

ECE 510 OCE
BDDs and Their Applications

**Lecture 13. (1) Compatible Projection
(2) Implicit Formulation for Problems
in Boolean Networks**

May 9, 2000
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Overview

- Compatible projection operator
 - Background
 - Recursive algorithm
 - Applications (FSM state minimization, FSM decomposition, functional decomposition, classification of functions, etc.)
- Representations of Boolean networks
 - functional
 - structural
- Problems in Boolean networks
 - test pattern generation
 - fault localization
 - optimization (for the number of nodes, area, delay, testability)
 - eliminating combinational loops, etc

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Equivalence Relation

- Equivalence relation is a boolean function $E(x,y) : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$, which is true for codes $x=(x_1,x_2,\dots,x_n)$ and $y=(y_1,y_2,\dots,y_n)$ iff the corresponding encoded objects are equivalent

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Properties of Equivalence Relation

- Equivalence relation is reflexive, symmetric, and transitive
- Suppose the equivalence classes are $\{(00,01), (11), (10)\}$

	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	1	0
10	0	0	0	1

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Equivalence Class Characterization Relation

- Equivalence class characterization relation $\xi(x,y)$ selects exactly one representative from each equivalence class of states defined by equivalence relation $E(x, y)$
- $\xi(x, y)$ is a boolean function that is one for the codes x and y iff the equivalence class containing code x is represented by the state with code y
 $\xi(x, y) = \text{CProjection}(E(x, y), y_0)$,
 y_0 is the reset state encoded using variables y .

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Compatible Projection Operator

- Given an equivalence relation $E(x,y)$, the compatible projection is a boolean function selecting exactly one representative from each equivalence class and relating it all members of this class
- In particular, the selection function may find the element which has the shortest distance from the given vertex y_0

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Compatible Projection Pseudocode

```
bdd CProjection( bdd E(x,y), bdd  $\alpha(y)$  ) {  
  if (  $\alpha = 1$  ) return E;  
  if ( E = 0 ) return 0;  
  if ( E = 1 ) return  $\alpha$ ;  
   $y_i$  is the top variable in  $\alpha$ ;  
  if (  $\alpha_{y_i} = 0$  )       $\alpha_i = y_i$ ;  
  else /*if (  $\alpha_{y_i} = 0$  )*/  $\alpha_i = y_i$ ;  
   $\gamma = \exists_{y_i} E_{\alpha_i}$ ;  
  return  $\alpha_i$  & CProjection(  $E_{\alpha_i}$  ,  $\alpha_{\alpha_i}$  ) +  
     $\gamma \alpha_i$  & CProjection(  $E_{\alpha_i}$  ,  $\alpha_{\alpha_i}$  );  
}
```

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Complete Source Code for CProjection

```
bdd CProjection( bdd F, bdd Ref )  
{  
  assert( Ref != bddfals; ) // cofactors of F with respect to this literal  
  if ( Ref == bddtrue ) return F; bdd PosCofF = bdd_restrict( F, Literal );  
  if ( F == bddfals; ) return bddfals; bdd NegCofF = bdd_restrict( F, !Literal );  
  if ( F == bddtrue ) return Ref; // part of projection that is aligned with Literal  
  // check cache for ready-made results bdd PosPart = CProjection( PosCofF, NextRef );  
  
  bdd NextRef, Literal; // the domain where the projection  
  int CurVar = bdd_var( Ref ); // aligned with Literal exists  
  if ( bdd_low( Ref ) == bddfals; ) bdd Domain = bdd_exists( PosCofF, AllVars );  
  { // the top var is positive // part of projection that is not aligned with Literal  
    NextRef = bdd_high( Ref ); bdd NegPart = !Domain &  
    Literal = bdd_ithvar( CurVar ); CProjection( NegCofF, NextRef );  
  }  
  else if ( bdd_high( Ref ) == bddfals; ) bdd Result = bdd_ite( Literal, PosPart, NegPart );  
  { // the top var is negative // insert the result into cache  
    NextRef = bdd_low( Ref );  
    Literal = !bdd_ithvar( CurVar );  
  }  
  else // Ref is not a cube! return Result;  
    assert( 0 );  
}
```

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Applications of Compatible Projection

