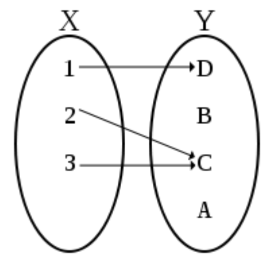
Homework #2 Name: ____________________________

If a question is wrong, or has no acceptable answer, do not mark any choice.
If a question has several correct answers, choose the most accurate/complete/informative one.
On a separate sheet, write a detailed justification of your choice.
You will be graded on the accuracy and precision of this justification only.
You will get 1 point for each correct answer and 0 points for missing or incorrect answers.
Your grade will be written on the back of this page.

1. Consider the function defined to the right.
The range of f is:

[-A-] Y
[-B-] $\{C, D\}$
[-C-] X
[-D-] $\{A, B\}$



2. Consider again the previous function:

[-A-] f is surjective
[-B-] f is injective
[-C-] f is bijective
[-D-] None of the above

3. Let $f : A \to B$ be a function such that for every $x \in A$ there is one and only one $y \in B$ such that $f(x) = y$.

[-A-] f is surjective
[-B-] f is injective
[-C-] f is bijective
[-D-] None of the above

4. Let x and y be rational numbers.

[-A-] $\lfloor x \rfloor + \lfloor y \rfloor = \lfloor x + y \rfloor$
[-B-] $\lfloor x \rfloor + \lfloor y \rfloor \leqslant \lfloor x + y \rfloor$
[-C-] $\lfloor x \rfloor + \lfloor y \rfloor \geqslant \lfloor x + y \rfloor$
[-D-] None of the above

5. Let $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x + 1$.

[-A-] f has no inverse
[-B-] f has an inverse
[-C-] $g(x) = (x - 1)/2$ is the inverse of f
[-D-] $g(x) = x/2 - 1$ is the inverse of f

6. Let $f : \mathbb{Q} \to \mathbb{Q}$ defined by $f(x) = 2x + 1$.

[-A-] f has no inverse
[-B-] f has an inverse
[-C-] $g(x) = (x - 1)/2$ is the inverse of f
[-D-] $g(x) = x/2 - 1$ is the inverse of f

7. Let R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $x\, R\, y$ iff $x < 3\, y$.

   [-A-] $10\, R\, 4$ and $4\, R\, 10$
   [-B-] $10\, R\, 4$ and not $4\, R\, 10$
   [-C-] not $10\, R\, 4$ and $4\, R\, 10$
   [-D-] not $10\, R\, 4$ and not $4\, R\, 10$

8. Let R and S be binary relations on $\mathbb{N}$ (repeat, $\mathbb{N}$) defined as follows:
   $x\ R\ y$ iff $x > y + 2$ and $x\ S\ y$ iff $x = y - 2$.
   Let $T = R \circ S$ (composition).

   [-A-] $(5, 2) \in T$ and $(5, 0) \in T$
   [-B-] $(5, 2) \in T$ and $(5, 0) \notin T$
   [-C-] $(5, 2) \notin T$ and $(5, 0) \in T$
   [-D-] $(5, 2) \notin T$ and $(5, 0) \notin T$

9. Let A be a set of n elements and R a binary relation on A.
   Assume that R is reflexive and $p = |R|$ (cardinality):

   [-A-] $p > n$
   [-B-] $p \geqslant n$
   [-C-] $p \geqslant 0$
   [-D-] $p > 0$

10. Let R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $x\, R\, y$ iff x is a factor of y.

   [-A-] R is reflexive
   [-B-] R is transitive
   [-C-] all of the above
   [-D-] none of the above

11. Let "$\sim$" be the relation on $\mathbb{N}_{60}$, defined by $x \sim y$ iff the leftmost digits of the decimal representation of x and y are the same. The relation "$\sim$" is an equivalence. The number of equivalence classes (size of the partition) induced by "$\sim$" is:

   [-A-] 1
   [-B-] 6
   [-C-] 10
   [-D-] 60