ECE 510 OCE
BDDs and Their Applications

Lecture 18.
Verification Using Specialized DDs

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Overview

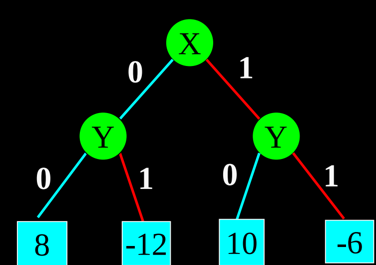
- Extensions of Binary Decision Diagrams
  - Multi-terminal BDDs (MTBDDs) (Clarke, DAC'93)
  - (Multiplicative) binary moment diagrams (BMDs/*BMDs) (Bryant, DAC'95)
  - Edge-valued BDDs (EVBDDs) (Lai, DAC'92)
  - Kronecker functional decision diagrams (OKFDDs) (Drechsler/Sarabi/Perkowski, DAC'94)
  - Kronecker multiplicative moment diagrams (K*BMDs) (Drechsler, EDTC'96)
- Word-level DDs vs. bit level DDs for verification
- Binary expression diagrams (Andersen/Hulgaard,1997)
- Advantages and limitations of the specialized DDs

Extensions of BDDs

- Representation of "pseudo-Boolean" functions: integer (real) number functions over Boolean functions
- Applications: integer linear programming, matrix multiplication, spectral transforms, word-level analysis of digital systems

Multi Terminal BDDs

X	Y	F
0	0	8
0	1	-12
1	0	10
1	1	-6



Generalization of Expansion

- Boole-Shannon expansion:

  $F = x' \,\&\, F_0 + x \,\&\, F_1$

- For numeric value functions, generalized to

  $F = (1\text{-}x) \,\&\, F_0 + x \,\&\, F_1$

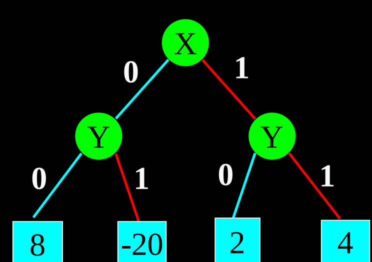
- Boole-Shannon expansion can be rewritten as

  $F = F_0 + x \,\&\, (F_1 - F_0)$

  where $F_x = F_1 - F_0$ is the binary moment (derivitive of F w.r.t. variable x)

Binary Moment Diagrams (BMDs)

X	Y	F
0	0	8
0	1	-12
1	0	10
1	1	-6



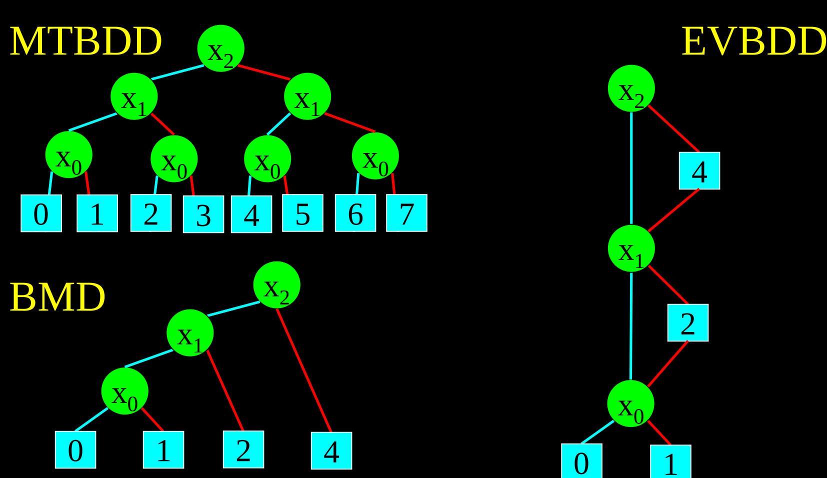
F = 8(1-x)(1-y) -12(1-x)y + 10x(1-y) - 6xy =

= 8 - 20y + 2x + 4xy

Edge-Valued BDDs

- An edge has an integer weight
- The weights are combined additively
- The value of the function is derived by following a path from the root to a leaf and summing the edge weights encountered

