ELectronics EXpress

IEICE Electronics Express, Vol.1, No.8, 211–216

Generation of multi-polarity helix transform over GF(3)

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Abstract: In this article, the new multi-polarity helix transform for ternary logic functions has been introduced. In addition, an extended dual polarity property that had been used to optimize Kronecker and quaternary Fixed-Polarity Reed-Muller (FPRM) expressions has been applied to generate efficiently the multi-polarity helix transform over GF(3). The experimental results for the transform are compared with the well known ternary Reed-Muller transform and it was found that the helix transform is quite efficient in terms of non-zero spectral coefficients and corresponding memory storage.

Keywords: Galois Field (3), multi-polarity helix transform, Reed-Muller.

Classification: Science and engineering for electronics

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DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
Published July 25, 2004

ELectronics EXpress

IEICE Electronics Express, Vol.1, No.8, 211–216

1 Introduction

Reed-Muller (RM) transform that represents an important class of AND-EXOR expressions had been successfully applied in many areas such as signal processing, fault detection and coding techniques, especially those concerned with group or block codes for error control [1]. Fixed-Polarity RM (FPRM) is the RM polynomial expansion in which each variable has the same form [1, 2, 3]. An n-variable ternary logic function can be expressed by 3^n different FPRM polynomial expansions, where each of them is canonical and can be differentiated from each other by its polarity number. FPRM polynomial expansions with different polarity numbers generally possess different computational complexity, which is measured by the number of nonzero spectral coefficients or the number of literals in the FPRM polynomial expansion. The polarity number, for which the number of the used computational complexity measure is smallest, is called the optimal polarity number.

A method that optimizes binary FPRM based on the relationship between two FPRMs was extended for the optimization of Kronecker expressions by introducing the term extended dual polarity in [4] and for the optimization of FPRM expressions over Galois Field (4) (GF(4)) in [5] as well as over Galois Field (5) (GF(5)) in [6]. As mentioned in [4], Kronecker expansions are potentially better than FPRMs in optimization of logic functions if the criterion is the number of non-zero terms. Therefore it is interesting to find novel Kronecker based transforms that are efficient in final polynomial representations of logic functions, have nice properties as well as can be calculated in efficient way what was done in [7].

In this article, the new ternary multi-polarity transform based on Kronecker product is introduced that is named helix transform [7] due to the symmetrical structure along the diagonal or reverse-diagonal in the transform matrices. Application of extended dual polarity for the efficient optimization of multi-polarity helix transform over GF(3) is also introduced. Experimental results showing big advantage in terms of non-zero spectral coefficients of the new multi-polarity helix transform when compared with ternary Reed-Muller transform are also presented in this paper.

2 Basic definitions

Definition 1. Let $\overrightarrow{F} = [F_0, F_1, ..., F_{3^n-1}]^T$ represent a column vector defining the truth vector of a ternary function $f\,(\overrightarrow{x_n})$ in a natural ternary ordering. The helix transform H_n is an $N \times N$ ($N = 3^n$) matrix with rows corresponding to minterms and columns corresponding to some switching ternary functions of n variables. If the sets of rows are linearly independent with respect to *ternary Galois Field*, then H_n has only one inverse in GF(3). The truth vector can be obtained by the following equation,

$$\overrightarrow{F} = H_n \overrightarrow{A}, \tag{1}$$

where $\overrightarrow{A} = [A_0, A_1, ..., A_{3^n-1}]^T$ is the spectral coefficient column vector for the particular transform matrix H_n. The spectral coefficient column vector

DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
Published July 25, 2004

IEICE Electronics Express, Vol.1, No.8, 211–216

can be reconstructed by the following equation,

$$\overrightarrow{A} = H_n^{-1}\overrightarrow{F}. \tag{2}$$

Definition 2. Let X be a matrix of order $m \times n$ and Y be a matrix of order $p \times q$, then their ternary Kronecker product, denoted by $Z = X \otimes Y$ that is executed over GF(3) is a partitioned matrix Z of order $(mp) \times (nq)$ defined as the following equation, where all the matrix elements belong to set {0,1,2}.

$$Z = X \otimes Y = \begin{bmatrix} x_{11}Y & x_{12}Y & \ldots & x_{1n}Y \\ x_{21}Y & x_{22}Y & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ x_{m1}Y & \ldots & \ldots & x_{mn}Y \end{bmatrix}. \tag{3}$$

Definition 3. The ternary helix transform matrix of size $N \times N$ $(N = 3^n)$ is created by applying $n-1$ times ternary Kronecker product to basic helix transform of size 3×3 where such basic ternary helix transform matrices fulfill Definition 1.

$$H_n = \overset{n-1}{\otimes} H_1;$$

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } H_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

3 Multi-polarity helix transform

In this section, the new ternary multi-polarity helix transform will be presented whose all matrices fulfill Definitions 1-3.

