CS 584/684 Spring 2017 Homework 9 – due noon, Friday, June 9 2017

Your solutions to problems 1-4 should be type-set in LaTeX and submitted in both `.tex` and `.pdf` form, with file names `hw9.tex` and `hw9.pdf`. These two files, plus any additional source files invoked from your `.tex` file (such as pictures), should be bundled together into a single `.zip` file named *your-last-name*`-tex-hw9.zip`.

Submit by emailing to `hamialex@pdx.edu` including the zip file as a separate attachment and including "`CS584 HW9`" in the subject line.

1. Show that the following problem is NP-Complete. (Hint: This does not require any fancy gadget designs.)

FEEDBACK VERTEX SET: "Given a directed graph $G = (V, E)$ and a positive integer $k \leq |V|$, is there a subset $V' \subseteq V$ with $|V'| \leq k$ such that every directed circuit in G includes at least one vertex from V'?"

2. Show that the following problem is NP-Complete. (Hint: Reduce from SUBSET SUM.)

PARTITION: "Given a finite set A and a positive integer $s(a)$ associated with each $a \in A$, is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$?"

3. Show that the following problem is NP-Complete. (Hint: Reduce from 3-SAT.)

FIREHOUSES: "Given an undirected graph G with positive integer distances on the edges, and two integers f and d, is there a way to select f vertices on G on which to locate firehouses, so that no vertex of G is at distance more than d from a firehouse?"

4. A boolean formula is in 2-conjunctive normal form (*2-CNF*) if it is in conjunctive normal form and each clause has exactly two distinct literals. Show that the following problem is in P.

2-CNF-SAT: "Given a set of n variables $x_1, x_2, \ldots, x_n$ and m clauses $l_{11} \vee l_{12}, l_{21} \vee l_{22}, \ldots, l_{m1} \vee l_{m2}$, where each literal l is either a variable or the negation of a variable, is there an assigment of boolean values to the variables that makes all the clauses true?"

(Hint: Reduce to a reachability problem on directed graphs, using the fact that $l_1 \vee l_2$ is satisfiable iff $\overline{l_1} \rightarrow l_2$ and $\overline{l_2} \rightarrow l_1$ are satisfiable.)