

ECE 510 OCE

BDDs and Their Applications

Lecture 7.

(1) BDDs for Symmetric Functions

(2) Constraint Satisfaction Problems

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Overview

- BDDs for symmetric functions
- Characteristic functions of sets and sets of sets
- Building BDD for Tuples "k out of n"
- Constraint satisfaction problems
- Example: N-queen Problem
- Binare Covering Problem
- Solution of BCP using Shortest Path on BDDs

BDDs for Symmetric Functions

- The value of a symmetric function depends only on the number of 1's in the input vector
- For n -var functions, there are $n+1$ possible numbers of 1's and so 2^{n+1} different functions
- Cofactors of symmetric functions are symmetric functions
- If the function is cofactored k times, there are no more than $n+1-k$ different cofactors
- The BDD size is limited by $n*(n+1-k) \sim O(n^2)$

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Characteristic Function of a Set

- Function $F: B^n \rightarrow B$, $B = \{0,1\}$, defines a subset of minterms of B^n , on which it is 1.
- Given a binary encoding of a set of elements, **characteristic function** of a *subset* of this set is a boolean function, which is 1 for minterms encoding the subset and 0 for other minterms.

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Example

- Problem:** Given the set $\{p_0, p_1, p_2, p_3, p_4, p_5\}$ and its encoding:

$$p_0 - \bar{x}_2 \bar{x}_1 \bar{x}_0 \quad p_2 - \bar{x}_2 x_1 \bar{x}_0 \quad p_4 - x_2 \bar{x}_1 \bar{x}_0$$

$$p_1 - \bar{x}_2 \bar{x}_1 x_0 \quad p_3 - \bar{x}_2 x_1 x_0 \quad p_5 - x_2 \bar{x}_1 x_0$$

find characteristic function of subset $\{p_0, p_2, p_3\}$ and represent the subset using BDD

- Solution:** Define a function over the encoding variables (x_0, x_1, x_2) such that it is equal to 1 for minterms representing subset $\{p_0, p_2, p_3\}$.

$$\Phi_{\{p_0, p_2, p_3\}}(x_0, x_1, x_2) = \bar{x}_2 \bar{x}_1 \bar{x}_0 + \bar{x}_2 x_1 \bar{x}_0 + \bar{x}_2 x_1 x_0$$

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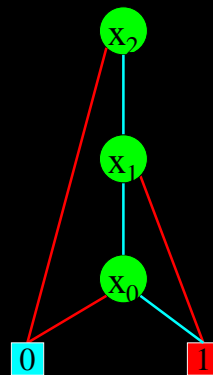
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BDD Representation of the Characteristic Function

$$F = \bar{x}_2 \bar{x}_1 \bar{x}_0$$

$$+ \bar{x}_2 x_1 \bar{x}_0$$

$$+ \bar{x}_2 x_1 x_0$$



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Char Function of Set of Subsets

- Let the set $S = \{p_1, p_2, \dots, p_n\}$ contain n elements. Assuming **positional notation**, the elements of the set are encoded using one variable per element
- **Example:** Suppose $n = 5$ and the subset is $\{p_1, p_3, p_4\}$. This subset can be represented by the characteristic function $\Phi_{\{p_1, p_3, p_4\}} = x_1 x_2' x_3 x_4 x_5'$
- **Example:** For the same set with $n = 5$, the set of all subsets containing exactly one element is represented by the characteristic function $\Phi_{S1} = x_1 x_2' x_3' x_4' x_5' + x_1' x_2 x_3' x_4' x_5' + x_1' x_2' x_3 x_4' x_5' + x_1' x_2' x_3' x_4 x_5' + x_1' x_2' x_3' x_4' x_5$

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Char Function of Tuples "k out of n"

- **Problem:** Given the set S of n elements, build a BDD for the characteristic function of the set of subjects containing exactly k out of n elements of set S
- The brute force approach results in creating the sum of $n!/k!(n-k)!$ cubes, each of which represents a characteristic function of a subset
- The intelligent approach uses a recursive BDD procedure to build the char function

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Building Char Function of Tuple Set

