CS321 Languages and Compiler Design I

Winter 2012

Lecture 14

Static Type Checking

Type checking means looking at a parsed program to make sure that:

- every piece of the program is well-typed;
- all identifiers are used in a type-consistent way.

Input:

- program AST or parse tree (or perhaps done during parsing)

Outputs:

- "OK" or typing error message(s)
- Perhaps type-annotated AST, perhaps per-identifier type information, etc.

Will see how to specify type-checking rules using

- Attribute grammars
- Inference rules

Environment and Symbol Tables

To type-check code that uses variables, function names, or other identifiers, must maintain an **environment** mapping identifiers to types.

- Environment is just a dictionary with key=identifer and value=type.
- Many possible representations: hash table, linked list or tree structure, etc.
- Current environment grows and shrinks as identifiers enter and leave scope.

It is often convenient to treat the current environment as a parameter of the type-checking code, so it can vary as we do a recursive traversal.

A more traditional approach is to treat the environment as a global whose contents are mutated as we traverse the program. In this case, the environment is usually called a **symbol table**.

Syntax-Directed Type Checking

Initially, we will show how to specify type-checking using attribute grammars over parse trees.

- Calculate a synthesized `type` attribute for each expression.
- Check that expressions are used correctly within other expressions and statements.
- Maintain environment information in an attribute or a global symbol table.

MOTIVATING INHERITED ATTRIBUTES

Sometimes it's convenient to make a node's attributes dependent on **siblings** or **ancestors** in tree.

Useful for expressing dependence on **context**, e.g., relating identifier **uses** to **declarations**. (This is especially important because CF grammar cannot capture such dependencies.)

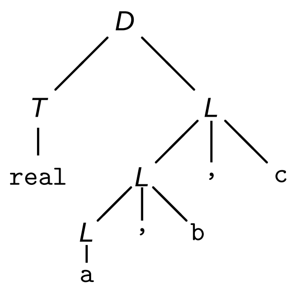
Example: Simple C-like Variable Declarations

$D \rightarrow T\ L$

$T \rightarrow$ `int` $|$ `real`

$L \rightarrow L_1$ `,id` $|$ `id`

Parse tree for `real a,b,c`:

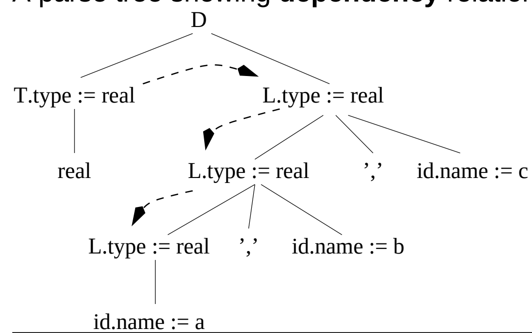


INHERITED ATTRIBUTE GRAMMAR

$D \rightarrow T\ L$	*L.type := T.type*
$T \rightarrow$ `int`	*T.type := integer*
$T \rightarrow$ `real`	*T.type := real*
$L \rightarrow L_1$ `,id`	{ *L_1.type := L.type; addsymb(*`id`*.name, L.type)* }
$L \rightarrow$ `id`	*addsymb(*`id`*.name, L.type)*

Here addsymb adds `id` and its type to symbol table, and *L.type* is an **inherited** attribute.

A parse tree showing **dependency** relations among attributes:



ATTRIBUTE EVALUATION

Dependency arrows for a dependency **graph**; we must evaluate attributes in **topological** order of dependency graph.

If attributes are defined on parse tree, may want to evaluate attributes while (or instead of) building the tree. This is **sometimes** possible:

- Saw how to evaluate **S-attributed** grammar, in which all attributes are synthesized, during bottom-up parsing; this method doesn't work for inherited attributes.
- Top-down parser can easily evaluate **L-attributed** grammars, in which attributes don't depend on their right ancestors. (Bottom-up parsers can sometimes handle these too, though with difficulty.) Example follows.
- For more complicated attribute grammars, might have to build some or all of tree **before** evaluating attributes.

ATTRIBUTE EVALUATION DURING RECURSIVE DESCENT

Each non-terminal function now takes **inherited** attribute values as **arguments** and return (record of) **synthesized** attribute value(s) as **result**.

