

of the evaluation phase. The results are summarised in Table 1 and show the proper behaviour of the ternary inverter.

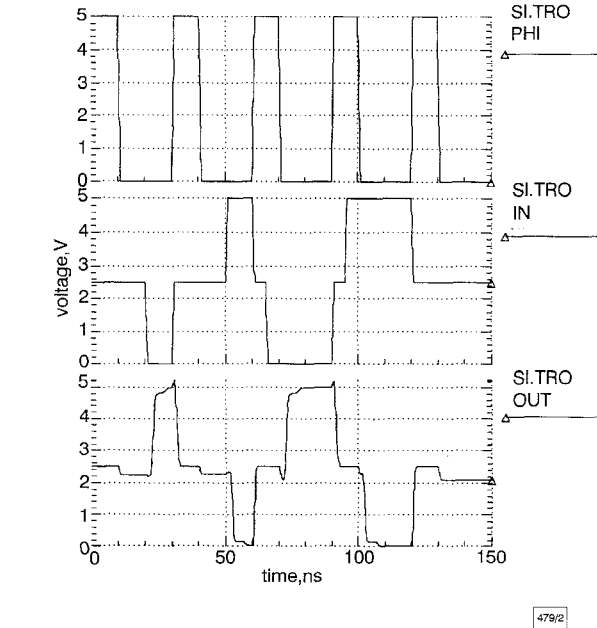


Fig. 2 Simulation of circuit
Signals *Phi*, *IN* and *OUT* are reported

Table 1: Simulation results

	Last signal to change	
	<i>Phi</i>	<i>IN</i>
t_{th}	ns	ns
t_{fl}	2.5	2.6
t_{hi}	2.0	2.1
t_{li}	1.3	
	3.5	

t_{th} (t_{fl}) = delay of gate for raise (fall) transition of output;
 t_{hi} and t_{li} are delay times for output to return to V_i from V_{dd} and gnd , respectively

Table 2: Truth table of three unary gates introduced

	STI	NTI	PTI
0	2	2	2
1	1	0	2
2	0	0	0

Logic design: To make it possible to realise any ternary function, at least two other unary gates are needed [1, 4]; the positive ternary inverter (PTI) and the negative ternary inverter (NTI) (Table 2). The circuit for the PTI gate is shown in Fig. 3; the NTI has a similar scheme. Their structures are easily derived from that in Fig. 1 and their behaviour is straightforward; it is important to note that the output is always brought to logic 1 during the pre-charge phase, to avoid charge sharing in cascaded gates.

The same structure used to realise the three inverters gives the ability to construct complex gates like the binary AND-OR-INVERT gate. Indeed, the scheme of the ternary inverter is very similar to the binary CMOS inverter, if the cluster of $Q1 - 4$ (or, similarly, $Q5 - 8$) is considered as a single switching element. This makes it easy to realise other ternary functions, combining the switches as in binary logic. These few circuits allow the use of the synthesis method presented in [1] for the implementation of any ternary function.

Conclusions: A new kind of circuit for ternary logic has been proposed. The most significant feature of it is that, using a standard CMOS process, only three supply voltages are needed (V_{dd} , $V_{dd}/2$ and gnd) and the logic levels do not suffer any degradation, allowing the maximum possible noise margins. The design of the ter-

nary inverters and more complex gates is easy and no static current flows through the gates, avoiding static power dissipation.

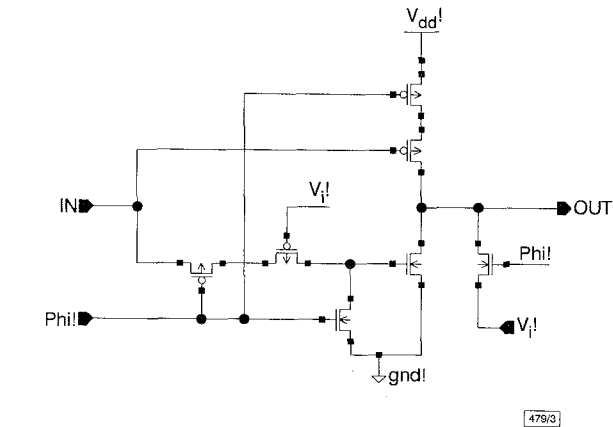


Fig. 3 Circuit scheme of positive ternary inverter

As compared to the dynamic logic presented in [4], this new structure eliminates many problems: no process customisation and body effect induced threshold variations are needed, and only three power supplies instead of four are used. The only disadvantage of the new logic is the higher number of transistors needed.

