1. Given the usual definitions

\[
\begin{align*}
    \text{map } f \left[ \right] & = \left[ \right] & \text{(map1)} \\
    \text{map } f \left( x:x:s \right) & = f \ x : \text{map } f \ xs & \text{(map2)} \\
    \left( f \ . \ g \right) \ x & = f \left( g \ x \right) & \text{(comp)}
\end{align*}
\]

prove that for any finite list \( xs \) and appropriately typed functions \( f \) and \( g \),

\[
\text{map } \left( f \ . g \right) \ xs = \text{map } f \ . \text{map } g \ xs
\]

2. Given the definitions

\[
\begin{align*}
    \text{reverse } \left[ \right] & = \left[ \right] & \text{(rev1)} \\
    \text{reverse } \left( x:x:s \right) & = \text{reverse } xs \ ++ \ \left[ x \right] & \text{(rev2)} \\
    \left[ \right] \ ++ \ ys & = ys & \text{(++1)} \\
    \left( x:x:s \right) \ ++ \ ys & = x\ : \left( xs \ ++ \ ys \right) & \text{(++2)}
\end{align*}
\]

prove that for any finite list \( xs \),

\[
\text{reverse } \left( \text{reverse } xs \right) = xs
\]

Hint: You’ll need to prove (by a separate induction) an auxiliary lemma relating \text{reverse} and ++. You may take as given (without the need for further proof), the two properties of \( ++ \) listed at the top of Hudak Table 11.2 (call them \( ++\text{assoc} \) and \( ++\text{nil} \) respectively).