Example revisited (with left-recursion removed):

```
class Ty {};
static Ty intTy = new Ty();  static Ty realTy = new Ty();

void D() { Ty ty = T(); L(ty); }
Ty T() {
   if (tok == INT) {
     tok = lex(); return intTy;
   } else if (tok == REAL) {
     tok = lex(); return realTy;
   } else error(); }
void L(Ty ty) {
   if (tok == ID) {
     addsymb(lexeme,ty); tok = lex();
   } else error();
   if (tok == ',') {
      tok = lex(); L(ty);} }
```

Avoiding Inherited Attributes

When using bottom-up parser (e.g., with `yacc` or `CUP`), it is desirable to avoid inherited attributes.

There are several approaches:

• Move the activity requiring the attribute to a higher node in the tree, by substituting a synthesized attribute for the inherited one, e.g.:

$D \rightarrow T\ L$	*for each* `id` *in L.list*
	addsymb(`id`*.name, T.type)*
$T \rightarrow$ `int`	*T.type := integer*
$T \rightarrow$ `real`	*T.type := real*
$L \rightarrow L_1$ `,` `id`	*L.list := append-list(*`id`*,*L_1*.list)*
$L \rightarrow$ `id`	*L.list := singleton-list(*`id`*)*

Avoiding Inherited Attributes (2)

• Can sometimes **rewrite** grammar, e.g.:

$D \rightarrow T$ `id`	{*D.type := T.type;*
	addsymb(`id`*.name,T.type)* }
$D \rightarrow D_1$ `,` `id`	{*D.type :=* D_1*.type;*
	addsymb(`id`*.name,D.type)* }
$T \rightarrow$ `int`	*T.type := integer*
$T \rightarrow$ `real`	*T.type := real*

Attributes on AST's

Attribute grammar method extends to **abstract** grammars (not intended for parsing), e.g., AST grammars.

• Same concept, but attribute evaluation always occurs after whole tree is built.

• Can use recursive descent as an attribute evaluation technique (regardless of how parsing was performed).

• Typical applications: typechecking, code generation, interpretation.

Why attribute grammars?

• **Compact**, convenient formalism.

• **Local** rules describe entire computation.

• Separate **traversal** from **computation**.

• (Purely **functional** rules can be evaluated in any order.)

Checking of E Language (Homework 1)

Can view checking process as evaluation of following attribute grammar, where

• *exp.ok* and *exps.ok* are synthesized boolean attributes indicating whether expression has checked successfully; and

• *exp.env* and *exps.env* are inherited environment attributes (with operators *empty*, *extend*, and *lookup*) containing entries for all in-scope variables.

$$
\begin{array}{lll r}
\textit{program} & \rightarrow & \textit{exp} & \textit{exp.env} := \textit{empty} \\
\\
\textit{exp} & \rightarrow & \textit{ID} & \textit{exp.ok} := \textit{lookup(exp.env,ID.name)} \\
& \rightarrow & \textit{NUM} & \textit{exp.ok} := \textit{true} \\
& \rightarrow & \textit{exp}_1\ \text{'+'}\ \textit{exp}_2 & \{\ \textit{exp}_1.\textit{env} := \textit{exp}_2.\textit{env} = \textit{exp.env}; \\
& & & \textit{exp.ok} := \textit{exp}_1.\textit{ok AND exp}_2.\textit{ok}\ \} \\
& \rightarrow & \textit{exp}_1\ \text{'–'}\ \textit{exp}_2 & \{\ \textit{exp}_1.\textit{env} := \textit{exp}_2.\textit{env} = \textit{exp.env}; \\
& & & \textit{exp.ok} := \textit{exp}_1.\textit{ok AND exp}_2.\textit{ok}\ \} \\
& \rightarrow & \textit{ID}\ \text{'='}\ \textit{exp}_1 & \{\ \textit{exp}_1.\textit{env} := \textit{exp.env}; \\
& & & \textit{exp.ok} := \textit{lookup(exp.env,ID.name) AND exp}_1.\textit{ok}\ \} \\
& \rightarrow & \texttt{if0}\ \textit{exp}_1\ \textit{exp}_2\ \textit{exp}_3 & \\
& & & \{\ \textit{exp}_1.\textit{env} := \textit{exp}_2.\textit{env} := \textit{exp}_3.\textit{env} := \textit{exp.env}; \\
& & & \textit{exp.ok} := \textit{exp}_1.\textit{ok AND exp}_2.\textit{ok AND exp}_3.\textit{ok}\ \} \\
& \rightarrow & \text{'\{'}\ \textit{vars}\ \text{';'}\ \textit{exps}\ \text{'\}'} & \{\ \textit{exps.env} := \textit{extend(exp.env,vars)}; \\
& & & \textit{exp.ok} := \textit{exps.ok}\ \} \\
\\
\textit{exps} & \rightarrow & \textit{exp} & \{\ \textit{exp.env} := \textit{exps.env}; \\
& & & \textit{exps.ok} := \textit{exp.ok}\ \} \\
& \rightarrow & \textit{exp}\ \text{';'}\ \textit{exps}_1 & \{\ \textit{exp.env} := \textit{exps}_1.\textit{env} := \textit{exps.env}; \\
& & & \textit{exps.ok} := \textit{exp.ok AND exps}_1.\textit{ok}\ \}
\end{array}
$$