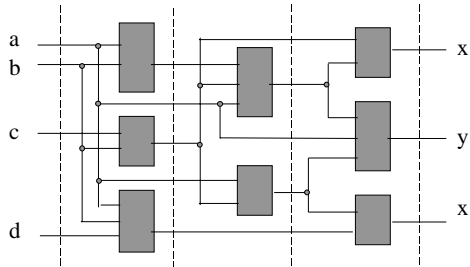
- FSM state minimization
 - selecting one state from a group of equivalent states
- FSM decomposition
 - counting number of blocks in implicitly represented partitions
 - finding partitions with maximum/minimum number of blocks
- Functional decomposition
 - finding exact or approximate column multiplicity
- Classification of boolean functions
 - deriving the number of NPN classes of functions depending on the given number of variables
- Other applications dealing with equivalence classes

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Boolean (Binary) Networks (Netlists)



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Functional/Structural Representations

- Functional representation characterizes the output behavior in response to inputs (possibly in presence of faults)
- Structural representation characterizes the network as interconnection of blocks belonging to one of the finite number of types

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A BDD-based approach to ATPG

- Deriving functional representation (sensitivity function)
 - Introducing modifiers
 - Encoding test points and fault models
- Solving testing problems using sensitivity function
 - Accounting for specific fault models
 - Stuck-at-k and stuck-at-any fault models
 - Other "combinational" fault models (shifting, bridging)
 - "Sequential" fault models (delay, stuck-open)
 - ATPG for multiple faults
 - Fault localization
 - Towards sequential ATPG

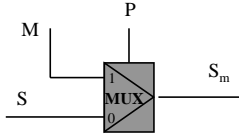
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Introducing Modifiers

- A modifier is a multiplexer controlled by additional variable P



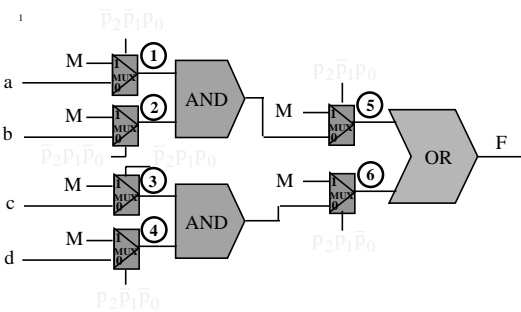
- Modifiers are *not* introduced in hardware. It is a way to formally describe selective change of signal S in the circuit by assuming that, under certain condition P, it is replaced by some other signal M

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Example: 2-AND-1-OR circuit



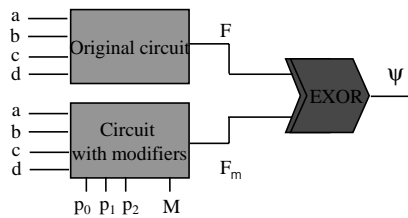
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Generalized Sensitivity Function

$$\psi(a,b,c,d,p_0,p_1,p_2,M) = F \oplus F_m$$



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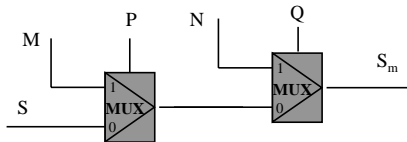
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$\psi(a,b,c,d,p_0,p_1,p_2,M)$ for $M = 1$

a	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
b	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
c	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0
d	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
p2	000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p1	001	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0
p0	011	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1
	010	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
	110	1	1	0	1	1	0	1	1	0	0	0	0	1	0	1
	111	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	101	1	1	0	1	1	0	1	1	0	0	0	0	1	0	1
	100	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0

Test Generation for Double Faults

- Let us introduce two modifiers controlled by signals P and Q



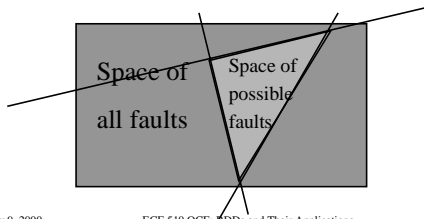
- This is a formal way to describe processes that occur in the circuit when two signals S_1 and S_2 in different locations are replaced by M and N

