CS 581: Theory of Computation
Mid-term exam follow-up exercise

James Hook

In this problem you will sketch the proof that a non-deterministic Turing machine can be simulated by a deterministic Turing machine and explore important consequences of the construction.

1. Give the definitions of configuration, computation history, the yields relation, and acceptance for deterministic and non-deterministic Turing machines. Define the language recognized by each type of machine. Define the language decided by each type of machine.

2. The construction of the deterministic machine that simulates the non-deterministic machine uses a search strategy to explore the possible computations of the non-deterministic machine. Describe that strategy.

3. Give the construction of the deterministic machine that simulates the non-deterministic machine as a language recognizer. The deterministic machine can have multiple tapes (the construction in the book uses 3 tapes). Please be explicit about how this construction implements the strategy you described above.

4. Argue the correctness of the construction by relating accepting computations of the non-deterministic machine to accepting computations of the simulator. In other words, prove $w \in \mathcal{L}(M)$ if and only if $w \in \mathcal{L}(M')$, where M and M' are the non-deterministic machine and the deterministic simulator.

5. Adapt the construction to show that deterministic and nondeterministic Turing machines decide the same class of languages. (I.e. modify part 3 to have the deterministic machine reject those strings that would be rejected by the non-deterministic machine.)

6. Argue the correctness of the construction in part 5.

7. Theorem 7.11 gives a time bound relating the time complexity of non-deterministic and deterministic Turing machines. Analyze the running time of the simulation you propose in part 5. Show that it achieves the time bound in Theorem 7.11.

8. Theorem 7.20 establishes the equivalence of two characterizations of the class NP. The proof of this theorem refers to the construction above. Elaborate the proof relating it to the construction in part 5.