CHECKING TYPES FOR A RICHER LANGUAGE

Consider a simple language of declarations, statements, and expressions.

$P \rightarrow D\ ;\ S \quad \{\ S.env = D.env;\ \}$

Actions for declarations synthesize environment attributes:

$$
\begin{array}{ll}
D \rightarrow \epsilon & \{\ D.env := empty\ \} \\
D \rightarrow \texttt{id}\ :\ T_1\ ;\ D_1 & \{\ D.env := extend(D_1.env, binding(\texttt{id},\ T_1.type))\ \} \\
T \rightarrow \texttt{bool} & \{\ T.type := boolean\ \} \\
T \rightarrow \texttt{int} & \{\ T.type := integer\ \} \\
T \rightarrow \texttt{array of}\ T_1 & \{\ T.type := array(T_1.type)\ \} \\
T \rightarrow \texttt{pair}\ T_1\ T_2 & \{\ T.type := T_1.type \times T_2.type\ \}
\end{array}
$$

EXPRESSIONS

Actions for expressions **check** for compatible operands and **synthesize** attribute type:

$$
\begin{array}{ll}
E \rightarrow \texttt{num} & \{\ E.type := integer\ \} \\
E \rightarrow \texttt{id} & \{\ E.type := lookup(E.env,\texttt{id})\ \} \\
E \rightarrow (E_1,E_2) & \{\ E_1.env = E.env;\ E_2.env = E.env;\ E.type = E_1.type \times E_2.type\ \} \\
E \rightarrow E_1\ \texttt{div}\ E_2 & \{\ E_1.env = E.env;\ E_2.env = E.env; \\
& \quad \text{if not } (E_1.type = integer \text{ and } E_2.type = integer) \text{ then} \\
& \qquad \text{issue type error;} \\
& \quad E.type := integer\ \} \\
E \rightarrow E_1\ \texttt{or}\ E_2 & \{\ E_1.env = E.env;\ E_2.env = E.env; \\
& \quad \text{if not } (E_1.type = boolean \text{ and } E_2.type = boolean) \text{ then} \\
& \qquad \text{issue type error;} \\
& \quad E.type := boolean\ \}
\end{array}
$$

Issuing error might or might not stop the checking process. If it doesn't, try to choose a synthesized type value that prevents a cascade of messages from a single mistake.

