority and hence achieve a very small throughput with increasing β.

The results also show that policy employed techniques have lower γ or more balanced equilibria compared to β-based throughput models. Further, these schemes achieve a more predictable aggregate throughput. Combined with the fact that these schemes guarantee convergence for generic weights and can be designed to help network objective (*e.g.*, with NC), these policies represent an attractive solution for implementation in practical systems.

Impact of Δ on Network Controller (NC). Recall from Section VI that Δ is a threshold employed by the NC which allows a client to switch only if its movement can vary the NC potential by at least Δ. We evaluate the impact of Δ on NC performance by performing simulations on a network consisting of 100 clients and 9 BSs. The bar chart in Fig. 2(h) shows how increasing the value of Δ will affect the convergence time (left bars) and aggregate throughput (right bars). We observe that both of these two metrics decrease with an increasing value of Δ. The greater value of Δ puts more restrictions on the migration of clients and thus decreases the network convergence time. Similarly, the average aggregate throughout decreases since clients can frequently be trapped in equilibria with low aggregate throughput and cannot move due to high value of Δ. Therefore, NC can select an appropriate Δ for a given network deployment to balance between convergence time and the efficiency of equilibria (with respect to NC potential/objective).

VIII. Conclusions

We studied the dynamics of network selection games with priority-based service considering two models in our analysis: *(i)* a pure client-centric model, and *(ii)* a hybrid client-network control model. We provided conditions and mechanisms under which convergence to equilibria can be guaranteed. We also provided tight bounds on the convergence time properties of these games. Finally, we showed that applying a minimal

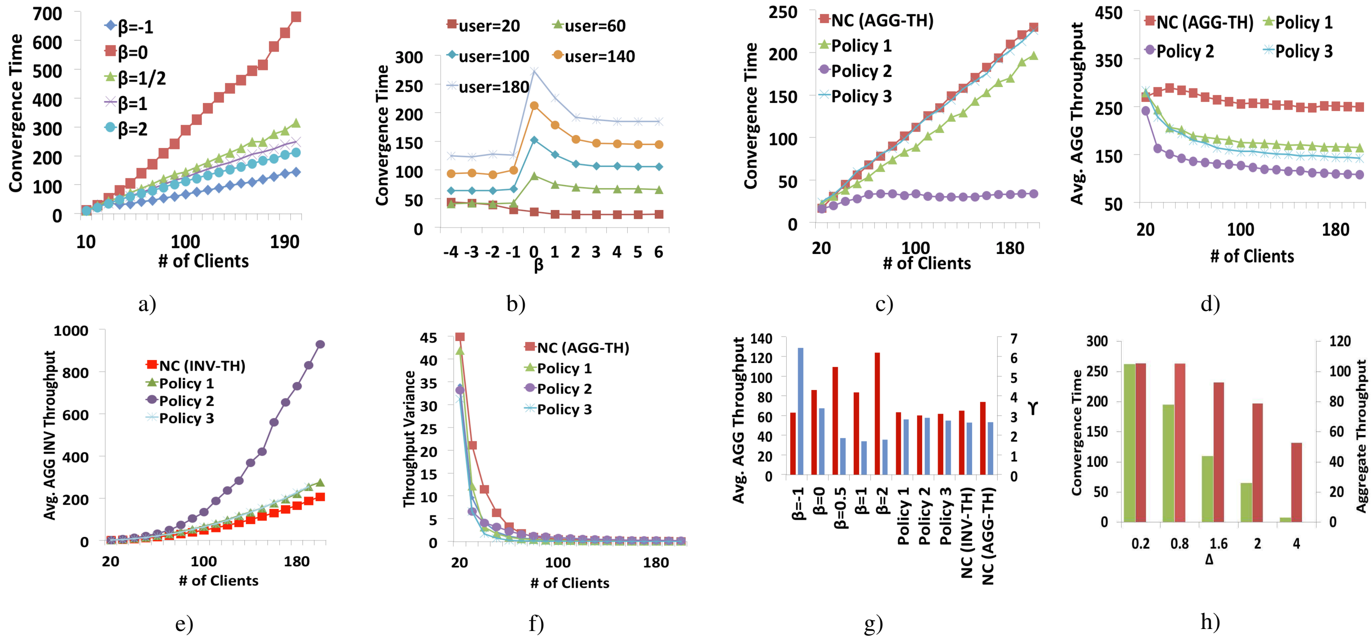

Fig. 2. Average convergence time to Nash equilibria in β-related throughput models (a),(b) and with client/NC based policies (c); Average aggregate throughput of clients at equilibria with client/NC based policies (d); Average aggregate inverse throughput with client/NC based policies (e); Aggregate throughput variance for client/NC based policies (f); Average aggregate throughput and γ across all schemes (g); Convergence time and average aggregate throughput vs. Δ for NC with aggregate throughput potential (h).

amount of network control can help clients converge to equilibria in a predictable manner.

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