

Sample solution for Pierce 9.2.2.

1. Let $\Gamma = f:\text{Bool} \rightarrow \text{Bool}$.

$$\frac{\frac{f:\text{Bool} \rightarrow \text{Bool} \in \Gamma}{\Gamma \vdash f:\text{Bool} \rightarrow \text{Bool}} \text{T-VAR} \quad \frac{\frac{\Gamma \vdash \text{false}:\text{Bool}}{\Gamma \vdash \text{if false then true else false}:\text{Bool}} \text{T-IF} \quad \frac{\Gamma \vdash \text{true}:\text{Bool}}{\Gamma \vdash \text{if false then true else false}:\text{Bool}} \text{T-TRUE} \quad \frac{\Gamma \vdash \text{false}:\text{Bool}}{\Gamma \vdash \text{if false then true else false}:\text{Bool}} \text{T-FALSE}}{\Gamma \vdash f(\text{if false then true else false}):\text{Bool}} \text{T-APP}$$

2. Let $\Gamma = f:\text{Bool} \rightarrow \text{Bool}$ and $\Gamma_1 = \Gamma, x:\text{Bool}$.

$$\frac{\frac{f:\text{Bool} \rightarrow \text{Bool} \in \Gamma_1}{\Gamma_1 \vdash f:\text{Bool} \rightarrow \text{Bool}} \text{T-VAR} \quad \frac{\frac{x \in \Gamma_1}{\Gamma_1 \vdash x:\text{Bool}} \text{T-VAR} \quad \frac{\Gamma_1 \vdash \text{false}:\text{Bool}}{\Gamma_1 \vdash \text{if } x \text{ then false else } x:\text{Bool}} \text{T-IF} \quad \frac{x \in \Gamma_1}{\Gamma_1 \vdash x:\text{Bool}} \text{T-VAR}}{\Gamma_1 \vdash f(\text{if } x \text{ then false else } x):\text{Bool}} \text{T-APP}}{\Gamma \vdash \lambda x:\text{Bool}.f(\text{if } x \text{ then false else } x):\text{Bool} \rightarrow \text{Bool}} \text{T-ABS}$$