CS 581: Theory of Computation
James Hook
Final exam.

This is a closed-notes, closed-book exam.

1. Consider the following languages. Without proof, attempt to classify the language as regular, context free but not regular, decidable but not context free, recognizable but not decidable, or not recognizable.

   (a) The empty set.
   (b) $\{1^a\#1^b\#1^c \mid ab = c\}$
   (c) $\{1^a\#1^b \mid a = b \bmod 3\}$
   (d) $\{1^a\#1^b\#1^c \mid a + b = c\}$
   (e) $\{1^a\#1^b \mid a = b!\}$
   (f) C programs that contain no dead code.
   (g) C programs that do not use a variable called "xyzzy".
   (h) Syntactically correct sentences in number theory (Sipser refers to these as sentences in the model (N,+, $\times$)).
   (i) Sentences in number theory (N,+, $\times$) provable in a reasonable proof system.
   (j) True sentences in number theory (Th(N,+, $\times$)).

2. Computation histories play a critical role in several of the fundamental results discussed in class.

   (a) Define a configuration.
   (b) Define start, accepting, and rejecting configurations.
   (c) Define the yields relation on configurations.
   (d) Define a computation history.
   (e) Give an example of an argument that uses computation histories. Summarize the result being proved. Sketch at a very high level what role computation histories served in the proof of that result.

3. Use diagonalization to prove directly that a language of your choice is not decidable.

4. Rice's theorem states that all non-trivial properties of the behavior of Turing machines are undecidable. The notion of property of behavior can be described as an index set. An *index set* is a set of Turing machine descriptions, I, with the property that if $L(M_1) = L(M_2)$ then $\langle M_1 \rangle \in I$ if and only if $\langle M_2 \rangle \in I$.

   For each of the following sets determine:

   - Is the set an index set (does it correspond to a language property)?
   - If it is an index set is it trivial?
   - If it is non-trivial describe a Turing machine with the property and another that does not have the property.
   - What can you conclude about the decidability of the set from Rice's theorem?

   (a) The set of all TMs with an odd number of states.

   (b) The set of all TMs.

   (c) The set of all TMs that accept all inputs.

   (d) The set of all TMs that are encoded by prime numbers.

   (e) The set of all TMs that decide the set (language) of prime numbers.

5. Prove the following incompleteness theorem [6.16 from Sipser]: Some true statement in Th(N,+,$\times$) is not provable.

   You may assume any of the other results from Chapter 6. Please identify what results you are assuming. (Note: it is not necessary to exhibit the paradoxical sentence to prove this form of the incompleteness theorem.)