

Problem 1. Prove that:

$$x^{\log_b y} = y^{\log_b x}$$

where b and x and y are positive numbers.

Problem 2. Prove that n^α is a smooth function for any constant $\alpha \geq 0$ and n ranging over \mathbb{N} .

Problem 3. What is the value of:

$$\sum_{j=1}^{\log_b n} (b^d/a)^j$$

when $b^d = a$ and $n = b^k$?