Let $H_n^{<k>}$ represent the k-th polarity of the transform H_n. For any number of variables, the generalized multi-polarity helix transform matrix $H_n^{<k>}$ is recursively defined as

$$H_n^{<k>} = \overset{n-1}{\otimes} H_1^{<k>}.$$

In general,

$$H_n^{<k=k_0k_1\ldots k_{n-1}>} = H_1^{<k_0>} \otimes H_1^{<k_1>} \otimes \ldots \otimes H_1^{<k_{n-1}>}.$$

All the elements in $H_n^{<k>}$ belong to the set {0, 1, 2}. The ternary multi-polarity helix transform $H_n^{<k>}$ can be derived from $H_n^{<0>}$ using 3-adic shift in the columns of $H_n^{<0>}$.

Similarly, the inverse transform matrix $(H_n^{-1})^{<k>}$ can be obtained by

$$(H_n^{-1})^{<k>} = \overset{n-1}{\otimes} (H_1^{-1})^{<k>}.$$

In general,

$$(H_n^{-1})^{<k=k_0k_1\ldots k_{n-1}>} = (H_1^{-1})^{<k_0>} \otimes (H_1^{-1})^{<k_1>} \otimes \ldots \otimes (H_1^{-1})^{<k_{n-1}>}.$$

For any number of variables, $(H_n^{-1})^{<k>}$ can be obtained from $(H_n^{-1})^{<0>}$, also by using 3-adic shift in the rows of $(H_n^{-1})^{<0>}$. Table I gives all the polarities of forward and inverse multi-polarity helix transform matrix when $n = 1$.

DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
Published July 25, 2004

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Table I. Ternary multi-polarity helix transform

	Forward	Inverse
$x^{<0>} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$	$H^{<0>} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$(H^{-1})^{<0>} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
$x^{<1>} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$	$H^{<1>} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$(H^{-1})^{<1>} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \end{bmatrix}$
$x^{<2>} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$	$H^{<2>} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$	$(H^{-1})^{<2>} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

4 Generation of multi-polarity helix transform

The notion of extended dual polarity property was introduced in [4, 5] and used for optimization of FPRM over GF(4) [5] and GF(5) [6] as well as Kronecker expressions [4]. In this section, it will be shown that the extended dual polarity property also applies to the ternary multi-polarity helix transform. The relationships between two helix transforms for ternary functions of dual polarities are also derived.

Definition 4. $P' = (p'_1, ..., p'_{i-1}, p'_i, p'_{i+1}, ...p'_n)$ is extended dual polarity for the given polarity $P = (p_1, ..., p_{i-1}, p_i, p_{i+1}, ...p_n)$ if $p'_j = p_j$, $j \neq i$ and $p'_i \neq p_i$.

Example: For polarity $P = (1,\ 2)$, the extended dual polarities are (0, 2) and (1, 0).

The relationship between spectra of two multi-polarity helix transforms is given by the following equation

$$\overrightarrow{A^{<p'>}} = (H_n^{-1})^{<p'>} \cdot \overrightarrow{F} = (H_n^{-1})^{<p'>} \cdot H_n^{<p>} \cdot \overrightarrow{A^{<p>}}.$$

If polarities p' and p are extended dual polarities as described in Definition 4, and $p'_i \neq p_i$, then the relationship can be presented as

$$\begin{aligned}\overrightarrow{A^{<p'>}} &= (H_n^{-1})^{<p'>} \cdot \overrightarrow{F} \\ &= \left(\left(\bigotimes_{j=1}^{i-1} (H_n^{-1})^{<p_j>}\right) \otimes (H_n^{-1})^{<p'_i>} \otimes \left(\bigotimes_{j=i+1}^{n} (H_n^{-1})^{<p_j>}\right)\right) \cdot \\ &\left(\left(\bigotimes_{j=1}^{i-1} H_n^{<p_j>}\right) \otimes H_n^{<p'_i>} \otimes \left(\bigotimes_{j=i+1}^{n} H_n^{<p_j>}\right)\right) \cdot \overrightarrow{A^{<p>}}\end{aligned}$$

Due to the properties of Kronecker product, the relation can be rewritten as

$$\overrightarrow{A^{<p'>}} = \left(I_i \otimes (H_n^{-1})^{<p'>} \cdot H_n^{<p>} \otimes I_{n-i}\right) \cdot \overrightarrow{A^{<p>}},$$

where I_k is identity matrix of order k.

