Improved Sample Solution for Pierce 12.1.5 T-If Case

The following is a much simpler way to prove this case than the elaborate inductive argument in the book’s sample solution.

Case T-If: \( t = \text{if } t_1 \text{ then } t_2 \text{ else } t_3 \) \( \Gamma \vdash t_1 : \text{Bool} \)
\( \Gamma \vdash t_2 : T \)
\( \Gamma \vdash t_3 : T \)

where \( \Gamma = x_1 : T_1, \ldots, x_n : T_n \)

Let \( \sigma = [x_1 \mapsto v_1] \cdots [x_n \mapsto v_n] \). By the induction hypothesis, we have \( R_{\text{Bool}}(\sigma t_1) \), i.e. \( \sigma t_1 \rightarrow^* v \) for some value \( v \). By preservation, \( v \) has type \( \text{Bool} \). Hence by Canonical Forms, \( v = \text{true} \) or \( v = \text{false} \). Suppose the former case; the latter case is completely symmetric. By repeated use of (E-If) followed by one use of (E-IfTrue), we have \( \sigma t \rightarrow^* \text{if } v \text{ then } \sigma t_2 \text{ else } \sigma t_3 \rightarrow \sigma t_2 \). By induction, \( R_T(\sigma t_2) \). Then by repeated use of Lemma 12.1.4 (\( \Leftarrow \)), \( R_T(\sigma t) \) as required.