Homework #1 Name: ____________________

If a question is wrong, or has no acceptable answer, do not mark any choice.
If a question has several correct answers, choose the most accurate/complete/informative one.
On a separate sheet, write a detailed justification of your choice.
You will be graded on the accuracy and precision of this justification only.
You will get 1 point for each correct answer and 0 points for missing or incorrect answers.
Your grade will be written on the back of this page.

1. Let $A = \mathit{false}$ and $B = \mathit{false}$.
   Let $X = A \text{ } \mathit{or} \text{ } (\mathit{not} \text{ } B)$ and $Y = \mathit{not}(A \text{ } \mathit{and} \text{ } B)$.

   [-A-] $X = \mathit{false}$ and $Y = \mathit{false}$
   [-B-] $X = \mathit{false}$ and $Y = \mathit{true}$
   [-C-] $X = \mathit{true}$ and $Y = \mathit{false}$
   [-D-] $X = \mathit{true}$ and $Y = \mathit{true}$

2. Let X be the proposition: $A \rightarrow B$.
   Let Y be the proposition: $B \rightarrow A$.

   [-A-] X is the antecendent of Y
   [-B-] X is the consequent of Y
   [-C-] X is the converse of Y
   [-D-] X is the contrapositive of Y

3. The proposition $A \leftrightarrow (A \vee B)$ is a:

   [-A-] tautology
   [-B-] contradiction
   [-C-] contingency
   [-D-] none of the above

4. Let $X = A \text{ } \mathit{or} \text{ } B$, where A and B are Boolean variables.

   [-A-] if A then X
   [-B-] if X then A
   [-C-] X if and only if A
   [-D-] None of the above

5. Let $X = \mathit{not} \text{ } (A \text{ } \mathit{and} \text{ } B)$, where A and B are Boolean variables.
   If $X = \mathit{true}$ then:

   [-A-] $A = \mathit{true}$
   [-B-] $B = \mathit{true}$
   [-C-] $A = \mathit{true}$ and $B = \mathit{true}$
   [-D-] None of the above

6. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$.

   [-A-] $\{2, 3\} = A \cup B$
   [-B-] $\{2, 3\} = A \cap B$
   [-C-] $\{2, 3\} = A - B$
   [-D-] $\{2, 3\} = A \oplus B$

7. Let $\mathbb{E}$ be the set of the even integers.
Which of the following sets is finite.

[-A-] $\mathbb{E} \cap \mathbb{E}$
[-B-] $\mathbb{E} \cup \mathbb{E}$
[-C-] $\mathbb{E} \oplus \mathbb{E}$
[-D-] None of the above

8. Let $P = \{x \mid x = 4\,k - 3, \text{ for } k \in \mathbb{N}\}$.

[-A-] $0 \in P$ and $1 \in P$
[-B-] $0 \in P$ and $1 \notin P$
[-C-] $0 \notin P$ and $1 \in P$
[-D-] $0 \notin P$ and $1 \notin P$

9. Let A and B be sets.
Suppose that $A - B \subseteq A$.

[-A-] $B \subseteq A$
[-B-] $B \supseteq A$
[-C-] $B = \varnothing$
[-D-] B can be any set

10. Let $X = \{1, 2\}$ be a set and $Y = 2^X$, the powerset of X.

[-A-] $|Y| = 2$
[-B-] $|Y| = 4$
[-C-] $|Y| \leqslant 4$
[-D-] $|Y| \geqslant 2$

11. Let B and P be the multisets (bags) made with the letters of the words “banana” and “panama”, respectively. The number of elements of $B \cup P$ is:

[-A-] 0
[-B-] 4
[-C-] 8
[-D-] 12

This is a sample question/answer

12. Let $A = \{1, (2, a), 3\}$ and $B = \{a, (3, a), 4\}$. Let C denote $A \times B$.

[-A-] $(2, a) \in C$ and $(3, a) \in C$
[-B-] $(2, a) \in C$ and $(3, a) \notin C$
[-C-] $(2, a) \notin C$ and $(3, a) \in C$
[-D-] $(2, a) \notin C$ and $(3, a) \notin C$

The correct answer is [-C-] because:
By definition $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$.
$2 \notin A$, hence $(2, y) \notin A \times B$ no matter what y is, and
$3 \in A$ and $a \in B$, hence $(3, a) \in A \times B$.