Functional Programming

- An alternative paradigm to imperative programming
- “First-class” functions
- Emphasis on pure (“functional”) computations (side effects restricted or prohibited)

- Haskell
- LISP
- Scala
- ML
- Scheme
Top-level Functions

So far, we’ve been implicitly assuming that all functions are declared separately at program top level, e.g.

```
(((f (x) (+ x 3))
  (g (y) (@ h (* y 2))))
 (h (z) (- (@ f z) 4))
)
(+ (@ f 1) (@ g 2))
```

- All function names are globally in scope.
- Functions may be (mutually) recursive.
- Only variable in function’s initial scope is its parameter.
- Functions are identified by name in applications.
- Function names can only appear in applications.
- "Main program" expression.
Almost Top-level Functions

- Some languages (e.g. C) only allow top-level functions.
- Other languages may have a top-level layer of modules or objects, with functions just inside. E.g. in Scala:

```scala
object LongLines {
  def processFile(filename: String, width: Int) {
    val source = Source.fromFile(filename)
    for (line <- source.getLines)
      processLine(filename, width, line)
  }

  private def processLine(filename: String, width: Int, line: String) {
    if (line.length > width)
      println(filename +": " + line)
  }
}
```

Source: Programming in Scala, First Edition by Martin Odersky, Lex Spoon, and Bill Venners
Nested Functions

- Many languages let us define **local** functions
- Inner function is only visible in scope of outer one, and can access variables bound in outer one. In Scala:

```scala
object LongLines {
  def processFile(filename: String, width: Int) {
    def processLine(line: String) {
      if (line.length > width)
        print(filename + "": "+ line)
    }
    val source = Source.fromFile(filename)
    for (line <- source.getLines)
      processLine(line)
  }
}
```

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First-class functions

What happens if we treat functions as just another kind of value that we can manipulate in expressions?

Slogan: functions are “first-class” values (just like integers or booleans or …) if they can be:

- bound to variables
- passed to or from other (“higher-order”) functions
- defined by unnamed program literals
- stored in data structures
Functions as Parameters

- Allows us to parameterize by behaviors
- Particularly useful for working over collections

```scala
def filter(p: Int => Boolean, xs: List[Int]): List[Int] = xs match {
  case Nil => Nil
  case (y::ys) => if (p(y)) y::filter(p, ys)
                  else      filter(p, ys)
}

def even(x: Int): Boolean = x % 2 == 0
def evens(xs: List[Int]) = filter(even, xs)
val v = evens(List(1,2,3,4))  // yields List(2,4)
```
Anonymous functions

- No need to name a function that is used just once

- Typically as an actual parameter:

```scala
def filter(p: Int => Boolean, xs: List[Int]): List[Int] =
  xs match {
    case Nil => Nil
    case (y::ys) => if (p(y)) y::filter(p, ys)
                    else      filter(p, ys)
  }

def evens(xs: List[Int]) = filter(x => x%2==0, xs)
```

- But ok anywhere:

```scala
val even = (x: Int) => x%2==0
```
Nested functions

A nested function (named or anonymous) can reference parameters of the enclosing function.

def filter(p: Int => Boolean, xs: List[Int]): List[Int] =
    def f(xs: List[Int]): List[Int] = xs match {
        case Nil => Nil
        case (y::ys) => if (p(y)) y::f(ys) else f(ys)
    }
    f(xs)

def multiplesOf(n: Int, xs: List[Int]) =
    filter(x => x%n==0, xs)

def evens(xs: List[Int]) = multiplesOf(2, xs)

def multsOf3(xs: List[Int]) = multiplesOf(3, xs)
Functions as results

A function can also be returned as the result of a function call. Here we use this to refactor filter:

```scala
  case Nil => Nil
  case (y::ys) => if (p(y)) y::f(ys) else f(ys)
}
  f _
}
def multiplesOf(n:Int): List[Int] => List[Int] = filter(x => x%n==0)

def evens = multiplesOf(2)
val v = evens(List(1,2,3,4)) // yields List(2,4)
```
Curried Functions

- Like filter, any multi-parameter function can be coded as a nest of single-parameter functions each returning a function.
- Such “Curried” functions can be either partially or fully applied.
- Scala has extra syntactic sugar for them, e.g.

```scala
def compose[A](f: A=>A, g:A=>A)(x:A) = f(g(x))

def multsOf6 = compose(evens, multsOf3)
val v = multsOf6(List.range(0,6)) // yields List(0,6)
val u = compose(evens, multsOf3)(List.range(0,6)) // same
```
Currying is especially useful when passing partially applied functions to other higher-order functions.

```scala
  def g(xs: List[A]) : List[B] = xs match {
    case Nil => Nil
    case (y::ys) => f(y)::g(ys)
  }
  g _
}

def pow(n:Int)(b:Int) : Int =
  if (n==0) 1 else b * pow(n-1)(b)

val a = map (pow(3)) (List(1,2,3)) // gives List(1,8,27)
```
Abstracting another pattern

Consider the following problems:

**Sum a list of integers:**
```scala
def sum[l:List[Int]] : Int = l match {
  case Nil => 0
  case h::t => h + sum(t)
}
```

**Product of a list:**
```scala
def prod[l:List[Int]] : Int = l match {
  case Nil => 1
  case h::t => h * prod(t)
}
```

**Length of a list:**
```scala
def len[A](l:List[A]) : Int = l match {
  case Nil => 0
  case _::t => 1 + len(t)
}
```

**Copy of a list:**
```scala
def copy[A](l:List[A]) : List[A] = l match {
  case Nil => Nil
  case h::t => h::copy(t)
}
```
Folding over lists

We can abstract over the common inductive pattern displayed by these examples:

```scala
def foldr[A,B] (c: (A,B) => B, n:B) (l:List[A]) : B = l match {
  case Nil => n
  case h::t => c (h,foldr(c,n)(t))
}

val sum = foldr[Int,Int] ((x,y) => x+y,0) _
val prod = foldr[Int,Int] (_*_,1) _
def len[A] = foldr[A,Int] ((_,y) => 1+y,0) _
def copy[A] = foldr[A,List[A]] (_::_,Nil) _
```

Function to apply to each element and previously computed result

Value to return for empty list

Compute a value of type B from a list of values of type A working from tail to head (i.e. from right to left)

The `foldr` function is Curried to make it convenient to partially apply it.

Scala short-hand for `(x,y) => x*y`
Visualizing folds

We can view $\text{foldr}(c, n)(l)$ as replacing each `::` constructor in $l$ by $c$ and the Nil constructor by $n$

$$l = x_1 :: (x_2 :: (\ldots :: (x_n :: \text{Nil})\ldots))$$

$$\text{foldr}(_+_,0)(l) = x_1 + (x_2 + (\ldots + (x_n + 0)\ldots))$$

We can also define a $\text{foldl}$ that accumulates a value from the left; this will sometimes be more efficient

In some languages $\text{fold}$ is called $\text{reduce}$, because we “reduce” a list of values to a single value. Similar ideas appear in “map-reduce” frameworks for organizing massively parallel computations.