Fault Localization

- Let $\Psi(a,b,c, p_0,p_1,p_2)$ be a vector of sensitivity functions for each output
- If $\Psi_i(a,b,c, p_0,p_1,p_2) = 1$, then observing the response of i-th output of the function to the test (a,b,c) detects fault (p_0,p_1,p_2)
- If $\Psi_i(a,b,c, p_0,p_1,p_2) = 0$, then observing the response of i-th output of the function to the test (a,b,c) does not detect fault (p_0,p_1,p_2) , assuming this fault exists in the circuit

Fault Localization for 3-Output Circuit

- After applying one test pattern, the search space is reduced



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Finding the Shortest Test Sequence to Localize a Fault

- Compute relation $F(a, b, c, p_0, p_1, p_2, y_0, y_1, y_2)$, which combines tests, faults, and output patterns
- For each test, group faults producing the same output pattern into equivalence classes
- Find next test by applying one of the following greedy heuristics at each iteration:
 - Select a test that splits faults into as many equivalence classes as possible
 - Select a test that splits faults into classes, such that the maximum number of faults in each class is minimized
- Reduce relation F after each iteration

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ATPG for Sequential Circuits

- Compute the sensitivity function for combinational logic
- Compute the set of reachable states
- Find the set of combinational redundant and sequentially non-excitable faults
- Build the product machine of the fault-free FSM and the FSM with faults encoded by variables p
- Perform reachability analysis on the product machine to determine test sequences for all faults that can be detected (the remaining faults are non-distinguishable; they change STG but never manifest at the outputs)

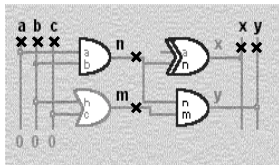
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Structural Representation of Networks

- a and n are in relation AND2-direct
- n and a are in relation AND2-reverse
- a and b are in relation AND2-adjacent...



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Encoding Gates and Signals

- Assign each signal ("cut point"), each type of gate, and relation a unique binary code (label)
 - a - $\bar{x}_2\bar{x}_1\bar{x}_0$ b - $\bar{x}_2\bar{x}_1x_0$ n - $\bar{x}_2x_1\bar{x}_0$
 - AND2 - $\bar{g}_1\bar{g}_0$ OR2 - \bar{g}_1g_0 EXOR2 - $g_1\bar{g}_0$
 - Direct - $\bar{a}_1\bar{a}_0$ Reverse - \bar{a}_1a_0 Adjacent - $a_1\bar{a}_0$
- Create products for each couple of objects (cut points in the network) and multiply these products by labels, defining their relationship
- Compute a bdd equal to the sum of these products

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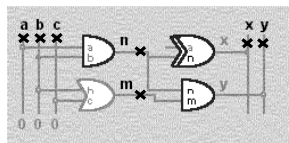
Example

- $R(x_2, x_1, x_0, y_2, y_1, y_0, g_1, g_0, a_1, a_0) =$

$$\bar{x}_2\bar{x}_1\bar{x}_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0\bar{a}_1\bar{a}_0 + \bar{x}_2\bar{x}_1\bar{x}_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0a_1\bar{a}_0 +$$

$$\bar{x}_2\bar{x}_1x_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0\bar{a}_1\bar{a}_0 + \bar{x}_2\bar{x}_1x_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0a_1\bar{a}_0 +$$

$$\bar{x}_2\bar{x}_1\bar{x}_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0\bar{a}_1a_0 + \bar{x}_2\bar{x}_1\bar{x}_0\bar{y}_2\bar{y}_1\bar{y}_0\bar{g}_1\bar{g}_0a_1a_0 + \dots$$



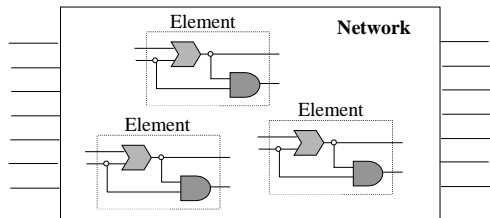
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Problem1: Locating Elements

- Find all occurrences of the combination of AND-OR gate in the network of logic gates



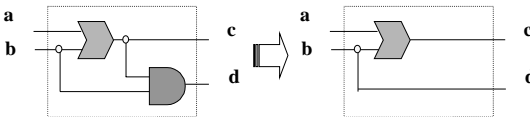
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Problem2: Replacing Elements

Replace a given element with another one in all places where it occurs. For example:
 $c = a + b$ $d = (a + b) \& b = a \& b + b = b$



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