CS 581: Theory of Computation
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Mid-term exam

This is a closed-notes, closed-book exam.

1. True or False. [10 points]

    (a) The intersection of a regular language and a context free language is a regular language.

    (b) The intersection of a regular language and a context free language is a context free language.

    (c) The union of regular languages is regular.

    (d) The complement of a context free language is a context free language.

    (e) The complement of a regular language is regular.

2. Context Free construction [10 points]

    Show that the language $\{a^i b^j c^{i-j} | i, j \geq 0, i \geq j\}$ is context free. Justify your construction.

3. Pumping for Regular languages [15 points]

    Use the pumping lemma to show that $\{a^i b^j c^{i+j} | i, j \geq 0\}$ is not regular.

4. Right Linear Grammars [25 points]

    A grammar G is *right linear* if every production is of one of the following forms: $S \to \epsilon$, $A \to aB$, or $A \to a$ (where S is the sentential symbol, A and B are arbitrary variables, and a is an arbitrary terminal).

    For example the following grammar generates 0^+1^+:

    $$
    \begin{aligned}
    S &\to 0A \\
    &| \quad 0B \\
    A &\to 0A \\
    &| \quad 0B \\
    B &\to 1B \\
    &| \quad 1
    \end{aligned}
    $$

    Show that L is generated by a right linear grammar if and only if L is a regular language.

    Suggestion: In my construction I show that if L is of finite index then L is generated by a right linear grammar and I show that if L is generated by a right linear grammar then L is recognized by an NFA. You are free to use other approaches.

5. Facts about Turing Machines [15 points]

   (a) Give the formal definition of a Turing machine.

   (b) Define a configuration.

   (c) Define starting, accepting, rejecting, and halting configurations.

   (d) Define acceptance for a deterministic Turing machine.

   (e) When is a Turing machine a decider?

   (f) Define acceptance for a nondeterministic Turing machine.

   (g) Define rejection for a nondeterministic Turing machine.