Example: $F = X = 4x_2 + 2x_1 + x_0$

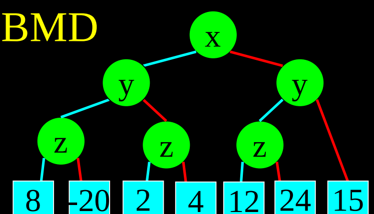


Multiplicative BMDs

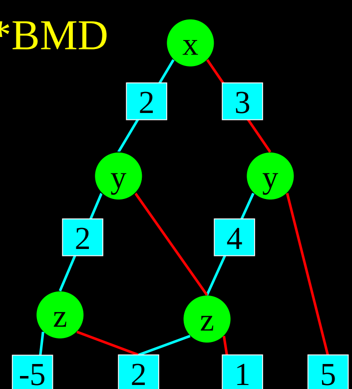
- An edge has an integer (rational) weight
- The weights are combined multiplicatively
- The value of the function is derived by following a path from the root to a leaf and multiplying the edge weights encountered

Example: F=8-20z+2y+4yz+12x+24xz+15xy

BMD



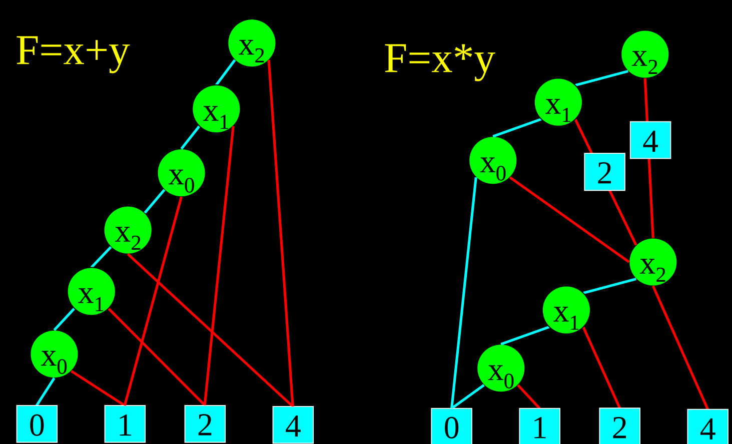
*BMD

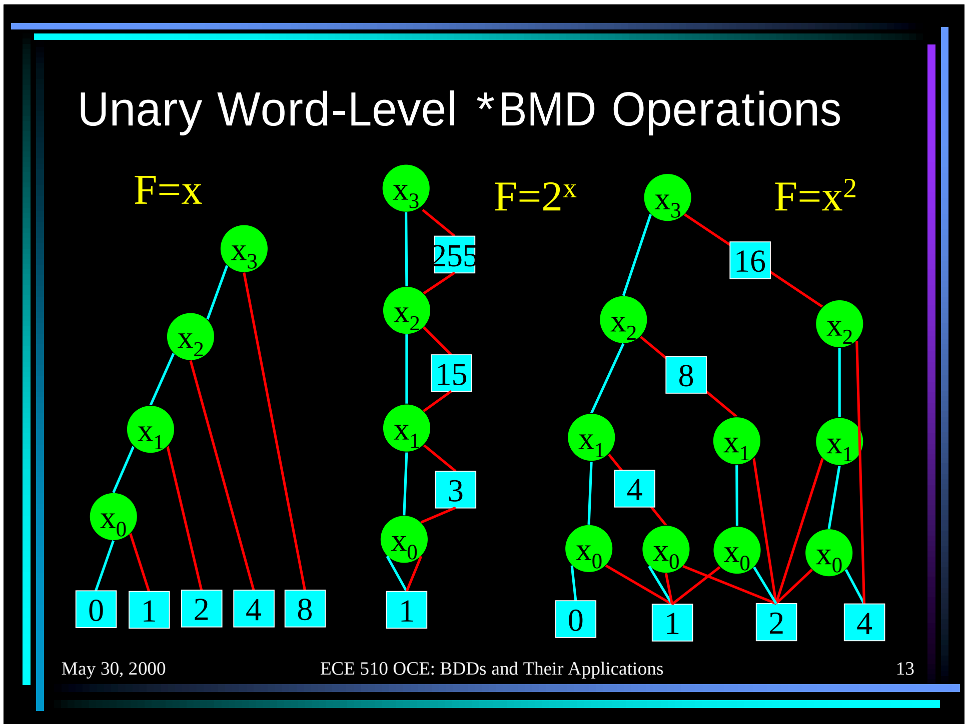


Word-Level Operation Complexity

Form	x	x+y	x*y	x^2	c^x
MTBDD	exp	exp	exp	exp	exp
EVBDD	lin	lin	exp	exp	exp
BMD	lin	lin	quadr	quadr	exp
*BMD	lin	lin	lin	quadr	lin
K*BMD	lin	lin	lin	quadr	lin