MORE EXPRESSIONS

$$
\begin{array}{ll}
E \rightarrow E_1\ [\ E_2\ ] & \{\ E_1.env = E.env;\ E_2.env = E.env; \\
& \quad \text{if } (E_1.type = array(T) \text{ and } E_2.type = integer) \text{ then} \\
& \qquad E.type := T; \\
& \quad \text{else issue type error}\ \} \\
E \rightarrow E_1.fst & \{\ E_1.env = E.env; \\
& \quad \text{if } E_1 = T_1 \times T_2 \text{ then} \\
& \qquad E.type := T_1; \\
& \quad \text{else issue type error;} \\
E \rightarrow E_1 < E_2 & \{\ E_1.env = E.env;\ E_2.env = E.env; \\
& \quad \text{if not } (E_1.type = integer \text{ and } E_2.type = integer) \text{ then} \\
& \qquad \text{issue type error;} \\
& \quad E.type := boolean\ \} \\
E \rightarrow E_1 = E_2 & \{\ E_1.env = E.env;\ E_2.env = E.env; \\
& \quad \text{if not } ((E_1.type = boolean \text{ or } E_1.type = integer) \\
& \qquad\quad \text{and } E_1.type = E_2.type) \text{ then} \\
& \qquad \text{issue type error;} \\
& \quad E.type := boolean\ \}
\end{array}
$$

Checking Statements

In most languages, statements don't have a type, so no point in synthesizing an attribute. Actions just check component types:

$S \rightarrow$ `id` `:=` E_1	{ E_1.*env* = S.*env*; *if* E_1.*type* $\neq$ *lookup*(S.*env*,`id`) *then* *issue type error* } *(Must also check that* `id` *is an l-value that can be assigned into.)*
$S \rightarrow$ `if` E_1 `then` S_1	{ E_1.*env* = S.*env*; S_1.*env* = S.*env*; *if* E_1.*type* $\neq$ *boolean then* *issue type error* }
$S \rightarrow S_1$ `;` S_2	{ S_1.*env* = S.*env*; S_2.*env* = S.*env*; }

Procedure/Function Definitions and Calls

Can describe type of function as $type_1 \times type_2 \times \ldots \times type_n \rightarrow type$

$D \rightarrow$ `id` `(` F_1 `)` `:` T_1 `;` D_1	{ D.*env* := *extend*(D_1.*env*, *binding*(`id`,F_1.*type* $\rightarrow$ T_1.*type*)) }
$F \rightarrow$ `id` `:` T_1	{ F.*type* := T_1.*type*}
$F \rightarrow$ `id` `:` T_1 `,` F_1	{ F.*type* := T_1.*type* $\times$ F_1.*type*}

$E \rightarrow$ `id` `(` A_1 `)`	{ A_1.*env* = E.*env*;
	if lookup(E.*env*,`id`) = $T_1 \rightarrow T_2$ *then*
	if A_1.*type* $\neq T_1$ *then*
	issue type error
	E.*type* := T_2
	else
	issue type error }

$A \rightarrow E_1$	{ E_1.*env* = A.*env*; A.*type* := E_1.*type* }
$A \rightarrow E_1$ `,` A_1	{ E_1.*env* = A.*env*; A.*type* := E_1.*type* $\times$ A_1.*type* }

Type Conversions

Implicit conversions (or "**coercions**") occur as a result of applying semantic rules of the language, e.g., perhaps evaluating `r + i`, where `r` is a real and `i` is an integer, causes implicit conversion of the fetched value of `i` to a real before the additon. This complicates type-checking:

$E \rightarrow E_1$ + E_2	{ E_1.*env* = E.*env*; E_2.*env* = E.*env*; *case* (E_1.*type*,E_2.*type*) *of* *(integer,integer): E.type := integer* *(integer,real):* *(real,integer):* *(real,real): E.type := real* *otherwise: issue type error* }

The relationship between integer and real is a special case of **subtyping** (more later).

Typing Judgments

A more compact way to specify typing rules is by using **inference rules** or **judgments** in the style of mathematical logic.

Each judgment for expressions has the form

$$TE \vdash e : t$$

Intuitively this says that expression e has type t, under the assumption that the type of each variable used in e is given by the *type environment* TE.

We write $TE(x)$ for the result of looking up x in TE, and $TE + \{x \mapsto t\}$ for the type environment obtained from TE by extending it with a new binding from x to t.

The key point is that an expression is well-typed **if-and-only-if** we can derive a typing judgment for it.