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Forward and inverse transformations between Haar wavelet and arithmetic functions

B.J. Falkowski

Mutual conversions between Haar wavelet and arithmetic transforms are presented. The new relations allow direct calculation of an arithmetic spectrum from a Haar wavelet spectrum and vice versa without the need to back the original function. As both arithmetic and Haar wavelet transforms are used widely in many areas, these results should further increase the scope of their applications.

Introduction: Mutual relations between different transforms have been investigated for many years. It is known, for example that the Walsh transform is a special case of an abstract Fourier transform [1, 2] and that Walsh functions can be expressed either by Rademacher functions [1 - 3], or Haar functions [2, 3] or they can be defined in a recursive way by the Kronecker product of Hadamard matrices [1 - 4]. It is frequently useful to calculate the spectrum of a function by means of some other known spectrum of the function without needing to regain the original function. For example, it is possible to directly convert Walsh and Haar spectra [2, 5].

Both the Haar wavelet transform (non-normalised version of the transform where only signs are entered into the transform matrix) and the arithmetic transform have been used in many applications of logic design [1, 2, 6, 7]. Therefore it is interesting not only theoretically, but also practically, to state their mutual relations, as presented in this Letter. As the arithmetic transform is related to the Reed-Muller transform, and the latter has been used in image processing [8], one can expect applications of the arithmetic transform in image processing as well.

Arithmetic and Haar transforms: The matrix of order $N = 2^n$ for the arithmetic transform is defined as [6, 7]

$$A_N = \begin{bmatrix} A_{N/2} & 0 \\ -A_{N/2} & A_{N/2} \end{bmatrix} \quad A_1 = 1 \quad N = 2, 3, \dots$$

Also $A_N = A_2 \otimes A_{N/2}$ for $N = 2, 3, \dots$

The non-normalised Haar transform H_N of order $N = 2^n$ can be defined recursively as [2, 3]

$$H_N = \begin{bmatrix} H_{N/2} \otimes [1 \ 1] \\ I_{N/2} \otimes [1 \ -1] \end{bmatrix} \quad \text{and } H_1 = 1$$

where $I_{N/2}$ is an identity matrix of the order of $N/2$. In the above equations, the symbol ' \otimes ' denotes the right-hand Kronecker product.

For an n -variable Boolean function $F(x_1, x_2, \dots, x_n)$, the Haar and arithmetic spectra (a column vector of dimension $2^n \times 1$) is given by $H = [H_N]F$ and $A = [A_N]F$ where H is the Haar spectrum and A is the arithmetic spectrum. From the above definitions it is obvious that the first two rows of $[H_N]$ are the global basis functions $H_0(x)$ and $H_1(x)$, respectively. All subsequent rows are constituted by the local basis functions $H_l^{(k)}(x)$ in ascending order of l and k . $l = 1, 2, \dots$ is known as a degree of the Haar function describing the number of zero crossings, and $k = 1, \dots, 2^l$ is an order of the Haar function describing the position of the subset l within a function. In the arithmetic transformation matrix $[A_N]$, all but the last row are local basis functions.

