CS 581: Theory of Computation
Fall 2009
Mid-term exam
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This is a closed-notes, closed-book exam.

1. PDA construction

   (a) Construct a PDA accepting the language

   $$A = \{w | w \text{ has an equal number of } a\text{'s and } b\text{'s}\}$$

   (b) Justify your construction

   (c) Illustrate a computation of your machine on the string *aabbba*.

2. Not Regular

   Consider the language

   $$A = \{w | w \text{ has an equal number of } a\text{'s and } b\text{'s}\}$$

   Use this language to demonstrate three techniques for showing that A is not regular.

   (a) Show A is not regular using the pumping lemma.

   (b) Show A is of infnite index.

   (c) Show A is not regular by using closure properties and the fact that $\{a^i b^i | i \geq 0\}$ is not regular.

3. Shuffle Let $A, B \subseteq \Sigma^\star$ be languages. Define the *shuffle* of A and B, $A \odot B$ as follows:

   $$A \odot B = \{x_1 y_1 \cdots x_k y_k \mid x_1 \cdots x_k \in A \text{ and } y_1 \cdots y_k \in B, x_i, y_i \in \Sigma^\star\}$$

   For example, $\{000\} \odot \{111\}$ includes the strings 000111, 111000, 101010, 010101, 011100, ...

   Define the *shuffle closure* of A, $A^\otimes$, as follows:

   $$\begin{array}{lcl} A^{\odot^0} & = & \{\epsilon\} \\ A^{\odot^{n+1}} & = & A^{\odot^n} \odot A \\ A^\otimes & = & \bigcup_{i \geq 0} A^{\odot^i} \end{array}$$

   (a) Show the regular sets are closed under shuffle ($\odot$).

   (b) Show the regular sets are *not* closed under shuffle closure ($\otimes$).