It is clear that for any spectrum of polarity p, its dual polarity spectrum vector $\overrightarrow{A^{<p'>}}$ can be calculated very efficiently by only calculating the results of multiplication between two helix transforms when $n = 1$. Table II gives all possible matrix products for $(H_n^{-1})^{<p'>} \cdot H_n^{<p>}$.

DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
Published July 25, 2004

IEICE Electronics Express, Vol.1, No.8, 211–216

Table II. Relationships between different polarities

$\left(H^{-1}\right)^{<0>} \cdot H^{<1>} = \left(H^{-1}\right)^{<1>} \cdot H^{<2>} = \left(H^{-1}\right)^{<2>} \cdot H^{<0>} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
$\left(H^{-1}\right)^{<0>} \cdot H^{<2>} = \left(H^{-1}\right)^{<1>} \cdot H^{<0>} = \left(H^{-1}\right)^{<2>} \cdot H^{<1>} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

5 Experimental results

In this section, the ternary multi-polarity helix transform has been implemented using Microsoft Visual C++ and run on PIII 500 MHz computer with 128 MB RAM. It is run on some benchmark functions that have been modified to represent ternary functions instead of original MCNC and IWLS'93 binary benchmark functions [7]. The translation from binary to ternary cases has been done by changing every 2 input (output) bits in binary files to an input (output) symbol in ternary files. If the number of input and/or output variables is odd, then a zero bit is added behind the binary cubes to make it even. For input (output), -- is taken as -, 00 is taken as 0, 01 is taken as 1 and 10 is taken as 2, whereas 11 is not used (taken as 0) for ternary case.

Table III presents the experimental results on some ternary benchmark functions. The third column shows the number of non-zero spectral coefficients for the best polarity ternary helix transform. They are compared with the experimental results for the best polarity ternary Reed-Muller transform that are given in the fourth column of Table III. The used ternary benchmark functions have more than one output. It can be seen that for the functions shown in the first column with their outputs pointed out in the second column, the ternary multi-polarity helix transform can obtain smaller number of non-zero spectral coefficients after searching out the best polarity for majority of cases when compared with the best polarity ternary Reed-Muller transform. The last column of this table also shows the decreasing and sometimes increasing rates in the number of non-zero spectral coefficients when the best polarity ternary Reed-Muller transform is compared with the best polarity helix transform.

DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
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Table III. Number of non-zero spectral coefficients

Ternary functions	Output	$H^{-1}_{<best>}$	$TRM^{-1}_{<best>}$	Decrease rate
apex4	O_1	22	51	56.9%
	O_2	63	78	19.2%
	O_3	63	77	18.2%
	O_4	62	79	21.5%
	O_5	59	79	25.3%
	O_6	77	76	−1.3%
	O_7	60	85	29.4%
	O_8	69	82	15.9%
	O_9	23	52	55.8%
	O_{10}	37	51	27.5%
clip	O_1	63	62	−1.6%
	O_2	88	75	−17.3%
	O_3	38	41	7.3%
ex1010	O_1	112	133	15.8%
	O_2	103	134	23.1%
	O_3	105	136	22.8%
	O_4	105	127	17.3%
	O_5	107	137	21.9%
inc	O_1	20	17	−17.6%
	O_2	28	30	6.7%
	O_3	16	22	27.3%
	O_4	21	23	8.7%
	O_5	7	2	−250%

6 Conclusions

In this article, novel Kronecker based transform called ternary multi-polarity helix transform has been introduced which has very regular structure and thus resulting efficient calculation. Kronecker based extended dual polarity properties [4, 5, 6] are revised and applied for efficient calculation of ternary multi-polarity helix transform. The presented properties and relationships give an efficient method to optimize the corresponding spectral polynomial expansions based on multi-polarity helix transform of any ternary function. The comparison of experimental results between ternary multi-polarity helix transform and ternary Reed-Muller transform are also discussed, and they show that for almost all the cases of ternary benchmark functions, our new ternary multi-polarity helix transform is more efficient than ternary Reed-Muller transform in terms of bigger number of zero spectral coefficients.

DOI: 10.1587/elex.1.211
Received June 08, 2004
Accepted July 05, 2004
Published July 25, 2004