

CS 581: Theory of Computation
James Hook
Final exam.

This is a closed-notes, closed-book exam.

1. If at first you don't succeed...
 - (a) [10 points] Define a PDA that recognizes palindromes (strings that read the same written forward and backward, such as "Wasilla's all I saw" or "a man a plan a canal panama") by final state and empty stack.
 - (b) [15 points] Show that your machine accepts the following strings.
 - i. ϵ
 - ii. 010
 - iii. 0110
2. [25 points] Prove that the set of palindromes is not regular.
3. [25 points] From first principles prove that A_{TM} is undecidable.
4. [25 points] In the proof of the incompleteness theorem we needed the notion of representability to show that there must be a formula in the theory of arithmetic that corresponds to the Kleene T predicate (or equivalently Sipser's $\phi_{M,w}$).
 - (a) [5 points] Sketch the definition of representability.
 - (b) [10 points] Discuss how representability is used in the incompleteness argument presented in class. Address the questions:
 - i. Is it needed to argue that truth is undecidable?
 - ii. Is it needed to formulate an unprovable sentence?
 - (c) [10 points] In class we sketched an induction proof that all the partial recursive functions were representable. Show that if g is a representable k -ary function and h_1, \dots, h_k are representable l -ary functions that $f = go[h_1, \dots, h_k]$ is representable.