Sum/Product *BMD Representation




Unary Word-Level *BMD Operations
F=x
F=2^x
F=x^2
x3
x2
x1
x0
0
1
2
4
8
255
15
3
16
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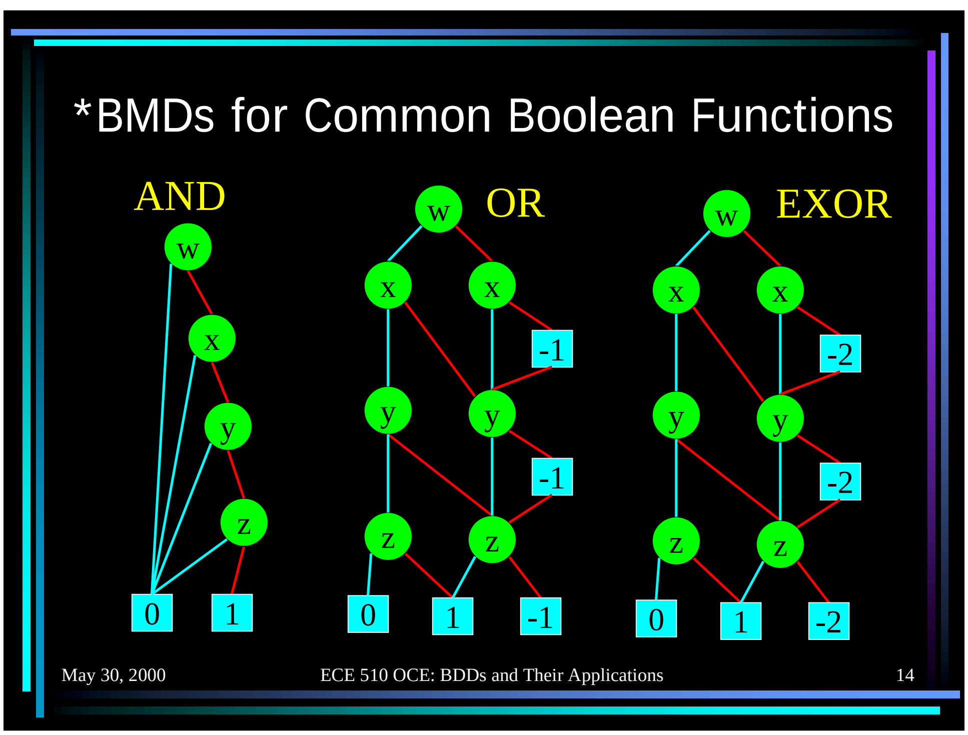
*BMDs for Common Boolean Functions
AND
OR
EXOR
w
x
y
z
0
1
-1
-2
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Kronecker Multiplicative (K*)BMDs

- An edge has two integer (rational) weights
- The weights are combined additively and multiplicatively
- The value of the function is derived by following a path from the root to a leaf and adding/multiplying edge weights encountered
- Three types of expansion are used: Shannon, Positive Davio, Negative Davio
- Normalization rules are quite complicated

Expansion Rules

- Shannon Expansion

  $<(a,m),F> = a + m((1-x)*F0 + x*F1)$
- Positive Davio Expansion

  $<(a,m),F> = a + m(F0 + x*F1)$
- Negative Davio Expansion

  $<(a,m),F> = a + m(F0 + (1-x)*F1)$

K*BMD Normalization Rules

- There is only one leaf labeled 0
- The low-edge of a node always has additive weight 0 and multiplicative weight 1
- The multiplicative weight at the high-edge is 1, if F_0 = 0. If the high-edge points to the leaf, then the multiplicative weight is normalized to 1

*BMD vs. K*BMD for Integer Encoding