Mutual relations between Haar and arithmetic functions: The first set of relations will be given for arithmetic functions and non-normalised Haar wavelet functions, as both have applications in logic design. As we anticipate the application of the arithmetic transform in other areas, the relations between arithmetic functions and standard Haar functions will also be given. The way in which the basic functions are entered into the transform matrix is called an ordering of the transform. There are many possible orderings for both transforms [1, 2, 6, 7]. Our derivations are independent of ordering, since by applying a permutation matrix, the presented equations can be expressed for any ordering. The value of each basis orthogonal function does not depend on their ordering in the transform matrix as long as the proper subscripts and superscripts of the basis functions remain constant during the permutation. However, to make this Letter consistent with other presentations, Hadamard ordering is used for both transforms. A_i denotes an i th arithmetic function and the Haar functions follow the definition from the previous paragraph. Kaczmarz gave the definition of Walsh functions by Haar wavelet functions for the first eight functions ($n = 3$) [3]. The definitions for higher values of n can be derived in a similar way. Here we will give similar definitions to express arithmetic functions by non-normalised Haar functions and vice versa.

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_{12} \\ A_3 \\ A_{13} \\ A_{23} \\ A_{123} \end{bmatrix} = \frac{1}{8} \times \begin{bmatrix} 1 & 1 & 2 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & -4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & -8 & 0 & 0 \\ 0 & -2 & -2 & 2 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & -8 & 0 \\ 0 & 0 & 4 & -4 & 4 & -4 & -4 & 4 \\ 0 & 0 & 0 & 0 & -8 & 8 & 8 & -8 \end{bmatrix} \times \begin{bmatrix} H_0 \\ H_1 \\ H_2^{(1)} \\ H_2^{(2)} \\ H_3^{(1)} \\ H_3^{(2)} \\ H_3^{(3)} \\ H_3^{(4)} \end{bmatrix}$$

On the other hand, the non-normalised Haar functions can be obtained in terms of the arithmetic functions as follows:

$$\begin{bmatrix} H_0 \\ H_1 \\ H_2^{(1)} \\ H_2^{(2)} \\ H_3^{(1)} \\ H_3^{(2)} \\ H_3^{(3)} \\ H_3^{(4)} \end{bmatrix} = \begin{bmatrix} 8 & 4 & 4 & 2 & 4 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & -4 & -2 & -2 & -1 \\ 0 & 0 & -2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 & 0 & -2 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_{12} \\ A_3 \\ A_{13} \\ A_{23} \\ A_{123} \end{bmatrix}$$

When the relations between normalised Haar functions and arithmetic functions are investigated, the general form of the above equations remain similar. However, when arithmetic functions are expressed in terms of Haar functions the difference is the division by $\sqrt{2}$ of each Haar function of the second group, and by 2 of each Haar function of the third group. For example, the first equation for normalised Haar functions would read

$$A_0 = H_0 + H_1 + 2/\sqrt{2}H_2^{(1)} + 4/2H_3^{(1)} \\ = H_0 + H_1 + \sqrt{2}H_2^{(1)} + 2H_3^{(1)}$$

The normalised Haar functions can be expressed by similar relations to the ones presented for non-normalised Haar functions, taking into account that each Haar function of the second group has to be divided by $\sqrt{2}$ and of the third group by 2 (as previously stated). For example, $H_2^{(2)}/\sqrt{2} = -2A_2 - A_{12} - 2A_{23} - A_{123}$ and $H_3^{(1)}/2 = -A_1$, etc. When the relations for higher groups of normalised Haar functions and arithmetic functions are investigated, such relations can be obtained from similar equations for non-normalised Haar functions by dividing the non-normalised Haar functions (starting from group 1) by a number equal to the number of square roots multiplied n times, where n is the group number of Haar functions. Hence for $n = 1$ we have division by $\sqrt{2}$, for $n = 2$ by 2, for $n = 3$ by $2/\sqrt{2}$, etc. The basic equations for higher groups are not given here since the method for their generation is obvious from the equations already presented. As for the recursive definitions for Grey code ordered Walsh functions, higher order matrices can be obtained by using operations of shift and copy [4].

Conclusion: In this Letter, new mutual relations between Haar wavelet and arithmetic transforms are presented. Since the equations are given for both types of Haar function, the presented derivations may be useful for applications of both transforms in areas other than logic design. On the other hand, applications of the arithmetic transform in logic design are much wider than those of the Haar transform. With the equations presented in this Letter, it is possible to transfer the known results of spectral logic design in the arithmetic domain [6, 7] to the Haar domain, and to compare the efficiency of both approaches in different applications.

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