CS 581: Theory of Computation
Spring 2009
Mid-term exam
James Hook

This is a closed-notes, closed-book exam.

1. [25 points]

   (a) Build a DFA to recognize the decimal numbers divisible by 3. You may regard ϵ as a representation of 0.

   (b) Recall the concept of indistinguishable elements in a language from the Myhill Nerode theorem. For the language described in (a), group the single digit numbers into the minimal number of sets of indistinguishable elements. That is, you should have some number of sets, each of which contains elements that are indistinguishable. Any pair of elements that are drawn from distinct sets should be distinguishable.

   (c) Extend the construction in the previous section from a partition of the single digit numbers to a partition of all strings over the digits 0 through 9 into maximal sets of indistinguishable elements.

   (d) What is the index of the language of decimal numbers divisible by 3?

   (e) Is your answer to part (a) minimal? Justify your answer.

2. [25 points] Closure Properties

   Which of the following are closed under complement?

   - Regular languages
   - Context Free languages
   - Turing decidable languages
   - Turing recognizable languages

   For each class answer yes or no. If the answer is yes give a brief sketch of the construction (full proof is not required). If the answer is no please give a counter example. Counter examples from reading, homework or lecture are preferred.

3. [25 points] Basic Turing Machine Definitions

   Consider the nondeterministic Turing Machine:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$$

   Where

$$\begin{array}{lcl} Q & = & \{q_0, q_1, q_a, q_r\} \\ \Sigma & = & \{0, 1\} \\ \Gamma & = & \Sigma \cup \{b\} \\ \delta(q_0, 0) & = & \{(q_0, 0, R), (q_1, 0, R)\} \\ \delta(q_0, 1) & = & \{(q_0, 1, R)\} \\ \delta(q_1, 0) & = & \{(q_r, 0, R)\} \\ \delta(q_1, 1) & = & \{(q_a, 1, R)\} \end{array}$$

   (a) What is the starting configuration of this machine on input 001?

   (b) Show a series of configurations for this machine that begins with the starting configuration given above and ends in an accepting configuration.

   (c) Show a series of configurations for this machine that begins with the starting configuration given above and ends in a rejecting configuration.

   (d) Give the definition of acceptance for a non-determinsitic TM.

   (e) Give the definition of rejection for a non-deterministic TM.

   (f) Does M accept or reject 001?

4. [25 points] Pumping Lemma for Context Free Languages

   State and prove the pumping lemma for context free languages. You may assume the results of relevant homework assignments. Please identify such assumptions.