

ECE 510 OCE
BDDs and Their Applications

Lecture 4. BDD-based
Representations

April 6, 2000
Alan Mishchenko

Overview

- BDD-based representation of functions, functions with dcs, relations, minterms, cubes, sets, sets of sets, state machines, partitions, set systems, graphs, covering tables, matrices - what else?:)
- Common features of all successful BDD-based representations
- Detailed discussion of project possibilities
- Selecting representation for your problem

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Incompletely Specified Functions

- Completely specified function (CSF) is a mapping $B^n \rightarrow B$, where $B = \{0,1\}$, $n > 0$
- Incompletely specified function (ISF) is a set of two CSFs, one represents ON-set (F1), while the other represents DC-set (FDC)
- Alternatively, an ISF can be represented by the interval $(F1, F2)$, where F1 is ON-set and F2 is the sum of ON-set (F1) and DC-set (FDC)

$$F1 \leq F \leq F2$$

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Relations

- Relation is a mapping $B^n \rightarrow B^m$, where $B = \{0,1\}$, $n > 0$, $m > 0$.
- If $m = 1$, a relation is a function
- For example, $F(x_1, x_2, y_1, y_2, y_3)$, $n = 2$, $m = 3$

x_1	x_2	y_1	y_2	y_3
0	0	1	-	0
0	1	0	1	-
1	0	1	1	0
1	1	0	1	-

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Relations Are Reducible to Functions

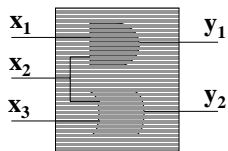
- Relation over variables (x_1, x_2, \dots, x_n) and (y_1, y_2, \dots, y_m) can be represented as a boolean function, which is 1 for a given minterm iff this minterm represents related assignments of variables (x_1, x_2, \dots, x_n) and (y_1, y_2, \dots, y_m) .

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Example



This is a relation with $n = 3$, $m = 2$

x_1	x_2	x_3	y_1	y_2
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

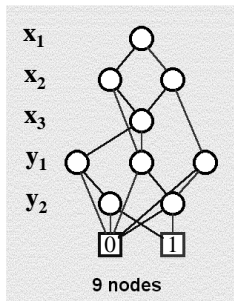
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Example (continued)

x_1	x_2	x_3	y_1	y_2	F
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	1	1	1
1	1	1	1	1	1
other					0



Minterms and Cubes

- Given a function $F(x_1, x_2, \dots, x_n)$, a product of *all* its variables in arbitrary polarities is a minterm
- A product of its variables, which does not necessarily include all variables, is a cube
- Each minterm is a cube; the reverse is not true

Implicit Cube Representation

- To represent cubes of n-variable function $F(x_1, x_2, \dots, x_n)$, two sets of n vars are used: signature variables $S = (s_1, s_2, \dots, s_n)$ and polarity variables $P = (p_1, p_2, \dots, p_n)$
 s_i variable is true iff i-th variable is present in the cube; p_i variable is true iff i-th variable enters this cube in positive polarity
- For example, cube $x_2x_3'x_4$ is represented by the pair [(0111), (1101)]

BDD-based Representation of Relations Between Cubes

- Characteristic function of the cube is $\chi(s,p)(x) = \prod_k (s_k \Rightarrow (x_k = p_k))$
- Cubes $C_1(s_1,p_1)$ and $C_2(s_2,p_2)$ are identical iff $\prod_k [(s_{1k} = s_{2k}) \ \& \ (s_{1k} \Rightarrow (p_{1k} = p_{2k}))]$
- Cube $C_1(s_1,p_1)$ contains cube $C_2(s_2,p_2)$ iff $\prod_k [s_{1k} \Rightarrow (s_{2k} \ \& \ (p_{1k} = p_{2k}))]$

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Characteristic Function of a Set

- Function $F: B^n \rightarrow B, B = \{0,1\}$, defines a subset of minterms of B^n , on which it is 1.
- Given a binary encoding of a set of elements, characteristic function of a *subset* of this set is a boolean function, which is 1 for minterms encoding the subset and 0 for other minterms.

