Functional Programming

- An alternative paradigm to imperative programming
- "First-class" functions
- Emphasis on pure ("functional") computations (side effects restricted or prohibited)

Languages:
- Haskell
- LISP
- Scala
- ML
- Scheme
Top-level Functions

- So far, we’ve been implicitly assuming that all functions are declared separately at program top level, e.g.

```
(((f (x) (+ x 3))
  (g (y) (@ h (* y 2))))
 (h (z) (- (@ f z) 4))
)
(+ (@ f 1) (@ g 2))
```

- All function names are globally in scope.
- Functions may be (mutually) recursive.
- Only variable in function’s initial scope is its parameter.
- Functions are identified by name in applications.
- Function names can only appear in applications.
- "Main program" expression
- Top-level functions
Almost Top-level Functions

- Some languages (e.g. C) only allow top-level functions.
- Other languages may have a top-level layer of modules or objects, with functions just inside. E.g. in Scala:

```scala
object LongLines {
  def processFile(filename: String, width: Int) {
    val source = Source.fromFile(filename)
    for (line <- source.getLines)
      processLine(filename, width, line)
  }

  private def processLine(filename: String, width: Int, line: String) {
    if (line.length > width)
      println(filename +": " + line)
  }
}
```

Source: Programming in Scala, First Edition
by Martin Odersky, Lex Spoon, and Bill Venners
Nested Functions

Many languages let us define local functions

Inner function is only visible in scope of outer one, and can access variables bound in outer one. In Scala:

```scala
object LongLines {
  def processFile(filename: String, width: Int) {
    def processLine(line: String) {
      if (line.length > width)
        print(filename +": "+ line)
    }
    val source = Source.fromFile(filename)
    for (line <- source.getLines)
      processLine(line)
  }
}
```

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First-class functions

- What happens if we treat functions as just another kind of value that we can manipulate in expressions?

- Slogan: functions are “first-class” values (just like integers or booleans or …) if they can be:
  - bound to variables
  - passed to or from other (“higher-order”) functions
  - defined by unnamed program literals
  - stored in data structures
Functions as Parameters

- Allows us to parameterize by behaviors
- Particularly useful for working over collections

```scala
def filter(p: Int => Boolean, xs: List[Int]): List[Int] = 
  xs match {
    case Nil => Nil
    case (y::ys) => if (p(y)) y::filter(p,ys) 
    else      filter(p,ys)
  }

def even(x: Int): Boolean = x%2==0
def evens(xs: List[Int]) = filter(even,xs)
val v = evens(List(1,2,3,4))  // yields List(2,4)
```
Anonymous functions

- No need to name a function that is used just once
- Typically as an actual parameter:

```scala
def filter(p: Int => Boolean, xs: List[Int]): List[Int] = 
  xs match {
    case Nil => Nil
    case (y::ys) => if (p(y)) y::filter(p, ys)
      else filter(p, ys)
  }
}

def evens(xs: List[Int]) = filter(x => x%2==0,xs)
```

- But ok anywhere:

```scala
val even = (x: Int) => x%2==0
```
Nested functions

A nested function (named or anonymous) can reference parameters of the enclosing function.

```scala
def filter(p: Int => Boolean, xs: List[Int]): List[Int] = 
  def f(xs: List[Int]): List[Int] = xs match {
    case Nil => Nil
    case (y::ys) => if (p(y)) y::f(ys) else f(ys)
  }
  f(xs)

def multiplesOf(n:Int, xs: List[Int]) = 
  filter(x => x % n == 0, xs)

def evens(xs: List[Int]) = multiplesOf(2, xs)
def multsOf3(xs: List[Int]) = multiplesOf(3, xs)
```
Functions as results

- A function can also be returned as the result of a function call. Here we use this to refactor filter:

```scala
def filter(p: Int => Boolean): List[Int] => List[Int] =
  def f(xs: List[Int]): List[Int] = xs match {
    case Nil => Nil
    case (y::ys) => if (p(y)) y::f(ys) else f(ys)
  }
  f _

def multiplesOf(n:Int): List[Int] => List[Int] =
  filter(x => x%n==0)

def evens = multiplesOf(2)
val v = evens(List(1,2,3,4)) // yields List(2,4)
```
Curried Functions

- Like filter, any multi-parameter function can be coded as a nest of single-parameter functions each returning a function.

- Such "Curried" functions can be either partially or fully applied.

- Scala has extra syntactic sugar for them, e.g.

```scala
def compose[A](f: A=>A, g:A=>A)(x:A) = f(g(x))

def multsOf6 = compose(evens, multsOf3)
val v = multsOf6(List.range(0,6)) // yields List(0,6)
val u = compose(evens, multsOf3)(List.range(0,6)) // same
```
Curried Functions

Currying is most useful when passing partially applied functions to other higher-order functions.

```scala
  def g(xs:List[A]) : List[B] = xs match {
    case Nil => Nil
    case (y::ys) => f(y)::g(ys)
  }
  g _
}

val a = map (pow(3)) (List(1,2,3)) // gives List(1,8,27)
```

```scala
def pow(n:Int)(b:Int) : Int =
  if (n==0) 1 else b * pow (n-1)(b)
```

```scala
val a = map (pow(3)) (List(1,2,3)) // gives List(1,8,27)
```
Abstracting another pattern

Consider the following problems:

Sum a list of integers:
```scala
def sum (l:List[Int]) : Int = l match {
  case Nil => 0
  case h::t => h + sum(t)
}
```

Multiply a list of integers:
```scala
def prod (l:List[Int]) : Int = l match {
  case Nil => 1
  case h::t => h * prod(t)
}
```

Calculate the length of a list (of any type):
```scala
def len[A](l:List[A]) : Int = l match {
  case Nil => 0
  case _::t => 1 + len(t)
}
```

Copy a list (of any type):
```scala
def copy[A](l:List[A]) : List[A] = l match {
  case Nil => Nil
  case h::t => h::copy(t)
}
```

Query: How does `copy` differ from the identity function `(x => x)`?
Folding over lists

We can abstract over the common inductive pattern displayed by these examples:

```scala
def foldr[A,B] (c: (A,B) => B, n:B) (l:List[A]) : B = l match {
  case Nil => n
  case h::t => c (h,foldr(c,n)(t))
}

val sum = foldr[Int,Int] ((x,y) => x+y,0) _
val prod = foldr[Int,Int] (_*_,1) _
def len[A] = foldr[A,Int] ((_,y) => 1+y,0) _
def copy[A] = foldr[A,List[A]] (_::_,Nil) _
```

Function to apply to each element and previously computed result

Value to return for empty list

Curried for convenient application

Scala short-hand for 

Compute a value of type B from a list of values of type A working from tail to head (i.e. from right to left)
Visualizing folds

- We can view \( \text{foldr}(c,n)(l) \) as replacing each :: constructor in \( l \) by \( c \) and the \text{Nil} constructor by \( n \)

\[
l = x_1 :: (x_2 :: (\ldots :: (x_n :: \text{Nil})\ldots))
\]

\[
\text{foldr}(_+_\_)(l) = x_1 + (x_2 + (\ldots + (x_n + 0)\ldots))
\]

- We can also define a \text{foldl} that accumulates a value from the left; this will sometimes be more efficient

- In some languages \text{fold} is called \text{reduce}, because we “reduce” a list of values to a single value. Similar ideas appear in “map-reduce” frameworks for organizing massively parallel computations.