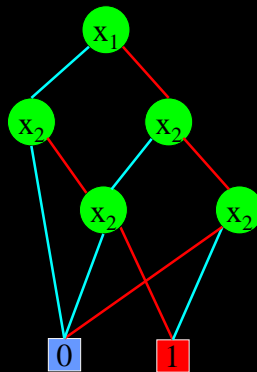
```
bdd Tuples( int k, int n)
{
  if ( k < 0 || n < k )      return bddfals;
  if ( k == 0 && n == 0 )   return bddtrue;
  // check cache for results
  bdd F0 = Tuples( k,  n-1 );
  bdd F1 = Tuples( k-1, n-1 );
  bdd Res = bdd_ite( bdd_ithvar(n), F1, F0 );
  // insert into cache
  return Res;
}
```

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BDD for Tuple(2, 3)



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Constraint Satisfaction Problems

- The parameters of SAT problems are represented using binary variables and the requirements for a solution are expressed as boolean formulas over the binary variables
- When all these formulas are multiplied, we get a BDD whose path lead to constant 1 for those assignments that satisfy the problem
- A SAT problem is solved by analyzing this BDD (for example, by counting the number of paths, or finding the shortest path)

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N-Queen Problem

- Is it possible to place n queens on a $n \times n$ chess board so that no queen can be captured by another queen? If yes, how many different placements are possible?
- Let us encode the presence/absence of a queen in each cell by a binary variable (altogether we need $n \times n$ binary variables)
- Now, the requirements of not capturing can be expressed in terms of these variables

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Encoding the Constraints

- There is no more than one queen in each row, each column, and each diagonal
 $X_{ij} \Rightarrow \prod(X_{ik'}), 1 \leq k \leq n, k \neq j; X_{ij} \Rightarrow \prod(X_{mj'}), 1 \leq m \leq n, m \neq i, \text{ etc.}$
- There is a queen in each row
 $X_{i1} + X_{i2} + \dots + X_{in}$
- After multiplying the constraint, we get the BDD representing all solutions
- **Example:** for 8 queens, there 2450 nodes, and 92 solutions
- **Reference:** S. Minato. "Calculation of Unate Cube Set Algebra Using ZBDDs", DAC'94.
<http://www.sigda.acm.org/Archives/ProceedingArchives/>

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Binare Covering Problem (BCP)

- Given a covering table and the objective function $\min[\sum_j(w_j x_j)]$, BCP is formulated as follows
- Find a subset S of columns of minimum cost (according to the objective function) such that for every row f_j either (1) $\exists_j: (f_{ij}=1) \ \& \ (F_j \in S)$
or (2) $\exists_j: (f_{ij}=0) \ \& \ (F_j \notin S)$
- This problem arises in incompletely-specified FSM state minimization, technology mapping, decomposition of functions with DCs, BDD minimization, etc.
- BCP is reduced to the problem of finding the shortest path on the BDD representing the product of all the constraints

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Example of BCP

x_1	x_2	x_3	x_4	x_5	x_6
1	-	1	-	-	-
-	1	-	1	-	1
-	-	0	1	1	-
-	-	-	-	-	0
0	-	-	-	-	0
-	-	1	0	1	-

- Solution 1:

$\{x_1, x_2\}$

- Solution 2:

$\{x_3, x_4\}$

BCP in State Minimization of ISFSMs

- Covering constraints

$$(c_1 + c_{11})(c_1 + c_2 + c_5)(c_2 + c_3 + c_5 + c_6 + c_7 + c_8)$$

$$(c_1 + c_2 + c_4 + c_6 + c_{10})(c_1 + c_4)(c_3 + c_7 + c_9 + c_{12})$$

$$(c_3 + c_8 + c_9 + c_{11})(c_4 + c_{10})$$

- Closure constraints

$$(c_2' + c_1)(c_2' + c_{11})(c_2' + c_1 + c_4)(c_3' + c_2 + c_6)$$

$$(c_3' + c_4)(c_4' + c_1)(c_4' + c_1)(c_6' + c_{11})(c_6' + c_1 + c_4)$$

$$(c_7' + c_2 + c_6)(c_8' + c_2 + c_6)(c_8' + c_3 + c_9)(c_9' + c_4)$$

- Solution

$$c_1 = c_4 = c_5 = c_9 = 1$$