

CS 581: Theory of Computation
Mid-term exam
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This is a take-home exam. You may use only class notes and the text. You are not to use any other materials. You are not to discuss the exam with anyone other than the instructor, who will be available by email occasionally during the examination period.

Please return the exam at the beginning of class on Tuesday, November 2, 2004.

1. [15 points] Addition as a regular language.

Sipser 1.25.

2. [30 points] Shuffle.

Let $A_1, A_2 \subseteq \Sigma^*$ be languages. Define the *shuffle* of A_1 and A_2 , $A_1 \odot A_2$ as follows:

$$A_1 \odot A_2 = \{x_1y_1 \cdots x_ky_k \mid x_1 \cdots x_k \in A_1 \text{ and } y_1 \cdots y_k \in A_2, x_i, y_i \in \Sigma^*\}$$

For example, $\{000\} \odot \{111\}$ includes the strings 000111, 111000, 101010, 010101, 011100, ...

Define the *shuffle closure* of A , A^\otimes , as follows:

$$\begin{aligned} L^{\odot 0} &= \{\epsilon\} \\ L^{\odot^{n+1}} &= L^{\odot^n} \odot L \\ L^\otimes &= \bigcup_{i \geq 0} L^{\odot^i} \end{aligned}$$

- (a) Show the regular sets are closed under shuffle (\odot).
- (b) Show the regular sets are *not* closed under shuffle closure (\otimes).
- (c) Show that the context free languages are not closed under shuffle.

3. [15 points] Pumping lemma application.

Sipser 2.18 c

4. [10 points] $\text{INFINITE}_{\text{DFA}}$

Sipser 4.5

5. [10 points] Countable triples

Sipser 4.8

6. [20 points] Turing-recognizable list of deciders is incomplete.

Sipser 4.21