CS558 Programming Languages – Fall 2021 – Suggested Study Question Solutions for Lecture 6b

1. If \( g \) is nested within \( f \), then its scope is limited to \( f \) (including perhaps other nested functions defined in \( f \)). Then there are only a limited number of ways in which \( g \) can be called:

   - \( g \) can be called directly by its name from within the body of \( f \). In this case, \( g \) returns before \( f \) returns.
   - \( g \) can be called directly by its name from within some function \( h \) defined locally in \( f \) (where \( h \) might be \( g \) itself, if \( g \) is recursive). But by an inductive argument, we can reason that \( h \) also cannot be called after \( g \) returns, so its call to \( g \) cannot be after \( f \) returns either.
   - Since downward funargs are allowed, \( g \) can be passed by \( f \) (or some \( h \), as above) as an argument \( p \) to some other function \( r \), and \( r \) can then call \( g \) indirectly under its alias \( p \). But since \( r \) must return before \( f \), this call to \( g \) must also occur before \( f \) returns.
   - Since only downward funargs are allowed, there is no other way for \( g \) to escape from \( f \): it cannot be returned or stored in a global variable that might be accessed after \( f \) returns.

2. (a)

```scala
case class Map[A,B](f: FCF[A,B]) extends FCF[List[A],List[B]]{
  def apply (xs:List[A]) : List[B] = {
    def g(xs:List[A]) : List[B] = xs match {
      case Nil => Nil
      case (y::ys) => f.apply(y)::g(ys)
    }
    g (xs)
  }
}
def pow(n:Int, b:Int) : Int = 
  if (n == 0) 1 else b * pow (n-1,b)
case class Pow(n:Int) extends FCF[Int,Int] {
  def apply (b:Int) = pow(n,b)
}
val v = Map.apply(Pow(3)).apply(List(1,2,3))
```

(b)

```scala
case class Compose[A,B,C] (f: FCF[B,C], g: FCF[A, B]) extends FCF[A,C] {
  def apply (x:A) : C = f.apply(g.apply(x))
}
val h = Compose(MkFCF((x:Int) => x> 3),MkFCF((y:Int) => y * 2))
```
3. (a)

def fac (n:Int) : Int = {
    def factcps(n:Int,k:Int => Int) : Int =
        if (n < 2)
            k(n)
        else factcps(n-1, v => k(n*v))
    factcps(n, v => v)
}

(b)

def fib (n:Int) : Int = {
    def fibcps(n:Int,k:Int => Int) : Int =
        if (n < 2)
            k(1)
        else fibcps(n-1, v1 => fibcps(n-2, v2 => k(v1+v2)))
    fibcps(n, v => v)
}