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Example

- Problem: Given the set $\{p_0, p_1, p_2, p_3, p_4, p_5\}$ and its encoding:
 $p_0 - \bar{x}_2 \bar{x}_1 \bar{x}_0 \quad p_2 - \bar{x}_2 x_1 \bar{x}_0 \quad p_4 - x_2 \bar{x}_1 \bar{x}_0$
 $p_1 - \bar{x}_2 \bar{x}_1 x_0 \quad p_3 - \bar{x}_2 x_1 x_0 \quad p_5 - x_2 \bar{x}_1 x_0$
 find characteristic function of subset $\{p_0, p_2, p_3\}$ and represent the subset using BDD
- Solution: Define a function over the encoding variables (x_0, x_1, x_2) such that it is equal to 1 for minterms representing subset $\{p_0, p_2, p_3\}$.

$$\Phi_{\{p_0, p_2, p_3\}}(x_0, x_1, x_2) = \bar{x}_2 \bar{x}_1 \bar{x}_0 + \bar{x}_2 x_1 \bar{x}_0 + \bar{x}_2 x_1 x_0$$

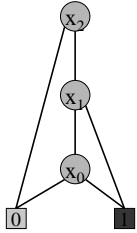
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BDD Representation of the Characteristic Function

$$\begin{aligned} \mathbf{F} &= \bar{x}_2 \bar{x}_1 \bar{x}_0 \\ &+ \bar{x}_2 x_1 \bar{x}_0 \\ &+ \bar{x}_2 x_1 x_0 \end{aligned}$$



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Set Manipulation

Operations on sets can be reduced to boolean operations on their characteristic functions

Empty set: $\chi_{\emptyset} = 0$

Union of sets: $\chi_{S \cup T} = \chi_S + \chi_T$

Intersection of sets: $\chi_{S \cap T} = \chi_S \& \chi_T$

Difference of sets: $\chi_{S - T} = \chi_S \& (\chi_T)'$

Subset relation ($S \subset T$): $\chi_{S - T} = \chi_S \& (\chi_T)' = 0$

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Sets of Sets

- If BDD representation of sets is known, let us apply it two times:
 - first, to represent elements of a set
 - next, to represent sets themselves
- To do this, we first encode the elements of a set; next encode sets themselves
- The simplest way to do this is to use a unique binary code for each set and represent it using labeling variables

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FSMs: Transition Relation

- FSM is $\{ I, O, S, \delta, \lambda \}$
(r inputs, m states, n outputs)
- Transition relation is a boolean function
 $T: B^r \times B^m \times B^m \rightarrow B, B = \{0,1\}$
such that $T(i, x, y) = 1$ iff state y can be reached in one transition from state x when input i is applied

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FSMs: Output Relation

- Output relation is a boolean function
 $O: B^r \times B^m \times B^n \rightarrow B$
such that $O(i, x, o) = 1$ iff output o can be produced when the FSM is in state x and input i is applied

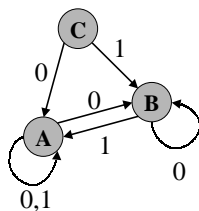
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Example

Ins	CS	code	NS	code
0	A	00	B	10
0,1	A	00	A	00
0	B	10	B	10
1	B	10	A	00
0	C	01	B	10
1	C	01	A	00



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Example (continued)