Sample Typing Judgments

Here are some of the rules from the attribute-grammar formalism transformed into judgments.

$$\frac{}{TE \vdash num : \textit{integer}} \text{ (Num)}$$

$$\frac{id \in dom(TE)}{TE \vdash id : TE(id)} \text{ (Var)}$$

$$\frac{TE \vdash e_1 : t_1 \quad TE \vdash e_2 : t_2}{TE \vdash \texttt{(}e_1\texttt{,}e_2\texttt{)} : t_1 \times t_2} \text{ (Pair)}$$

$$\frac{TE \vdash e_1 : \textit{integer} \quad TE \vdash e_2 : \textit{integer}}{TE \vdash e_1 \ \texttt{div} \ e_2 : \textit{integer}} \text{ (Div)}$$

$$\frac{TE \vdash e_1 : \textit{boolean} \quad TE \vdash e_2 : \textit{boolean}}{TE \vdash e_1 \ \texttt{or} \ e_2 : \textit{boolean}} \text{ (Or)}$$

More Expression Rules

$$\frac{TE \vdash e_1 : \textit{array}(t) \quad TE \vdash e_2 : \textit{integer}}{TE \vdash e_1\texttt{[}e_2\texttt{]} : t} \text{ (Subscript)}$$

$$\frac{TE \vdash e : t_1 \times t_2}{TE \vdash e\texttt{.fst} : t_1} \text{ (Fst)}$$

$$\frac{TE \vdash e_1 : \textit{integer} \quad TE \vdash e_2 : \textit{integer}}{TE \vdash e_1 \ \texttt{<} \ e_2 \ : \textit{boolean}} \text{ (LT)}$$

$$\frac{TE \vdash e_1 : \textit{integer} \quad TE \vdash e_2 : \textit{integer}}{TE \vdash e_1 \ \texttt{=} \ e_2 \ : \textit{boolean}} \text{ (EQI)}$$

$$\frac{TE \vdash e_1 : \textit{boolean} \quad TE \vdash e_2 : \textit{boolean}}{TE \vdash e_1 \ \texttt{=} \ e_2 \ : \textit{boolean}} \text{ (EQB)}$$

Statement Rules

Judgments for statements omit the result environment, and simply assert that the statement is well-typed.

$$\frac{TE \vdash e : t \quad TE(id) = t}{TE \vdash id \ \texttt{:=} \ e} \text{ (Assign)}$$

$$\frac{TE \vdash e : \textit{boolean} \quad TE \vdash s}{TE \vdash \texttt{if} \ e \ \texttt{then} \ s} \text{ (If)}$$

$$\frac{TE \vdash s_1 \quad TE \vdash s_2}{TE \vdash s_1\texttt{;}s_2} \text{ (Sequence)}$$

$$\frac{\vdash texp : t \quad TE + \{id \mapsto t\} \vdash s}{TE \vdash \texttt{var} \ id\texttt{:}texp\texttt{;} \ s} \text{ (Decl)}$$

Type Expression Rules

Judgments for type expressions just translate the external syntax for types into the internal representation:

$$\frac{}{\vdash \texttt{bool} : \textit{boolean}} \text{ (Bool)}$$

$$\frac{}{\vdash \texttt{int} : \textit{integer}} \text{ (Int)}$$

$$\frac{\vdash texp : t}{\vdash \texttt{array of} \ texp : \textit{array}(t)} \text{ (Array)}$$

$$\frac{\vdash texp_1 : t_1 \quad \vdash texp_2 : t_2}{\vdash \texttt{pair} \ texp_1 \ texp_2 : t_1 \times t_2} \text{ (Pair)}$$

Type Equivalence

When do two identifiers have the “same” type, or “compatible” types?

E.g., if `a` has type t_1, `b` has type t_2 and `f` has type $t_2 \to t_3$, how must t_1 and t_2 be related for these to make sense?

```
a := b
f (a)
```

To maintain **type safety** we must insist at a minimum that t_1 and t_2 are **structurally equivalent**.

Structural equivalence is defined inductively:

- Primitive types are equivalent iff they are exactly the same type.
- Cartesian product types are equivalent if their corresponding component types are equivalent. (Record field names are typically ignored.)
- Disjoint union types are equivalent if their corresponding component types are equivalent.
- Mapping types (arrays and functions) are the same if their domain and range types are the same.

Equivalence (continued)

Another way to say this: two types are equal if they have the same set of values.

