HW2 Additional sample solution information

Question 1. Contextual Semantics for Pure $\lambda$-calculus.

Following the outline on p. 2 of the Contextual Semantics handout, we need to define a basic computational stepping relation $\rightarrow_{cmp}$, a grammar of contexts $C$, and a contextual stepping relation $\rightarrow_{ctx}$.

The computational stepping relation is just given by rule E-AbsApp from the small-step semantics:

$$(\lambda x. t) v \rightarrow_{cmp} t[v/x] \quad \text{(E-AbsApp)}$$

The grammar is $C ::= [ ] \mid C \ t \mid v \ C$.

The contextual stepping relation is exactly as for the language of the handout, namely:

$$\frac{t \rightarrow_{cmp} t'}{C[t] \rightarrow_{ctx} C[t']} \quad \text{(E-Step)}$$

Question 5. Pierce 5.3.3

Here one really should start by defining the $size()$ function. The most plausible definition (which is consistent with the reasoning in Pierce’s sample solution) is to count the number of nodes in the AST of a term:

$$
\begin{align*}
size(x) &= 1 \\
size(\lambda x. t) &= size(t) + 1 \\
size(t_1 \ t_2) &= size(t_1) + size(t_2) + 1
\end{align*}
$$