Homework #4 Name: ____________________

If a question is wrong, or has no acceptable answer, do not mark any choice.
If a question has several correct answers, choose the most accurate/complete/informative one.
On a separate sheet, write a detailed justification of your choice.
You will be graded on the accuracy and precision of this justification only.
You will get 1 point for each correct answer and 0 points for missing or incorrect answers.
Your grade will be written on the back of this page.

Variations of some problems are suggested under the label "Extra". Instead of selecting a choice, compute a value or formula according to the problem. You should not turn in any variation, but you are welcome to discuss problems and solution in class and with the Teaching Assistant.

1. The close form of $\sum_{i=1}^{n}(2i+2)$ is:

   [-A-] n^2+3n-2
   [-B-] $n(n+1)+2n$
   [-C-] n^2+n
   [-D-] None of the above

   Extra:
   1. Replace the boundaries of the summation with 0 and $n+1$.
   2. Replace the multiplicative coefficient with $-2,-1,0,1,3$.
   2. Replace the additive coefficient with $-2,-1,0,1,3$.

2. The value of $\displaystyle\sum_{i=45}^{60} i$ is (remember Gauss):

   [-A-] 840
   [-B-] 850
   [-C-] 880
   [-D-] 900

   Extra:
   1. Replace the low bounary of the summantion with 25, 35, 40, 48.
   2. Replace the high boundaty with 60, 66, 70, 75. (watch out for odd/even).
   3. Replace i with $2i$, $2i+1$, $2i-1$.

3. How many times does the following program prints "hi" for $n=9$.

```
for (i=0; i<n; i++) {
  for (j=i+1; j<=n; j++) {
    print("hi");
  }
}
```

   [-A-] 45
   [-B-] 60
   [-C-] 90
   [-D-] none of the above

   Extra:
   1. Replace the outer loop high boundary with `i<=n`.

2. Replace the inner loop low boundary with `j=i` or `j=0`.
3. Replace the inner loop high boundary with `j<n`.
4. Replace n with $7, 8, 10, 12, 20$.

4. Let L be a language containing exactly l strings.
Let M be a language containing exactly m strings.
The number of strings in $L \cup M$ is:

[-A-] exactly $l + m$
[-B-] at least $l + m$
[-C-] at most $l + m$
[-D-] None of the above

Extra:
1. Replace $L \cup M$ with LM and $l + m$ with lm.
2. Add a language N with exactly n strings.
3. Consider $(L \cup M)N$ and $LN \cup M$.

5. Let $L = \{aa, b\}$ and $M = \{bb, a\}$ be languages over $\{a, b\}$.
Let $X = LML$.

[-A-] $a^0 \in X$
[-B-] $a^3 \in X$
[-C-] $a^5 \in X$
[-D-] $a^7 \in X$

Extra:
1. Consider languages with other strings, e.g, $L = \{aa, b, cc\}$ and similarly for M.
2. Consider other expressions for X, e.g., L^2M, L^2M^2, $L(L \cup M)$, etc.
3. Consider other strings for membership, e.g., a^2b^2, abc, etc.

6. Let A be an alphabet.
Which of the following is **not** a language over A.

[-A-] Λ
[-B-] $\varnothing$
[-C-] A
[-D-] $\{\Lambda\}$

7. Let L be an alphabet.

[-A-] $L^* = L^*L^*$
[-B-] $L^* \subseteq L^*L^*$
[-C-] $L^* \supseteq L^*L^*$
[-D-] None of the above

Extra:
1. Consider other languages and other expressions, e.g., $L^+ = L^+L^+$, $L^+ \subseteq L^+L^+$, $L^+ = L^*L^+$, $L^* = L^*L^+$, $L^n = L^m$, for $n < m$, $n \leq m$, etc.

8. Let $P(n)$ be a statement where n stands for a natural number.
In a proof by induction of P, the base case proves $P(k)$ where k is:

[-A-] zero
[-B-] zero or greater than zero
[-C-] strictly greater than zero
[-D-] None of the above

9. In a proof by induction of $P(n)$, you must prove:

[-A-] $P(k)$, for $k \geq 0$
[-B-] $P(k) \wedge P(k+1)$, for $k > 0$
[-C-] if $P(k)$, then $P(k+1)$, for $k > 0$
[-D-] if $P(k)$, then $P(k+1)$, for $k \geq 0$

10. Let $a_0 = 2$ and, for $n > 0$, $a_n = a_{n-1} + 2$ be a recurrence relation.
The close form of a_n is:

[-A-] $2(n-1)$
[-B-] $2n$
[-C-] $2(n+1)$
[-D-] n^2

Extra:
1. Choose $-2, -1, 0, 1, 3$ for a_0.
2. Choose $-2, -1, 0, 1, 2, 3$ as multiplicative coefficient of a_{n-1}.
3. Choose $-2, -1, 0, 1, 3$ as additive coefficient of a_{n-1}.

11. Let $a_0 = 0$ and, for $n > 0$, $a_n = a_{n-1} + 3$ be a recurrence relation.
The close form of a_n is:

[-A-] $3(n-1)$
[-B-] $3n$
[-C-] $3(n+1)$
[-D-] n^2

Extra:
Like the previous variation.