Transition relation

$$T(l, a_1, a_2, b_1, b_2) = i$$

$$i'a_1'a_2'b_1'b_2' + a_1$$

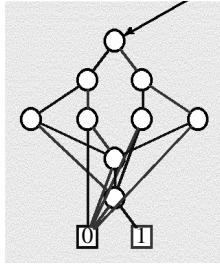
$$a_1'a_2'b_1'b_2' + b_1$$

$$i'a_1a_2'b_1'b_2' + a_2$$

$$ia_1a_2'b_1'b_2' + b_2$$

$$i'a_1a_2'b_1'b_2'$$

$$ia_1a_2'b_1'b_2'$$



Equivalence Relation

- Equivalence relation is a boolean function $E(x_1, x_2)$ such that $E(x_1, x_2) = 1$, iff x_1 and x_2 are equivalent.
- Equivalence relation is
 - reflexive: $\forall x E(x, x) = 1$
 - symmetric: $\forall x, y E(x, y) = E(y, x)$
 - transitive: $\forall x, y, z [E(x, y) \& E(y, z)] \Rightarrow E(x, z)$

Partitions

- Partition π on a set of elements S is a collection of disjoint subsets of S , whose set union is S
- It is written: $\pi = \{B_a\}$ such that
 - $B_a \cap B_b = \emptyset, \forall a \neq b$
 - $\cup\{B_a\} = S$
- Often used partitions are partitions of states of FSM $M = \{S, I, O, \delta, \lambda\}$ satisfying certain properties, for example, substitution property:

$$\forall s, t \in S: s \equiv t(\pi) \Rightarrow \forall a \in I \delta(s, a) \equiv \delta(t, a)(\pi)$$

Equivalence Relations and Partitions

- There is a one-to-one correspondence between equivalence relations and partitions
- Each equivalence relation corresponds to a partition of objects, such that each block of the partition consists of objects equivalent with respect to the given equivalence relation, and vice versa

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Example

- Problem: Given the set $\{p_0, p_1, p_2, p_3, p_4, p_5\}$ and its encoding:

$$p_0 - \bar{x}_2 \bar{x}_1 \bar{x}_0 \quad p_2 - \bar{x}_2 x_1 \bar{x}_0 \quad p_4 - x_2 \bar{x}_1 \bar{x}_0$$

$$p_1 - \bar{x}_2 \bar{x}_1 x_0 \quad p_3 - \bar{x}_2 x_1 x_0 \quad p_5 - x_2 \bar{x}_1 x_0$$

find characteristic function of partition

$\pi = \{ \{p_0, p_1\}, \{p_2, p_3, p_4\}, \{p_5\} \}$ and represent it using BDDs

- Solution: Define an equivalence relation $E(x_0, x_1, x_2, y_0, y_1, y_2)$ such that it is equal to 1 only for those pairs of minterms that correspond to codes of objects belonging to some block.

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Example (continued)

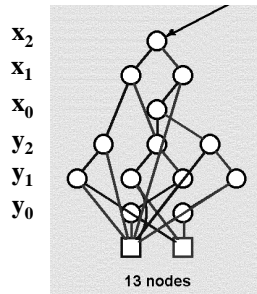
XY	000	001	010	011	100	101	110	111
000	1	1	0	0	0	0	0	0
001	1	1	0	0	0	0	0	0
010	0	0	1	1	1	0	0	0
011	0	0	1	1	1	0	0	0
100	0	0	1	1	1	0	0	0
101	0	0	0	0	0	1	0	0
110	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0

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Implicit Representation of Partition π



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Implicit Representation of Matrices

- Suppose each cell of the matrix contains one of a finite number of entries
- Encode columns and rows of the matrix using minterms composed of variables $X=(x_0, x_1, \dots, x_n)$ and $Y=(y_0, y_1, \dots, y_m)$
- For each type of entry, create a relation $M_i(X, Y)$ such that $M_i(X_1, Y_1)=1$, iff the matrix has an entry of this type on the intersection of column X_1 and row Y_1

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Properties of BDDs

- Canonicity
- Compactness (with some exceptions)
- Fast computation (with some exceptions)
- Represent a variety of discrete objects
 - Boolean functions and relations
 - Compositional sets and sets of sets
 - Partitions of states
 - Graphs and matrices
- Facilitate symbolic methods
 - Two-level minimization
 - State traversal of FSMs, etc.

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