

CS 457/557 Homework 5 – due 2pm, Tuesday, November 8, 2005

Hand in all your solutions on paper *only* (there is no need to email your proofs).

1. Given the usual definitions

<code>map f [] = []</code>	<i>(map1)</i>
<code>map f (x:xs) = f x : map f xs</code>	<i>(map2)</i>
<code>(f . g) x = f (g x)</code>	<i>(comp)</i>

prove that for any finite list `xs` and appropriately typed functions `f` and `g`,

$$\text{map } (f.g) \text{ xs} = (\text{map } f . \text{map } g) \text{ xs}$$

2. Given the definitions

<code>reverse [] = []</code>	<i>(rev1)</i>
<code>reverse (x:xs) = reverse xs ++ [x]</code>	<i>(rev2)</i>
<code>[] ++ ys = ys</code>	<i>(++1)</i>
<code>(x:xs) ++ ys = x:(xs ++ ys)</code>	<i>(++2)</i>

prove that for any finite list `xs`,

$$\text{reverse } (\text{reverse } xs) = xs$$

Hint: You'll need to prove (by a separate induction) an auxiliary lemma relating `reverse` and `++`. You may take as given (without the need for further proof), the two properties of `(++)` listed at the top of Hudak Table 11.2 (call them *(++assoc)* and *(++nil)* respectively).