

ECE 510 OCE BDDs and Their Applications

Lecture 2. Foundations

March 30, 2000

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Overview

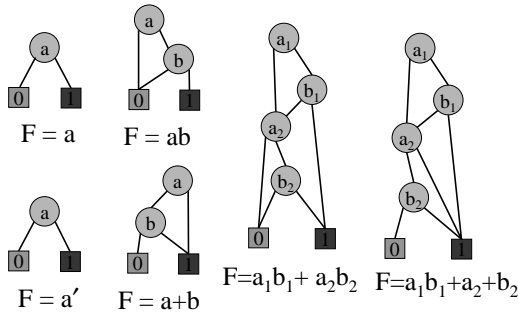
- Representations of boolean functions
- Canonical and non-canonical forms
- Decision trees and reduction rules
- Building already reduced diagrams
- Zero-Suppressed BDDs
- BDDs with complement edges
- Dynamic variable reordering

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2

Examples



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3

Known Representations

- Truth table
- SoP (DNF) and PoS (CNF)
- Decision tree
- Reduced decision tree
 - BDD
 - Zero-Suppressed BDD
 - BDD with complement edges

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4

Truth Table, SoP, and PoS

x_1	x_2	x_3	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- Sum-of-products

$$F = x_1'x_2x_3 + x_1x_2'x_3 + x_1x_2x_3$$

- Product-of-sums

$$F = (x_1 + x_2 + x_3) \& (x_1 + x_2 + x_3') \& (x_1 + x_2' + x_3) \& (x_1' + x_2 + x_3) \& (x_1' + x_2' + x_3)$$

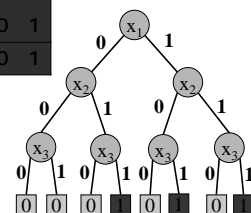
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5

Truth Table and Decision Tree

x_1	0	0	0	0	1	1	1	1
x_2	0	0	1	1	0	0	1	1
x_3	0	1	0	1	0	1	0	1
F	0	0	0	1	0	1	0	1



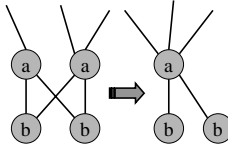
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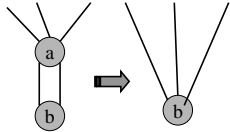
6

Reduction of Decision Tree

Rule 1: Merging Rule:
Nodes must be unique



Rule 2: Elimination Rule:
Redundant tests should not be present

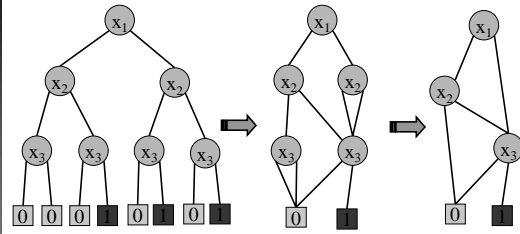


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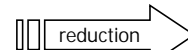
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7

Example of Decision Tree Reduction



Decision Tree



BDD

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8

Shannon Expansion

- A boolean function can be expanded with respect to any variable
- $$F(x, y, z) = x' \& F_{x'} + x \& F_x$$
- where $F_{x'}$ and F_x are positive (negative) cofactors
- $$F_{x'} = F(0, y, z), F_x = F(1, y, z)$$

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9

Building Already Reduced Tree

Build a reduced tree for F by calling **Build**(F , 1),
if variable ordering $x_1 < x_2 < x_3 < \dots < x_n$ is given

```
function Build ( F, i )
if ( i > n )
    if ( F == 0 ) return Node0
    else return Node1;
else
    v0 = Build ( Fx', i+1 );
    v1 = Build ( Fx, i+1 );
    return CreateNode ( i, v0, v1 );
```

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10

Creating a Unique Node

assuming there is a node table with
functions *CheckExists()* and *Insert()*

```
function CreateNode( Var, Fx', Fx )
if ( Fx' == Fx )
    return Fx';
else if ( CheckExists( Var, Fx', Fx ) )
    return the existing node;
else Insert( Var, Fx', Fx );
    return the new node;
```

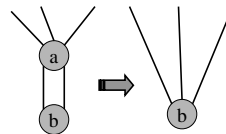
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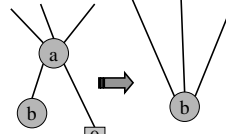
11

Zero-Suppressed BDDs

Rule 2: Elimination Rule:
Redundant tests should not be present



Rule 2: Elimination Rule:
Remove nodes with 1-edge pointing to 0-terminal



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12

Complement Edges

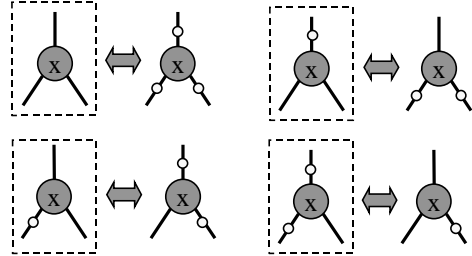
- BDD edges can carry attributes
- Using complementation as an attribute for edges saves BDD nodes and makes NOT a constant time operation
- To maintain canonicity, the use of attribute edges should be constrained

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13

Equivalent Pairs of Functions



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14