Recursive types are a challenge. Are these two types structurally equivalent?

```
type t1 = { a:int, b: POINTER TO t1 };
type t2 = { a:int, b: POINTER TO t2 };
```

Intuitively yes, but it’s (a little) tricky for a type-checking algorithm to determine this!

Type Names

Question of equivalence is more interesting if language has type **names**, which arise for two main reasons:

- As a convenient shorthand to avoid giving the full type each time. E.g.,

```
function f(x:int * bool * real) : int * bool * real = ...
type t = int * bool * real
function f(x:t) : t = ...
```

- As a way of improving program correctness by subdividing values into types according to their meaning **within the program**.

```
type polar = { r:real, a:real };
type rect = { x:real, y:real };
function polar_add(x:polar,y:polar) : polar ...
function rect_add(x:rect,y:rect) : rect ...
var a:polar; c:rect;
a := (150.0,30.0) (* ok *)
polar_add(a,a)  (* ok *)
c := a  (* type error *)
rect_add(a,c) (* type error *)
```

For this to be useful, some structurally equivalent types must be treated as **inequivalent**.

Name Equivalence

Simplistic idea: Two types are equivalent iff they have the same **name**.

Supports `polar`/`rect` distinction.

But pure name equivalence is very restrictive, e.g.:

```
type ftemp = real
type ctemp = real
var x:ftemp, y:ftemp, z: ctemp;
x := y; (* ok *)
x := 10.0; (* probably ok *)
x := z; (* type error *)
x := 1.8 * z + 32.0; (* probably type error *)
```

Different types now seem **too** distinct; can’t even convert from one form of real to another.

Name Equivalence (continued)

Also: what about unnamed type expressions?

```
type t = int * int
procedure f(x: int * int) = ...
procedure g(x: t) = ...
var a:t = (3,4)
g(a); (* ok *)
f(a); (* ok or not ?? *)
```

Because of these problems with pure name equivalence, most languages use **mixed** solutions.

C Type Equivalence

C uses structural equivalence for array and function types, but name equivalence for `struct`, `union`, and `enum` types. For example:

```
char a[100];
void f(char b[]);
f(a); /* ok */

struct polar{float x; float y;};
struct rect{float x; float y;};
struct polar a;
struct rect b;
a = b; /* type error */
```

A type defined by a `typedef` declaration is actually just an abbreviation for an existing type.

Note that this policy makes it easy to check equivalence of recursive types, which can only be built using `struct`s.

```
struct fred {int x; struct fred *y;} a;
struct bill {int x; struct fred *y;} b;
a = b; /* type error */
```


ML Type Equivalence

ML uses structural equivalence, except that each `datatype` declaration creates a new type unlike all others.

```
datatype polar = POLAR of real * real
datatype rect = RECT of real * real
val a = POLAR(1.0,2.0)  and  b = RECT(1.0,2.0)
if (a = b) ...  (* type error *)
```

Note that the mandatory use of constructors makes it possible to uniquely identify the types of literals.

Note that a datatype need not declare a record:

```
datatype fahrenheit = F of real
datatype celsius = C of real
val a = F 150.0  and  b = C 150.0
if (a = b) ... (* type error *)
fun convert(F x) = C(1.8 * x + 32.0) (* ok *)
```

For type abbreviation, ML offers the `type` declaration, which simply gives a new name for an existing type.

```
type centigrade = celsius
fun g(x:centigrade) = if x = b ... (* ok *)
```


Java Type Equivalence

Java uses nearly strict name equivalence, where names are either:

- One of eight built-in **primitive** types (`int`,`float`,`boolean`, etc.), or
- Declared classes or interfaces (**reference** types).

The only non-trivial type expressions that can appear in a source program are **array** types, which are compared structurally, using name equivalence for the ultimate element type. Java has no mechanism for type abbreviations.

Java types form a **subtyping** hierarchy:

- If class `A` extends class `B`, then `A` is a subtype of `B`.
- If class `A` implements interface `I`, then `A` is a subtype of `I`.
- If numeric type `t` can be coerced to numeric type `u` without loss of precision, then `t` is a subtype of `u`.

If T_1 is a subtype of T_2, then a value of type T_1 can be used wherever a value of T_2 is expected.